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Technical Note

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Reference:

Summary:

This Technical Note deals with irreversible hydraulic flight control systems with linear servo actuators. The simulation centers around the Electric Hydraulic Servocontrol (EHS) as used in Fly By Wire (FBW) flight control systems. A variant are Mechanic Hydraulic Servocontrols (MHS) which can be simulated by following the same principles. A nonlinear simulation approach is made possible by computer usage and the application of a simulation language.

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List of Abbreviations and Symbols

A	piston area (bore) of the actuator	[m^2]
A_r	surface area of the rudder	[m^2]
A_1	gap #1 area	[m^2]
A_2	gap #2 area	[m^2]
A_3	gap #3 area	[m^2]
A_4	gap #4 area	[m^2]
c_{d1}	loss coefficient for gap #1	
c_{d2}	loss coefficient for gap #2	
c_{d3}	loss coefficient for gap #3	
c_{d4}	loss coefficient for gap #4	
c_h	hingemoment coefficient	
d	diameter of valve spool	[m]
d_r	reduced damping coefficient of control surface	[Ns/m]
d_s	damping coefficient of aircraft structure	[Ns/m]
D	damping ratio of servo valve	
D_H	hydraulic diameter	[m]
E	modulus of compression	[N/m^2]
f_0	natural frequency of servo valve	[Hz]
F_{aero}	aerodynamic loads	[N]
F_c	transfer function of the controller	
$F_{Coulomb}$	Coulomb friction	[N]
F_{max}	maximum actuator force	[N]
F_p	piston force	[N]
HM	hingemoment	
I	servo valve current	[A]
$I_{analogue}$	servo valve current in case of analogue signal processing	[A]
k	gain of controller	[A/m]
k_s	gain of servo valve	[m/A]
L_1, L_2	length of input lever (see Fig. 3 and Fig. 4)	[m]
m	see Fig. 6	[m]
m_r	reduced mass of the control surface on the actuator (see Fig. 8)	[kg]
m_a	mass of actuator without piston	[kg]
n_z	load factor	
p_a	pressure in actuator chamber **a**	[N/m^2]
p_b	pressure in actuator chamber **b**	[N/m^2]
p_r	return pressure	[N/m^2]
p_0	supply pressure	[N/m^2]
q	dynamic pressure	[N/m^2]
Q_a	flow into actuator chamber **a**	[m^3/s]
Q_b	flow into actuator chamber **b**	[m^3/s]

Q_1	flow through gap #1 into the valve	[m^3/s]
Q_2	flow through gap #2 into the valve	[m^3/s]
Q_3	flow through gap #3 into the valve	[m^3/s]
Q_4	flow through gap #4 into the valve	[m^3/s]
Re	Reynolds Number	
Re_{crit}	critical Reynolds Number	
s	Laplace operator	
s_1	see Fig. 6	[m]
s_{1h}	horizontal gap opening of gap #1	[m]
s_{2h}	horizontal gap opening of gap #2	[m]
s_{3h}	horizontal gap opening of gap #3	[m]
s_{4h}	horizontal gap opening of gap #4	[m]
t	time	[s]
v	flow velocity through damping orifice	[m/s]
v	aircraft speed	[m/s]
v_c	design cruising speed of aircraft	[m/s]
v_1	flow velocity in gap #1	[m/s]
V_a	volume of the actuator chamber **a**	[m^3]
V_b	volume of the actuator chamber **b**	[m^3]
V_0	volume of an actuator chamber with actuator in mid position	[m^3]
x_a	movement of actuator relative to fixed aircraft reference	[m]
x_e	error of actuator position	[m]
x_i	input value for actuator position (demanded position)	[m]
x_{max}	maximum input / output value for actuator position	[m]
x_o	output value for actuator position (actual position)	[m]
x_p	movement of actuator piston relative to actuator housing	[m]
y	position of the valve spool relative to actuator housing	[m]

α_L	loss coefficient for turbulent flow	
Δp	pressure difference $p_a - p_b$	[N/m^2]
Δr	see Fig. 6	[m]
ν	kinematic viscosity of the fluid	[m^2/s]
ρ	fluid density	[kg/m^3]
ω_0	natural frequency of servo valve	[1/s]
ξ	loss coefficient	

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1 General Overview

Flight control systems are classified in reversible and irreversible flight control systems [1]. Reversible flight control systems are typically mechanized with cables, push rods or combinations thereof. With these coupling devices the surfaces or the surface connected tabs are directly operated.

Irreversible flight control systems are classified upon their power source which can be either hydraulic, electric, or pneumatic. A hydraulic drive system can be of rotary type (as in Airbus high lift systems or the trimmable horizontal stabilizer) or of linear type with servo actuators (as in Airbus primary flight controls).

This simulation only deals with irreversible hydraulic flight control systems with linear servo actuators. The simulation centers around the Electric Hydraulic Servocontrol (EHS) as used in Fly By Wire (FBW) flight control systems. A variant of this are systems with mechanical input: Mechanic Hydraulic Servocontrols (MHS). Although these two systems have in detail a different design, they are based upon the same general principles.

The equation for the simulation of the flight control system as derived in Chapter 2 follow a new approach. They are not linearized as classically done [2],[3]. The simulation presented here is able to model

- the servo valve with valve overlap and valve underlap individually for each gap,
- the fluid flow in the valve instead of merely calculating the flow from linearized characteristic curves of the valve [2],
- compressibility effects due to the variation of the volume in the actuator chambers when the actuator piston is moving,
- different nonlinear effects (whenever needed for special investigations) as e.g. digital control, Coulomb Friction, or pressure relief valve function.

This simulation approach is made possible by computer usage and the application of a simulation language called ACSL (Advanced Continuous Simulation Language). The ACSL program shown in this Technical Note can be seen as a basic approach for simulation of hydraulic flight control systems with linear servo actuators. The program has to be extended to account for specific details of interest as

- special parameter variations,
- switching between active and damping mode,
- switching between EHS and MHS [4].

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[1] Definition of reversible flight control system:
In a reversible flight control system, when the cockpit controls are moved, the aerodynamic surface controls move and vice versa. [1]
Definition of irreversible flight control system:
In an irreversible flight control system, when the cockpit controls are moved, the aerodynamic surface controls move and NOT vice versa. (In an irreversible flight control system an actuator moves the aerodynamic surface controls. The pilot merely 'signals' the actuator to move). [1]

[2] The possibility of modelling the flow is especially important when starting from scratch as in the pre development phase.

2 Derivation

2.1 Actuator Control Loop

The actuator control loop is the most inner control loop of aircraft handling. Further control loops do exist: a) for aircraft orientation in three–dimensional space and aircraft velocity; b) for aircraft guidance to the destination airport. Controlling aircraft orientation requires movable control surfaces. The correct position of these surfaces is ensured by the actuator control loop.

Fig. 1 shows an Electric Hydraulic Servocontrol (EHS) and its control loop in the environment of a FBW aircraft. For illustration, pitch control is taken as an example. The control loop receives a demanded position (input) $\mathbf{x_i}$ for the actuator. This value is calculated from control laws in the flight control computer. The flight control computer controls the servo valve by the servo valve current **I**. The actual actuator position $\mathbf{x_o}$ is measured and compared with the demanded actuator position in the flight control computer. The servo valve current **I** is calculated form the error

$$x_e = x_i - x_o \quad .$$

Using the Laplace operator **s**

$$I(s) = F_c(s) \cdot x_e(s) \quad .$$

$F_c(s)$ is the transfer function of the controller. In case of a digital controller as in a digital flight control computer, the servo valve current **I** is depending upon *computational delay* and *sampling time*. This influence can easily be incorporated using ACSL DISCRETE sections (see Chapter 3). The controller is often a P controller (proportional controller), but can also be a P+D controller (proportional plus derivative controller) or even a P+I+D controller (proportional plus integral plus derivative controller). In case of a P controller and *analogue signal processing*, the servo valve current is

$$I_{analogue} = k \cdot x_e \quad .$$

The position of the servo valve spool **y** can be calculated from dynamic response measurements of the servo valve as shown in Fig. 2. The servo valve can be approximated by a second order term. The Abex 410 servo valve yields a natural frequency $\mathbf{f_0} \approx 150$ Hz respectively $\omega_0 \approx 940$ 1/s and damping ratio $\mathbf{D} \approx 0.8$. For a valve Abex 410, the gain $\mathbf{k_s} = 0.0875$ m/A. Hence

$$y(s) = \frac{k_s}{\frac{1}{\omega_0^2} s^2 + \frac{2D}{\omega_0} s + 1} \cdot I(s) \quad .$$

Very similar is the situation for a Mechanic Hydraulic Servocontrol (MHS) which has a direct mechanical feed back loop. The MHS actuator can either be of moving body type or of fixed body type:

Fig. 3 shows an actuator of fixed body type with its control valve. The summing bar is connected to the control valve spool and to the actuator piston rod. The summing bar is moreover connected to the control mechanisms in the cockpit via aft quadrant, cable, and forward quadrant. When the input lever is moved a distance $\mathbf{x_i}$ (relative to the aircraft reference) towards the right, the control valve spool is also moved to the right. Note that forces to move the control valve spool are much less than forces required to move the actuator piston itself. Hydraulic fluid is supplied to the right actuator chamber and returns from the left actuator chamber. The actuator piston moves to the right until the summing bar has guided the control valve spool back in its mid position.

$$y = \frac{L_2}{L_1 + L_2} \cdot (x_i - x_a) - \frac{L_1}{L_1 + L_2} \cdot (x_o - x_a) \quad .$$

The movement of the actuator relative to the fixed aircraft reference $\mathbf{x_a}$ (see Fig. 8) depends on the stiffness of the actuator attachment to the aircraft structure. $\mathbf{x_a}$ can be neglected if the attachment is sufficiently stiff.

Fig. 4 shows an actuator of moving body type with its control valve. The input lever is solely connected to the control valve piston. The motion of the input lever relative to the moving actuator housing is

$$y = \frac{L_2}{L_1} \cdot (x_i - x_o) \quad .$$

2.2 Fluid Flow through the Valve

Calculations for each of the gaps between valve piston and valve housing (see Fig. 5) have to be performed to get the flows into and out of the actuator.

$$Q_a = Q_1 - Q_3$$

$$Q_b = Q_2 - Q_4$$

Net flows $\mathbf{Q_1}$, $\mathbf{Q_2}$, $\mathbf{Q_3}$, and $\mathbf{Q_4}$ into the valve are taken as positive numbers. In the same way, net flows $\mathbf{Q_a}$ and $\mathbf{Q_b}$ into an actuator chamber are taken as positive numbers. The flow through gap #1 is

$$Q_1 = A_1 \cdot c_{d_1} \cdot \sqrt{\frac{2}{\varrho} \cdot \left| p_0 - p_a \right|} \cdot sign\,(p_0 - p_a) \quad .$$

Similarly for gap #2,#3, and #4 the equations are:

$$Q_2 = A_2 \cdot c_{d_2} \cdot \sqrt{\frac{2}{\varrho} \cdot \left| p_0 - p_b \right|} \cdot sign\,(p_0 - p_b)$$

$$Q_3 = A_3 \cdot c_{d_3} \cdot \sqrt{\frac{2}{\varrho} \cdot \left| p_a - p_r \right|} \cdot sign\,(p_a - p_r)$$

$$Q_4 = A_4 \cdot c_{d_4} \cdot \sqrt{\frac{2}{\varrho} \cdot \left| p_b - p_r \right|} \cdot sign\,(p_b - p_r) \quad .$$

Absolute values of pressure differences and the sign function have to be applied to avoid negative arguments for the square root. The density ρ for Skydrol is about 980 kg/m^3. The approach shown here to calculate the valve flows assumes that 1.) there is no pressure loss in supply and return tubes and 2.) the supply pressure $\mathbf{p_0}$ does not depend on the amount of flow through the pump.

In the following, the gap area **A** and the loss coefficient $\mathbf{c_d}$ will be calculated. Equations are only given for gap #1 when they are identical in all four cases. When neglecting all occurrences of eccentricity, the gap areas **A** are part of a cone as indicated in Fig. 6:

$$\begin{aligned} A_1 &= A_{1_{cone,total}} - A_{1_{cone,top}} \\ &= \pi \cdot \left(\Delta r + \frac{d}{2} \right) \cdot (s_1 + m) - \pi \cdot \frac{d}{2} \cdot m \\ &= \pi \cdot \left(\Delta r \cdot s_1 + \Delta r \cdot m + \frac{d}{2} \cdot s_1 \right) \end{aligned}$$

with

$$\frac{\Delta r}{s_1} = \frac{\frac{d}{2}}{m} \qquad \text{or} \qquad \Delta r \cdot m = \frac{d}{2} \cdot s_1$$

$$A_1 = \pi \cdot (d + \Delta r) \cdot s_1$$

where

$$s_1 = \sqrt{{s_{1_h}}^2 + \Delta r^2} \qquad \text{and}$$

$$s_{1_h} = \frac{1}{2}\left(y_{01e} + y + \left| y_{01e} + y \right| \right) \qquad .$$

For gap 2, 3, and 4 we get

$$s_{2_h} = \frac{1}{2}\left(y_{02e} - y + \left| y_{02e} - y \right| \right)$$

$$s_{3_h} = \frac{1}{2}\left(y_{01a} - y + \left| y_{01a} - y \right| \right)$$

$$s_{4_h} = \frac{1}{2}\left(y_{02a} + y + \left| y_{02a} + y \right| \right) \qquad .$$

The loss coefficient $\mathbf{c_d}$ can be taken from [7] page 64 (see Fig. 7) or [8] page 2–56. The curve shown in Fig. 7 can be represented by

$$c_{d_1} = -\frac{1}{2} \cdot \left[\frac{-\alpha_L}{\sqrt{\mathrm{Re}_{crit}}} \cdot \sqrt{\mathrm{Re}_1} + \alpha_L + \left| \frac{-\alpha_L}{\sqrt{\mathrm{Re}_{crit}}} \cdot \sqrt{\mathrm{Re}_1} + \alpha_L \right| \right] + \alpha_L \, .$$

The critical Reynolds Number $\mathbf{Re_{crit}}$ marks the condition where the flow transits from laminar to turbulent flow. For turbulent flow $\mathbf{c_d} = \alpha_L$ = constant. For the valve under consideration the critical Reynolds Number $\mathbf{Re_{crit}} = 25$.

$$\mathrm{Re}_1 = \frac{\left| v_1 \right| \cdot D_{H_1}}{\nu} \qquad .$$

$\mathbf{D_H}$ is the hydraulic diameter. For a narrow slot (see [8] page 2–33)

$$D_{H_1} \approx 2 \cdot s_1 \qquad .$$

The kinematic viscosity ν for Skydrol LD is about $1.4 \cdot 10^{-5}$ m²/s.

The velocity v_1 of the fluid flow through the gap #1 is

$$v_1 = \frac{Q_1}{A_1} \quad .$$

Initial values for Q_1, Q_2, Q_3, and Q_4 have to be calculated to get the simulation started. For these initial flows c_d is set to the value for turbulent flow $c_d = \alpha_L$.

2.3 Equations of Motion

The effect of forces and masses are reduced to their direct effect on the actuator piston. The equation of motion follows from Fig. 8 showing the modell of an actuator attached to the flexible structure of the aircraft on one side and to the control surface on the other side. The aerodynamic loads are represented by the force F_{aero} which has to be countered by the piston force F_p. Friction is considered as being Coulomb Friction $F_{Coulomb}$ as well as velocity proportional friction. Control surface movement is

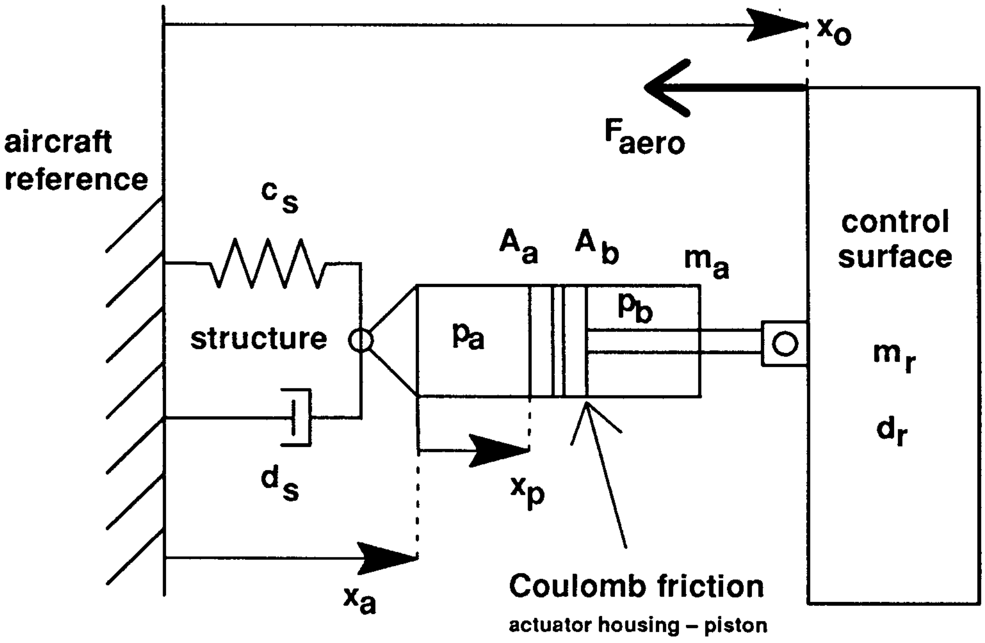

Fig. 8 Model of actuator and control surface.

$$x_o = x_a + x_p \quad .$$

x_a is the movement of the actuator relative to the fixed aircraft reference; x_p is the movement of the actuator piston relative to the actuator housing.

The equation of motion for the actuator is

$$m_a \cdot \ddot{x}_a + d_s \cdot \dot{x}_a + c_s \cdot x_a = F_{Coulomb} - F_p$$

with the mass of the actuator **m_a**, the damping coefficient of the aircraft structure **d_s**, and the stiffness of the structure **c_s**.

The piston force **F_p** introduced by the actuator is

$$F_p = p_a \cdot A_a - p_b \cdot A_b \qquad ;$$

or for actuators with equal areas **$A_a = A_b = A$**

$$F_p = A \cdot (p_a - p_b) \qquad .$$

The equation of motion for the control surface is

$$m_r \cdot \ddot{x}_o + d_r \cdot \dot{x}_o = F_p - F_{aero} - F_{Coulomb}$$

with the reduced mass of the control surface on the actuator piston **m_r** and the damping coefficient of the control surface **d_r**.

The aerodynamic force **F_{aero}** has to be taken from measured or calculated hingemoments. The hingemoment **HM** can be calculated from the dynamic pressure **q**, the surface area of the control surface **A_r**, the control surface chord, and the hingemoment coefficient **c_h**.

$$HM = c_h \cdot q \cdot A_r \cdot c$$

The hingemoment coefficient **c_h** depends on such design factors as
1.) hinge line location
2.) gap size
3.) nose shape
4.) trailing edge angle
5.) overhang
6.) horn size and shape
7.) surface deflection
8.) Reynold's Number
9.) Mach Number.

(For illustration see Fig. 9).

The variety of influencing factors make it difficult to perform the calculation. Details for calculating hinge moments are given in [9]. A first approximation yields

$$F_{aero} = x_o \cdot \frac{F_{\max}}{x_{\max}} \cdot \left(\frac{v}{v_c}\right)^2 \qquad \text{with} \qquad F_{\max} = A \cdot (p_0 - p_r) \quad .$$

The pressure in the actuator chamber **a** and actuator chamber **b** (**p_a** and **p_b**) are obtained differently for active and damping mode. For active mode

$$p_a = \int \dot{p}_a \, dt + p_{a_{ic}} \qquad \text{and} \qquad p_b = \int \dot{p}_b \, dt + p_{b_{ic}}$$

$$\dot{p}_a = \frac{E}{V_a} \cdot \left(Q_a - A \cdot \dot{x}_p\right) \qquad \text{and} \qquad \dot{p}_b = \frac{E}{V_b} \cdot \left(Q_a + A \cdot \dot{x}_p\right)$$

$$V_a = V_0 + A \cdot x_p \qquad \text{and} \qquad V_b = V_0 - A \cdot x_p \quad .$$

V_0 is the volume of an actuator chamber when the actuator piston is in its mid position. **V_a** and **V_b** are chamber volumes with displaced piston.

The two degree of freedom system consisting of actuator and control surface can be reduced to a one degree of freedom system. With the assumptions **$m_a = 0$** and **$d_s = 0$**

$$x_a = \frac{1}{c_s} \cdot \left(F_{Coulomb} - F_p\right) \quad .$$

The model is further simplified assuming a stiff structural attachment $\left(\infty \leftarrow c_s\right)$ yielding **$x_a = 0$** and **$x_p = x_o$** .

For damping mode

$$\Delta p = -\xi \cdot \frac{\varrho}{2} \cdot v^2 \cdot sign\left(\dot{x}_p\right) \quad .$$

The minus sign is introduced because the pressure difference opposes the flow through the orfice. ξ has to be found by experiments for the specific damping orfice used. The value for ξ will be in the range $\xi = [3;6]$ (see [8] page 2–50). The velocity of the fluid flow through the orfice is

$$v = \frac{Q}{A_{orfice}}$$

and for incompressible flow

$$Q = \dot{x}_o \cdot A$$

with the piston area **A** of the actuator. In this way the equation for the pressure difference in the damping mode converts to

$$\Delta p = -\xi \cdot \frac{\varrho}{2} \cdot \dot{x}_o^2 \cdot \left(\frac{A}{A_{orfice}} \right)^2 \cdot sign\left(\dot{x}_p\right) \quad .$$

3 The Simulation Language ACSL

ACSL is a FORTRAN–based language for continuous and discrete systems. The simulation system consists of two parts: a model definition program and run time analysis commands. The ACSL translator converts model definitions into FORTRAN simulation programs that use the ACSL's run time library to read and interpret commands interactively and perform the analysis of the model. The ACSL model definition contains the mathematical specification of the dynamics of the system. Fig. 10 illustrates the basic structure of an ACSL model definition. The model is contained between a PROGRAM statement and its matching END statement. The body of the model definition contains three sections: INITIAL, DYNAMIC and TERMINAL.

The INITIAL section is designed for computing initial conditions prior to running the simulation.

The DYNAMIC section contains code that is executed at every data recording interval and –nested within– any number of DERIVATIVE and DISCRETE sections.

DERIVATIVE sections contain differential equations which determine the continuous time history performance of the model. The differential equations are specified in integral form with ACSL's integrator: INTEG.

DISCRETE sections contain statements which describe changes in the behavior of the continuous part of the system at discrete intervals or discrete events.

The TERMINAL section is used to perform any computations required after the termination of the simulation of the model.

The ACSL run time library is loaded with each translated model to form a simulation program which reads and interprets run time analysis commands interactively. The PREPAR run time command defines a list of model variables which will be written out to a data file at each data logging interval. This file is used by the PLOT command. The PLOT command can plot any variable against any other variable (including time, of course) using the screen or preparing a postscript file.

For further details on ACSL language elements and statements refer to [10]. Coulamb Friction can be implemented in the ACSL program as shown in [10] Appendix A/13.

4 Simulation

4.1 Assumptions

For demonstration purpose, simulation results will be presented applying various simplifications: An Electric Hydraulic Servocontrol (EHS) is digitally controlled by a proportional controller. The sampling time is 0.0125s and the delay due to computation, relays, and solenoids is 0.03s. An Abex 410 servo valve controls the fluid flow. The equal piston areas of the actuator measure 15.5cm^2, the actuator stroke is 55mm, and the reduced mass on the piston rod is 315 kg. The actuator is connected to a conventional hydraulic system with supply pressure of 206 bar and return pressure of 3.5 bar. Pressure losses in the hydraulic system are neglected. It is assumed that the actuator is attached sufficiently stiff to the rigid aircraft structure. Coulomb friction is assumed to be absent. For aerodynamic loads a first approximation (see Page 13) is applied. Damping of the control surface is not considered.

4.2 Results

Simulation results for a step input commanding maximum actuator stroke are shown in Fig. 11 through 13. Following a delay of about 0.3s the pressure in the actuator chamber **a** increases from 103 bar to almost 200 bar. In the same time the pressure in chamber **b** decreases dramatically. Upon reaching the commanded position, the pressure difference levels off to a value necessary to counter the aerodynamic loads. Oscillation of pressure values is caused by a system consisting of the reduced mass and the elasticity of the hydraulic fluid in the actuator chambers. Oscillation of pressure values in chamber **b** – which is reduced in size – is more significant than in the enlarging chamber **a** . The fluid flow reaches a maximum of about 1.25 l/s. Fig. 13 compares the servo current **I** of a digital controller with the servo current $\mathbf{I_a}$ of an analogue controller.

Fig. 14 through 16 show simulation results for a ramp input commanding maximum actuator stroke within 0.25s. Pressure and flow variations are less dramatical compared to step input, Nevertheless, the general phenomena are the same.

A Bode plot can be obtained from frequency response analysis. This can be achieved by embedding the ACSL simulation program into an ACSL program published in [10] Appendix A/9. Phase and gain are calculated by forcing the nonlinear model with a sine wave. Changing frequency slowly allows successive points to be calculated. The method can be used to match hardware bench tests excited by sine wave generators. Because of nonlinearities, the excitation amplitude must be chosen correctly and the model must be in steady state while the measurements are made. Fig. 17 shows a Bode plot for the linear servo actuator with a frequency sweep of frequency ω from 1 1/s to 200 1/s. Frequency ω and gain are plotted on a logarithmic scale, whereas the phase is plotted on a linear scale. For $\omega = 50$ 1/s phase is 90°.

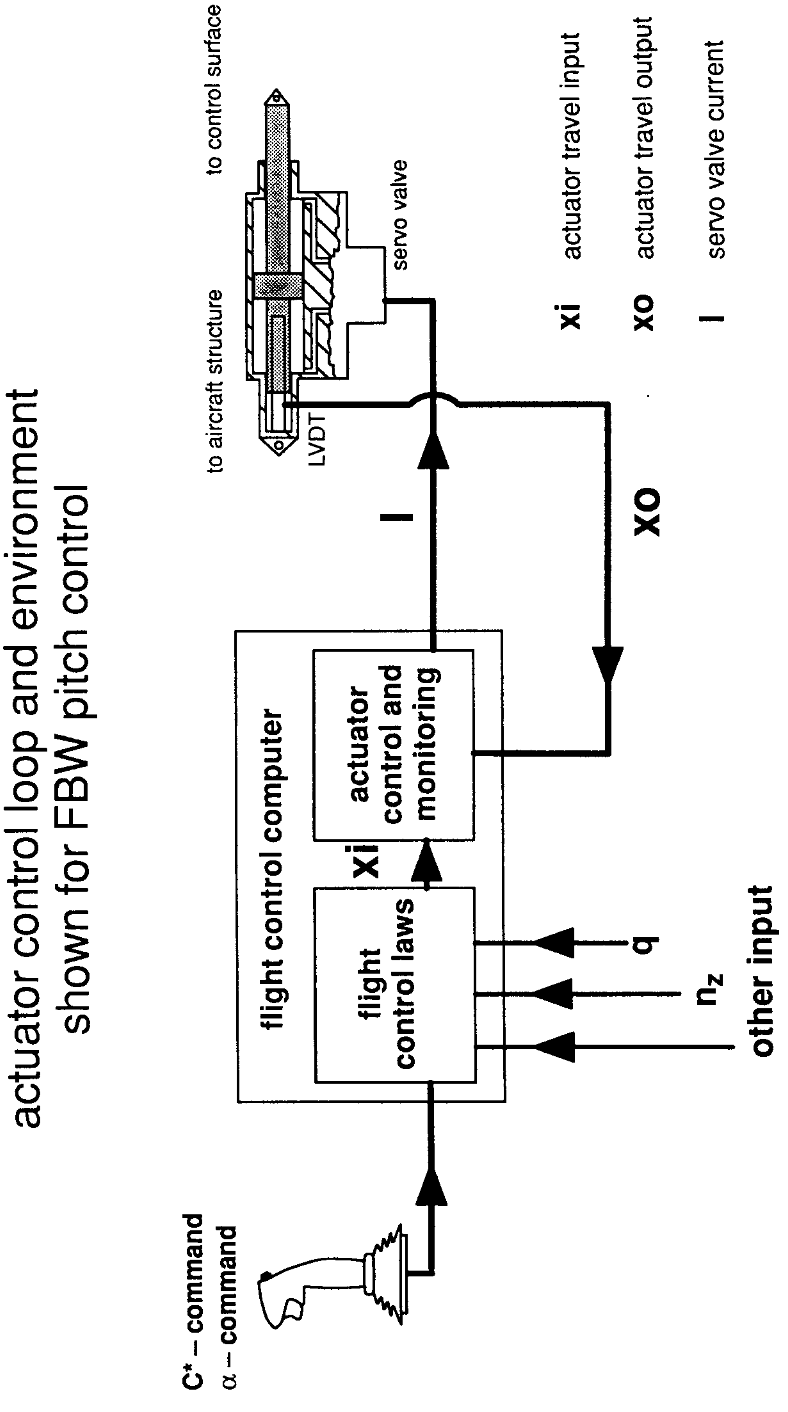

Fig. 1 *Actuator control loop.*

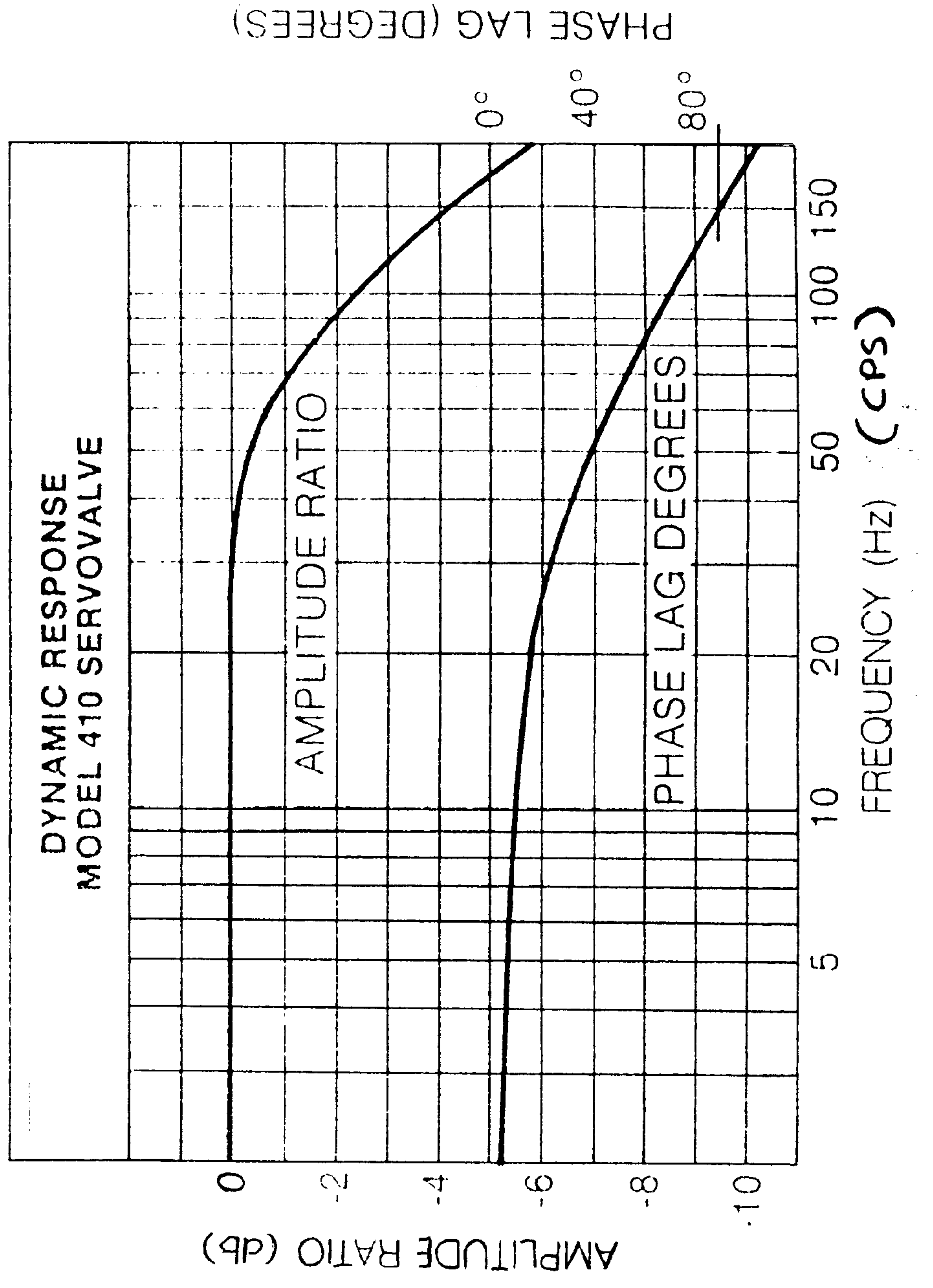

Fig. 2 *Abex Model 410 servo valve dynamic response.*

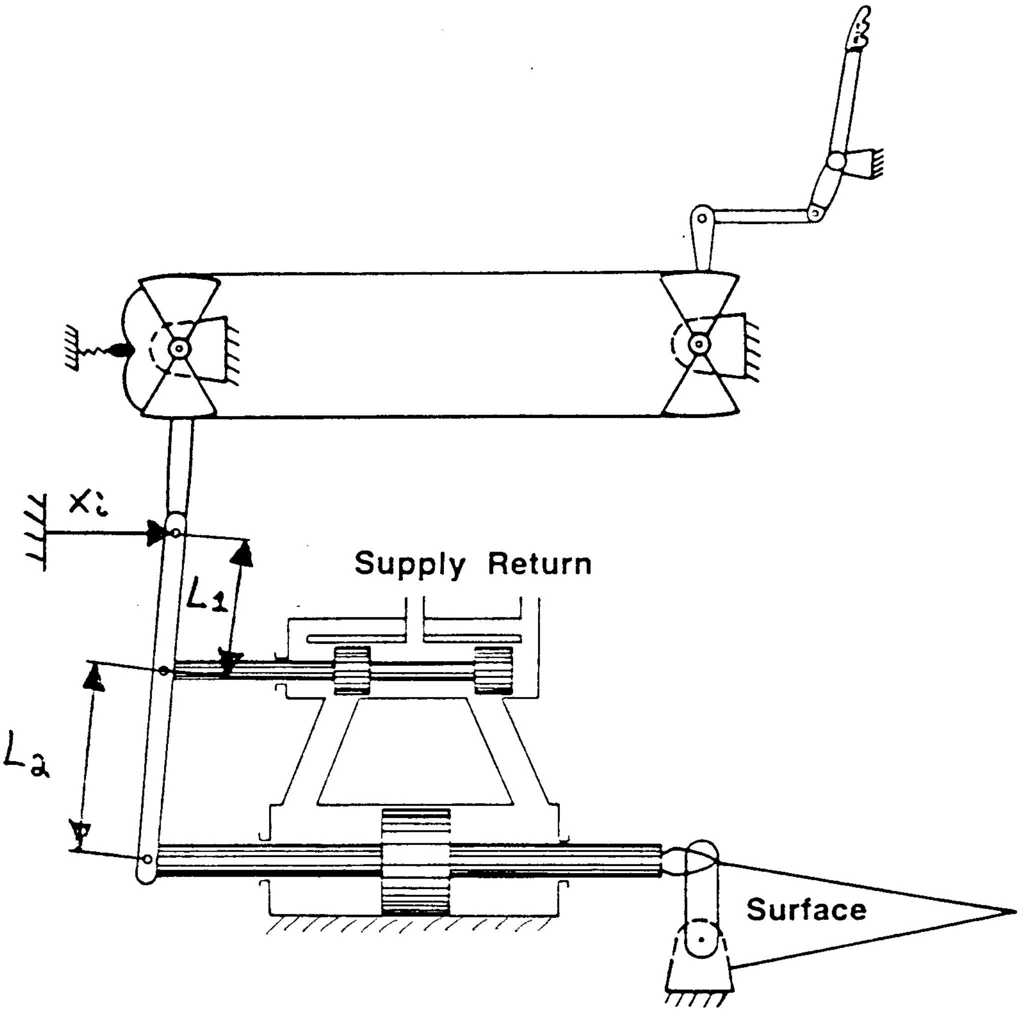

Fig. 3 *Mechanically signaled fixed body hydraulic actuator. [6]*

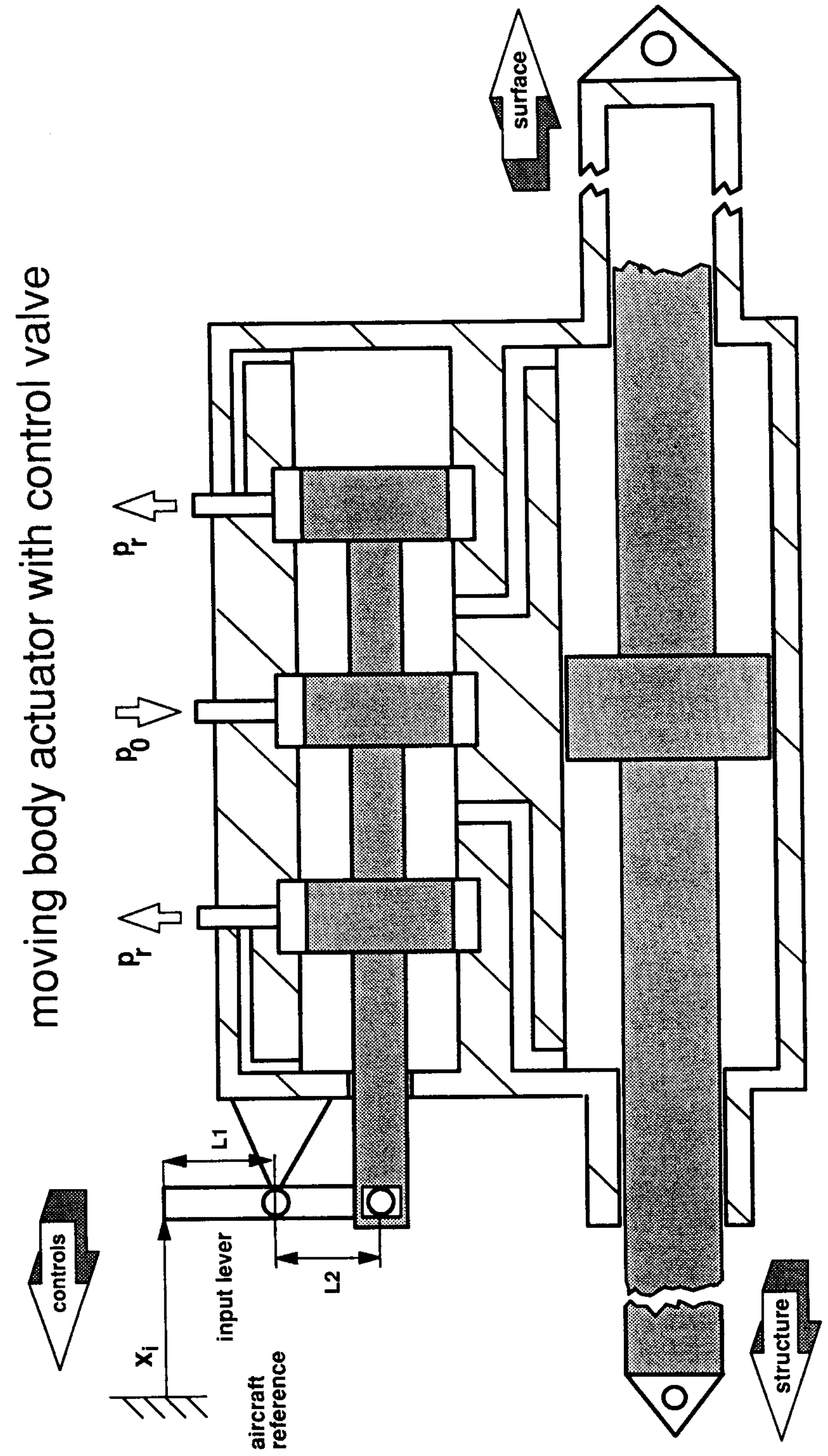

Fig. 4 *Mechanically signaled moving body hydraulic actuator.*

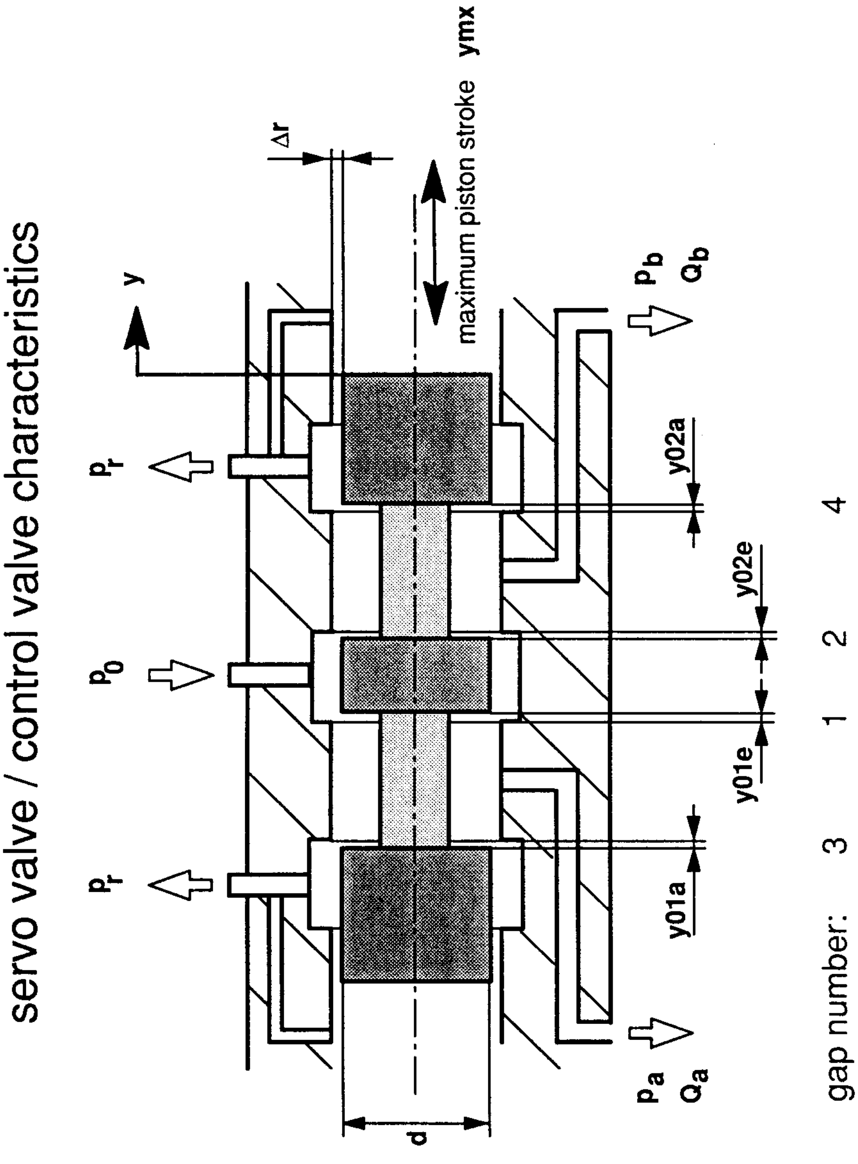

Fig. 5 *Valve characteristic.*

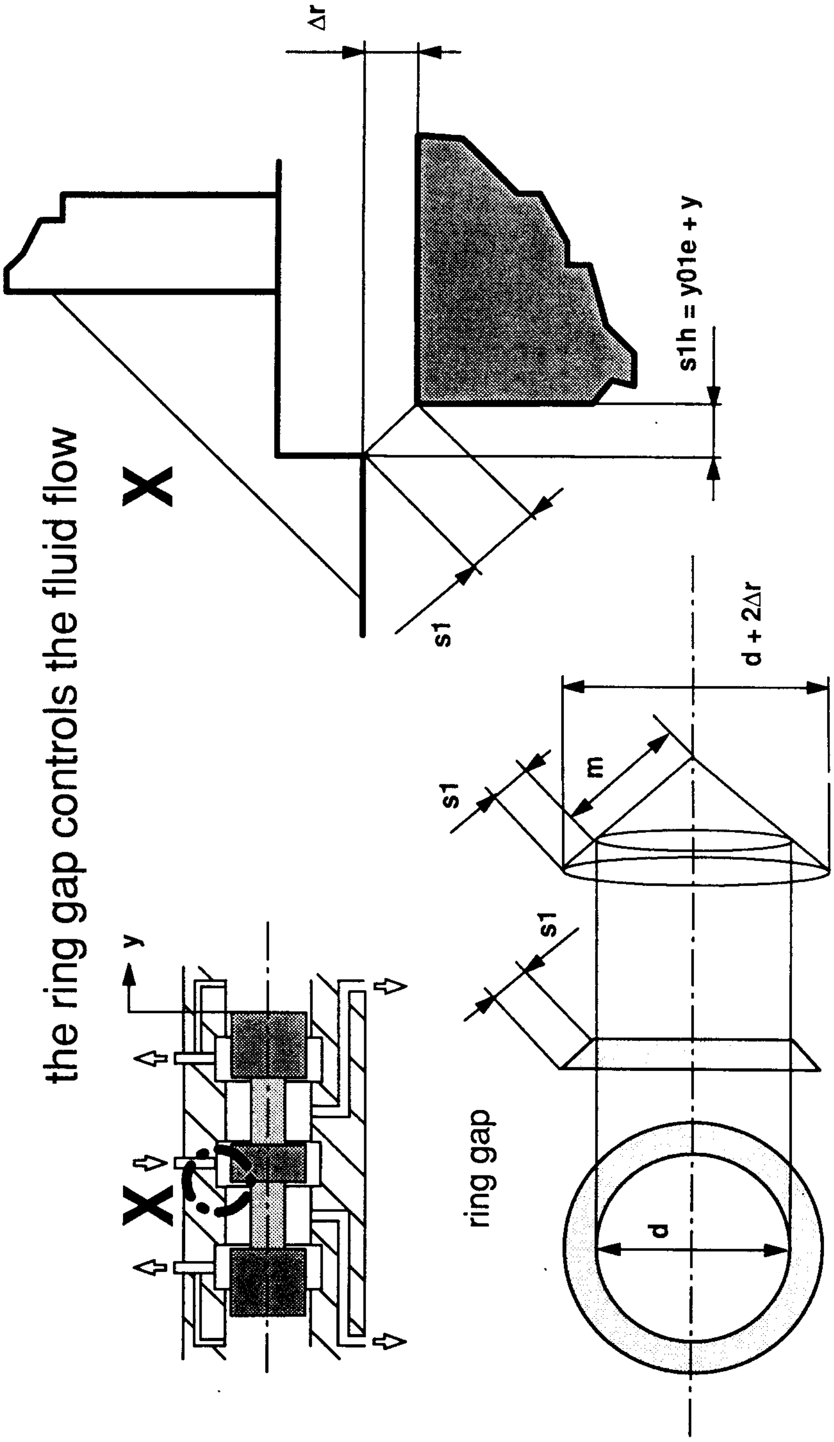

Fig. 6 *Ring gap controlling the fluid flow in a servo valve or control valve.*

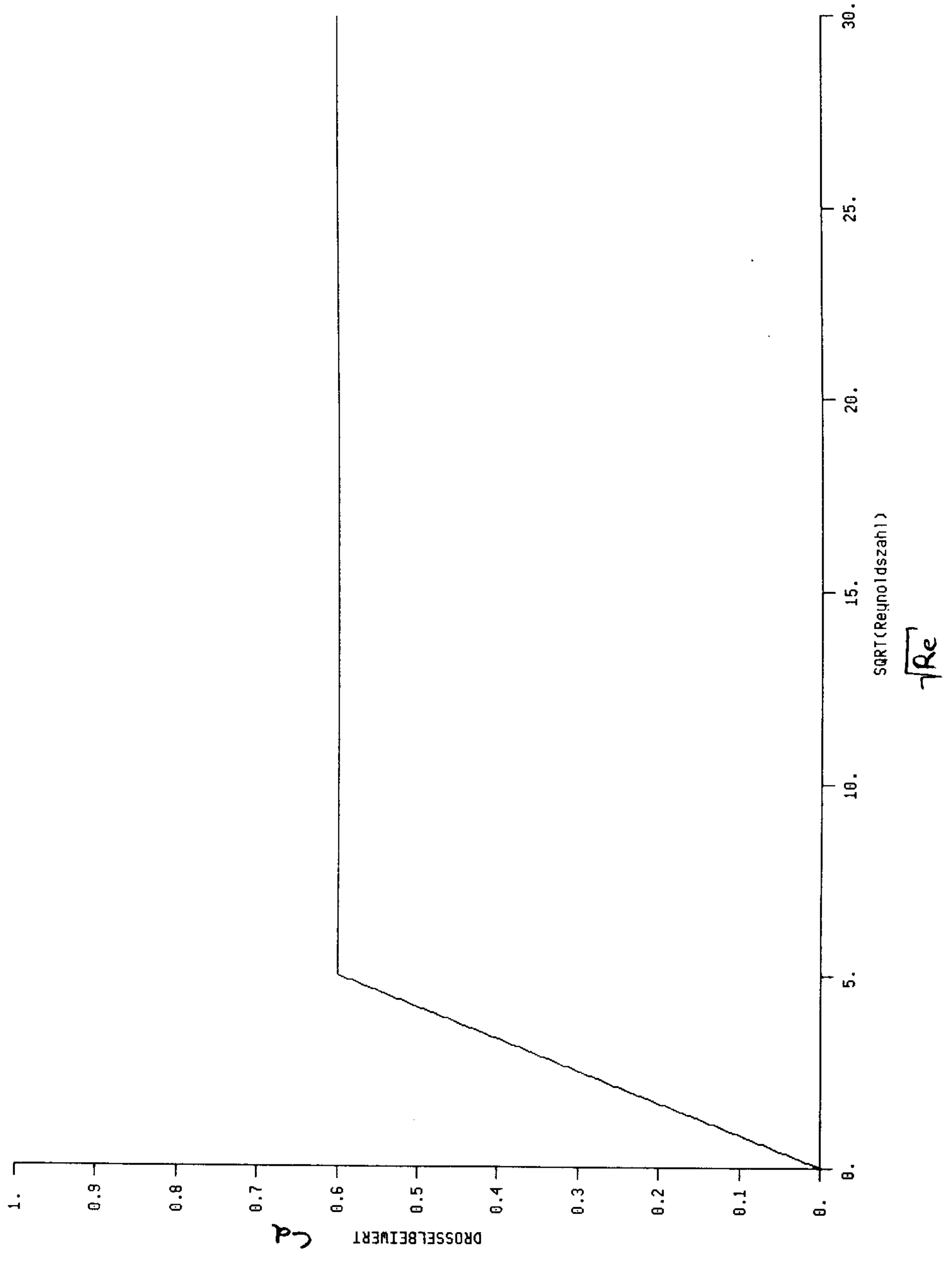

Fig. 7 *Loss coefficient* $\mathbf{c_d}$.

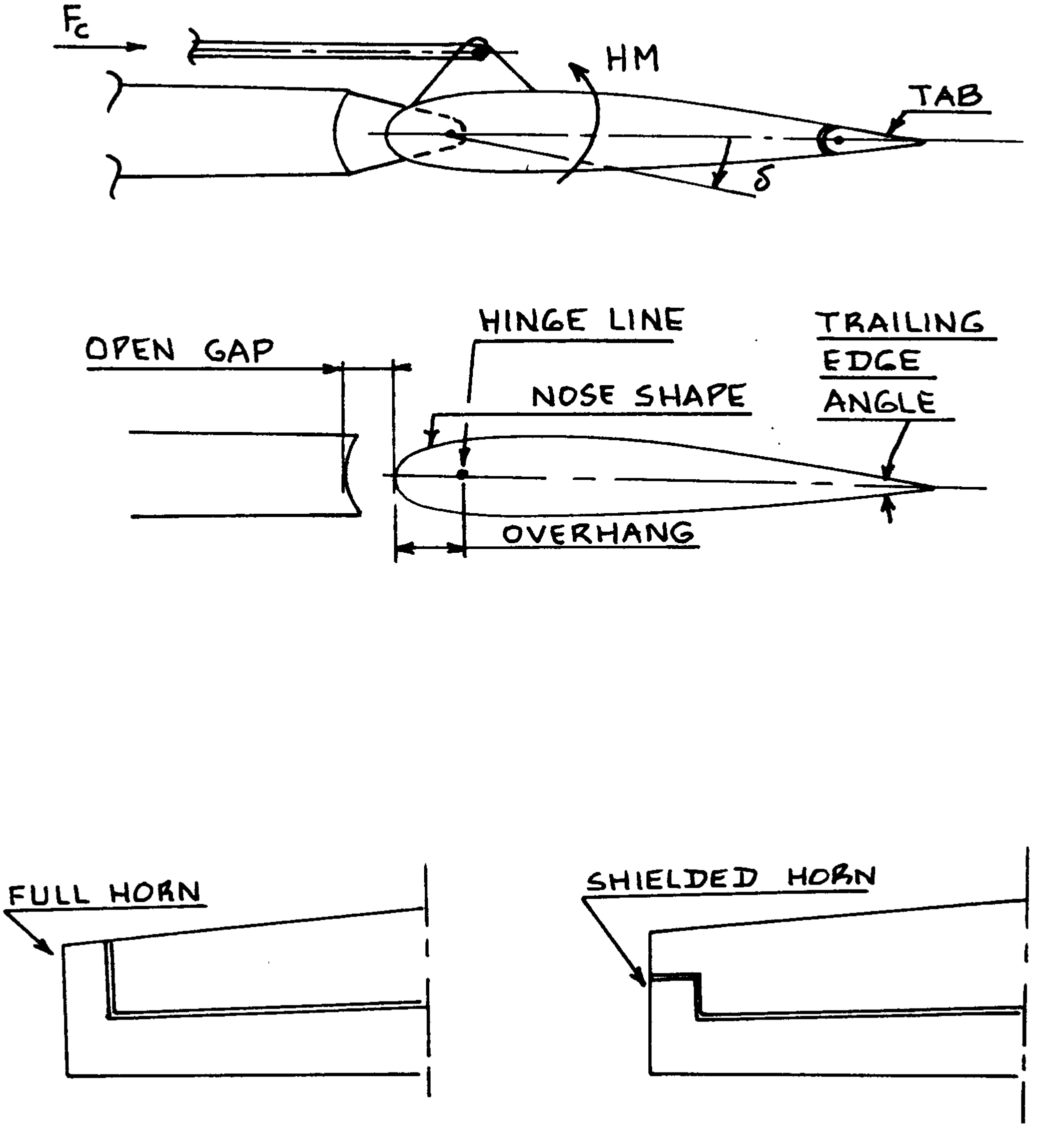

Fig. 9 *Design factors influencing the hingemoment. [1]*

```
PROGRAM
  INITIAL
   } Statements performed before the run begins.  State variables do not contain
     the initial conditions yet.
  END
  DYNAMIC
      DERIVATIVE
       } Statements needed to calculate derivatives of each INTEG
         statement.  The dynamic model.
      END
      DISCRETE
       } Statements executed periodically.
      END
   } Statements executed every communication interval.
  END
  TERMINAL
   } Statements executed when the termination condition TERMT becomes
     true.
  END
END
```

Fig. 10 *Structure of ACSL program. [10]*

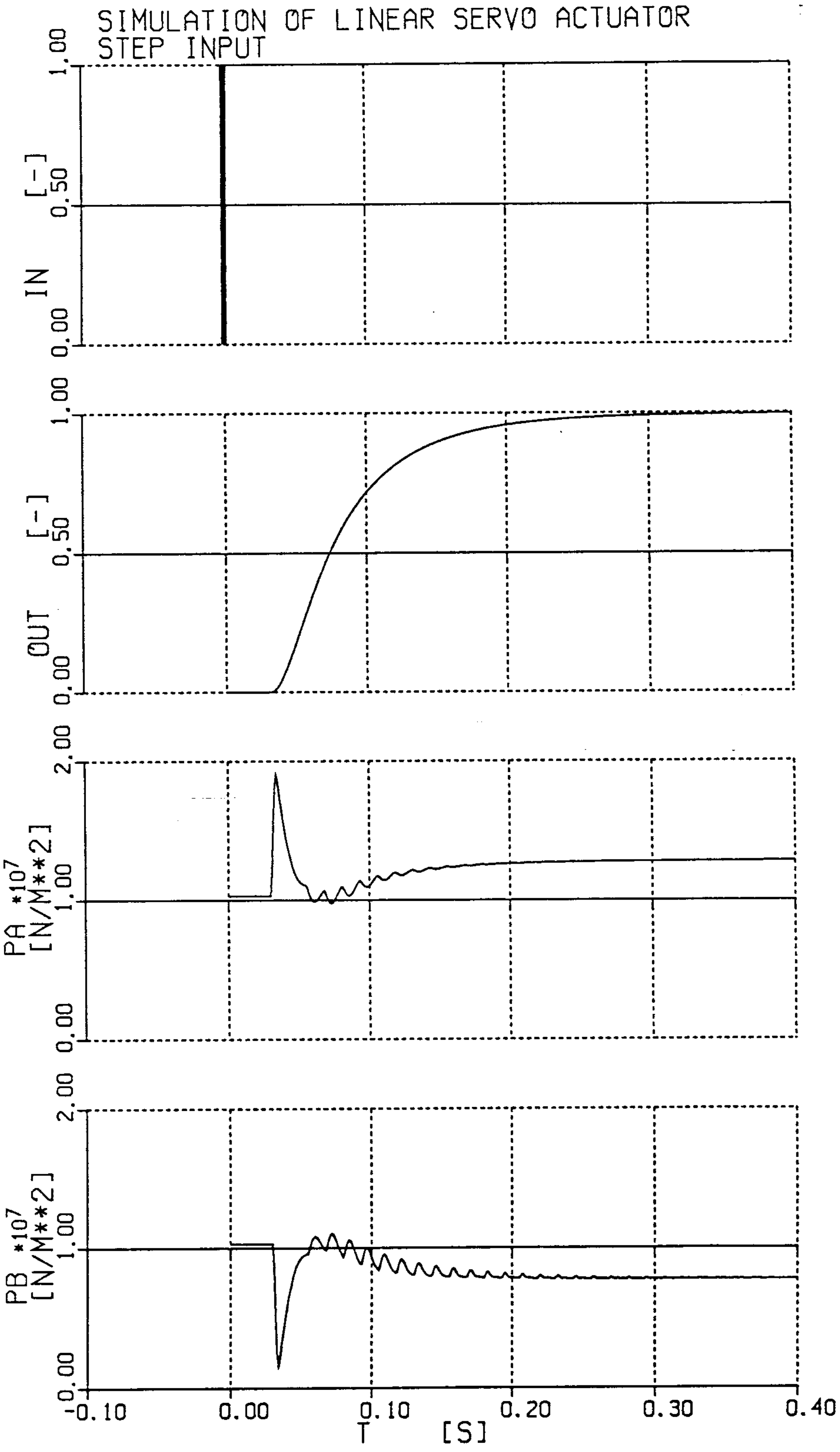

Fig. 11 *Pressure in actuator chambers for step input.*

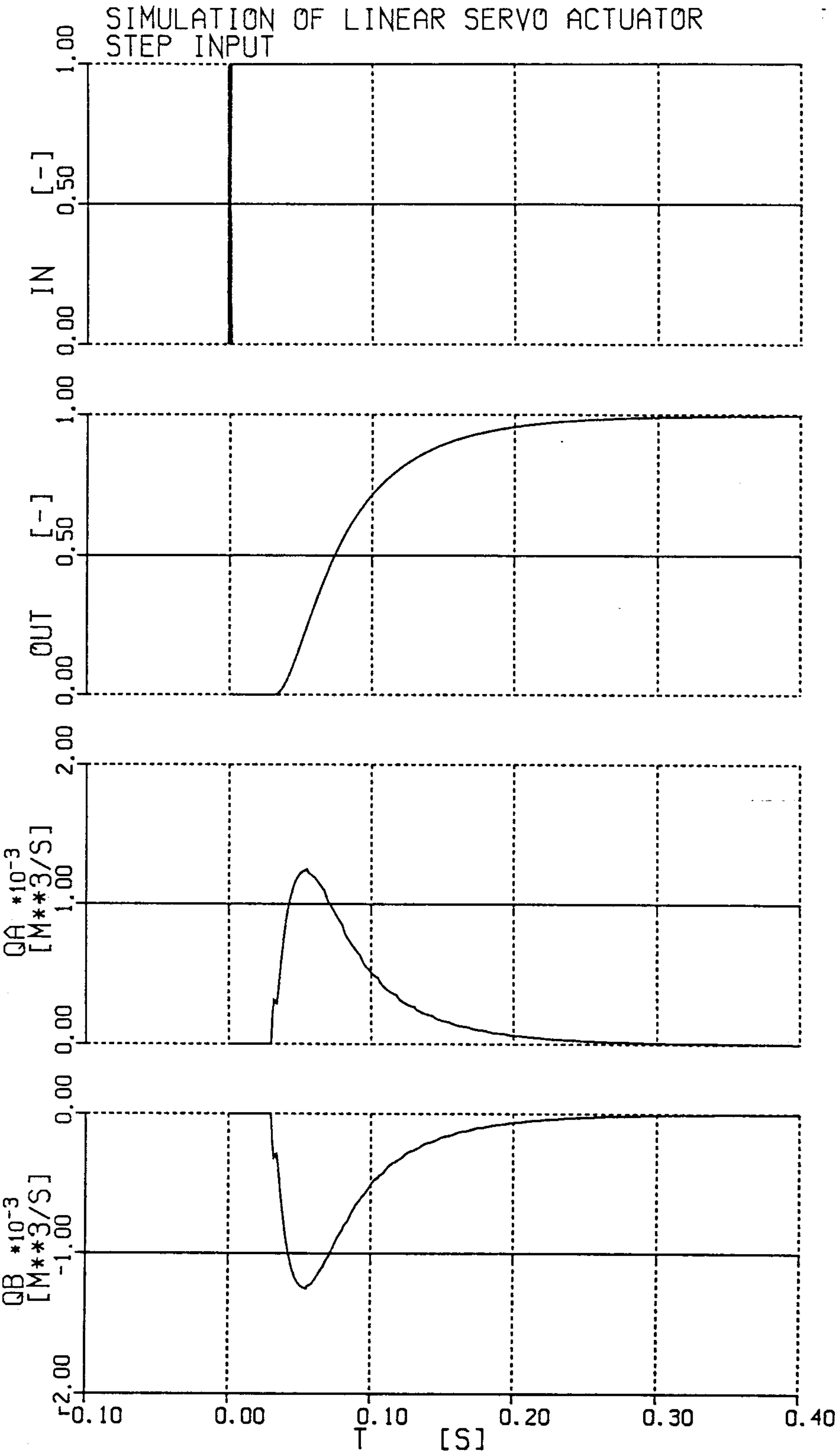

Fig. 12 *Fluid flow in actuator chambers for step input.*

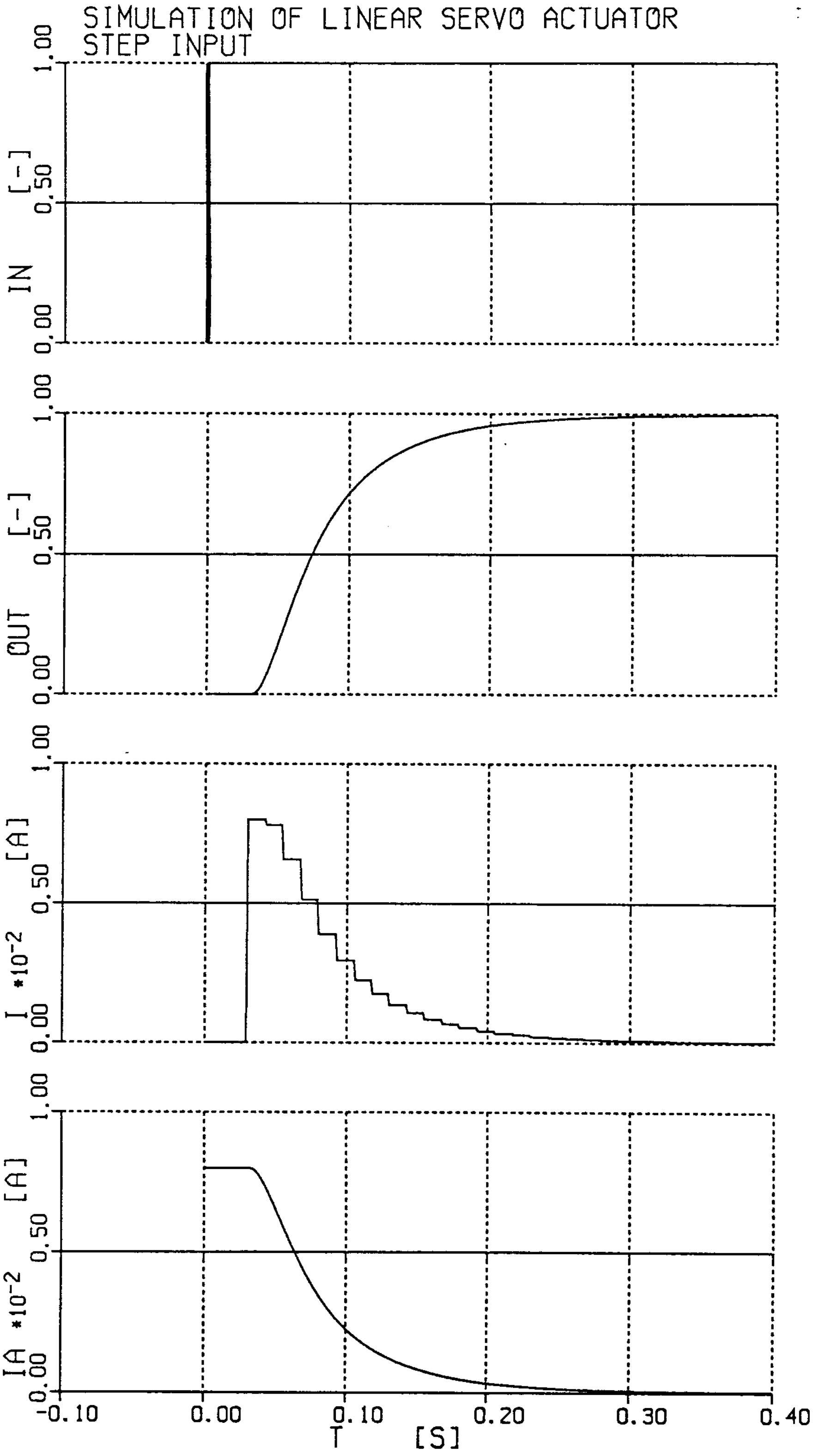

Fig. 13 *Servo valve current in case of digital and analogue controller for step input.*

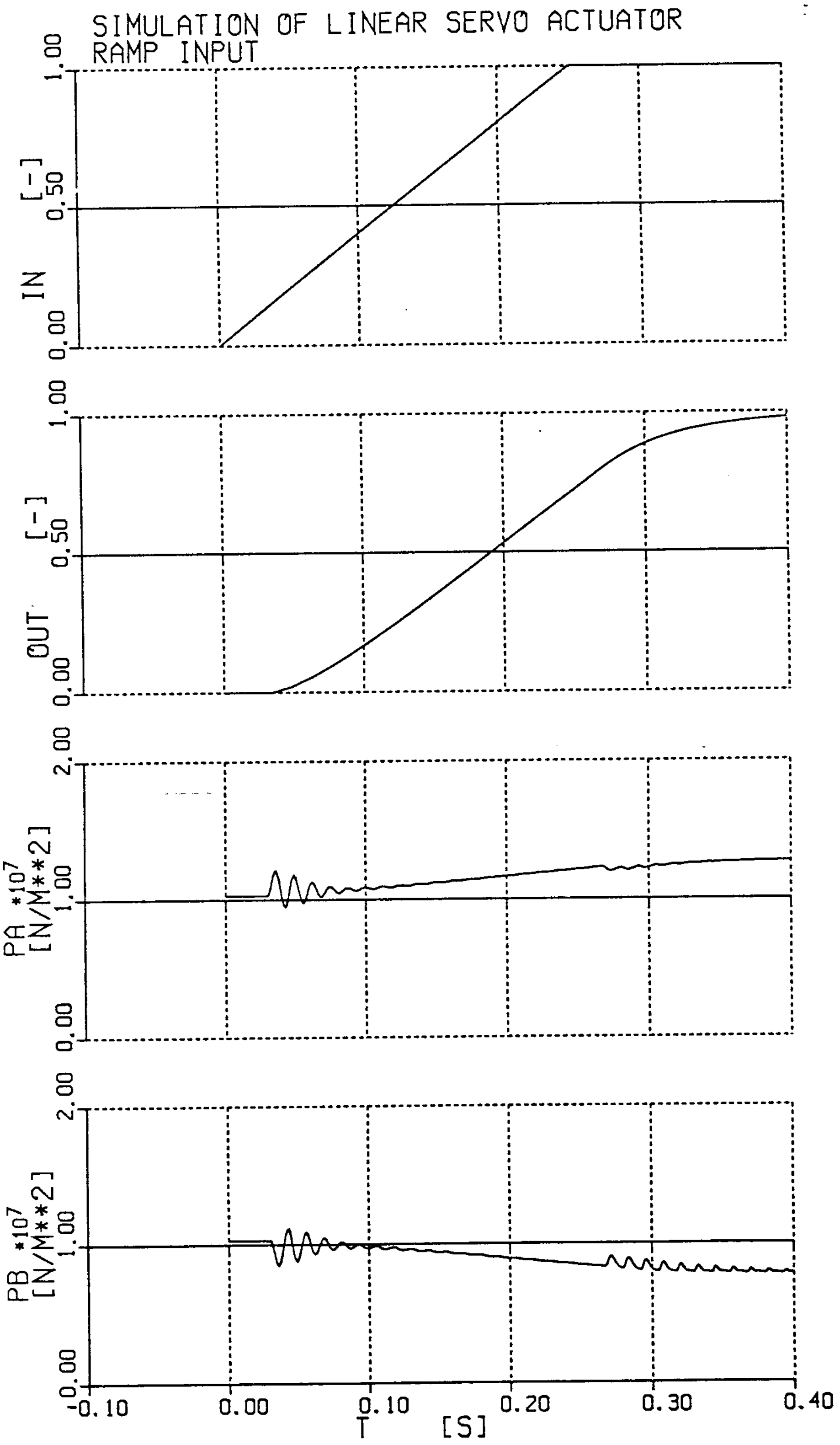

Fig. 14 *Pressure in actuator chambers for ramp input.*

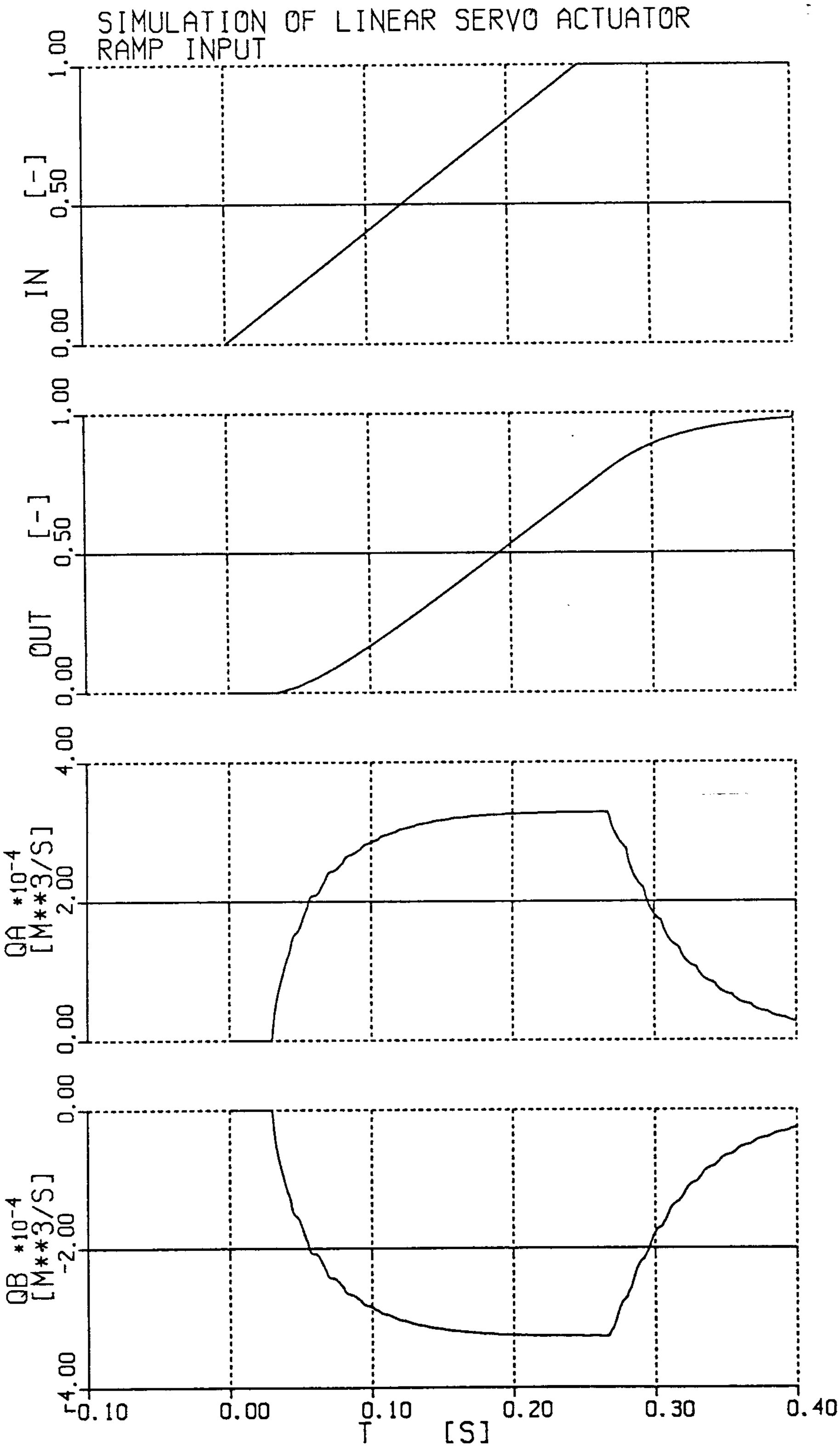

Fig. 15 *Fluid flow in actuator chambers for ramp input.*

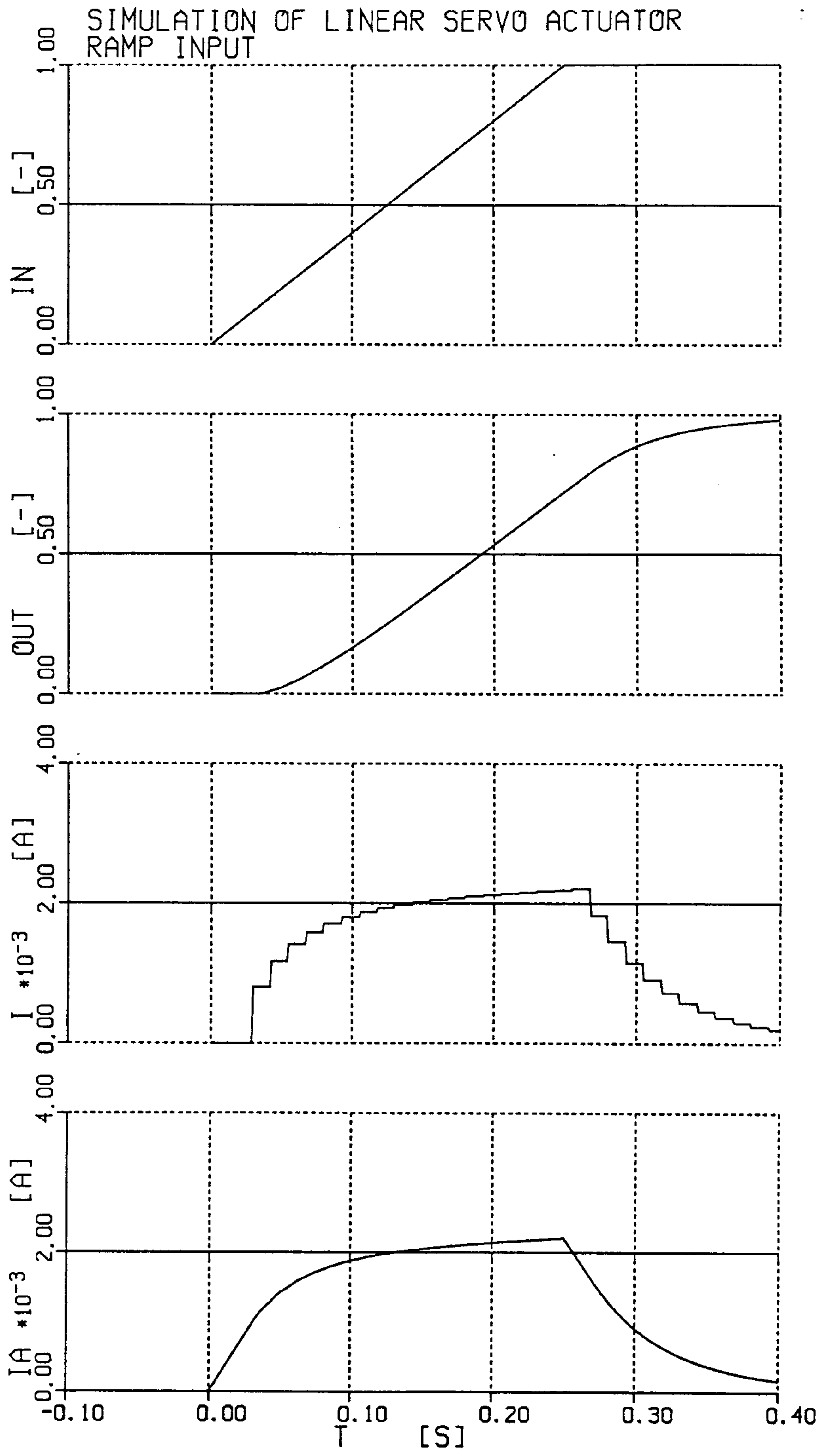

Fig. 16 *Servo valve current in case of digital and analogue controller for ramp input.*

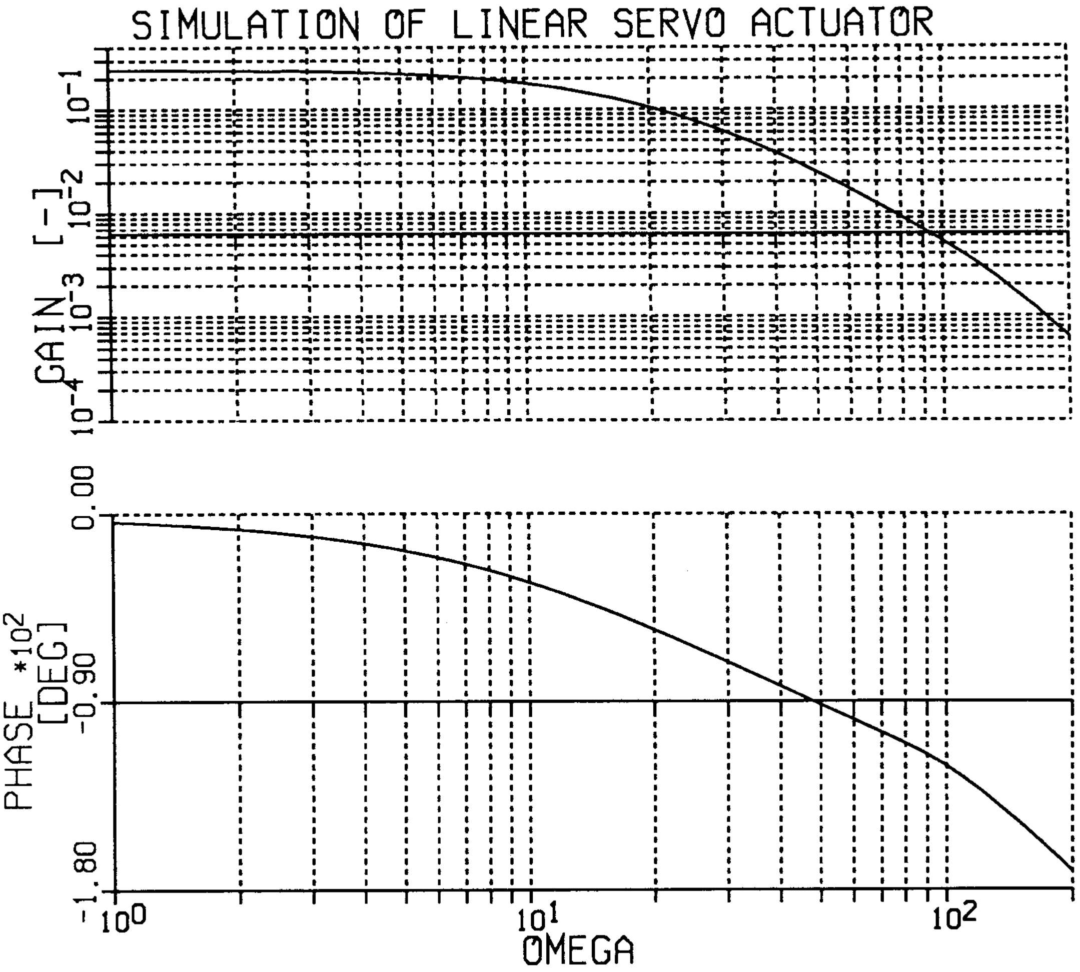

Fig. 17 *Bode plot.*

Appendix

On the following pages, programs are listed which were applied to obtain the simulation results presented in Chapter 4. The first program is a general program for simulation of a linear servo actuator (EHS). The second program listed, shows the general simulation program embedded in the ACSL frame program for frequency response analysis.

Other programs exist for investigations on switching between active and damping mode or for switching between EHS and MHS [4]. Parameter studies can easily be performed using the ACSL run time commands.

```
PROGRAM ehs
"-----------------------------------------------------------------------"
" PATHNAME   : /user/ds/acsl/ehs.csl                                     "
" ORIGINATOR : Dieter Scholz                                             "
"          AT: Deutsche Airbus, Hamburg, EV 52                           "
"          ON: Apollo DN 4000                                            "
"        DATE: 29 AUG  1990                                              "
" LAST UPDATE:                                                           "
" DESCENT    :                                                           "
" REMARK     : Example program for Technical Note: Simulation of         "
"              Hydraulic Flight Control Systems with Linear Servo        "
"              Actuators                                                 "
" FUNCTION   : Servo actuator with Re - dependant                        "
"              flow calculation, discrete sampling                       "
" INPUTS     : xoic,xodic,yic,iic,xomx,xomn,mr,dr,a,v0,                  "
"              xp0,xpr,vrel,k,ks,ymx,ymn,d,y01a,y01e,y02e,y02a,dlr,al,  "
"              sqrecr,ro,e,nu                                            "
" OUTPUT     : xo,pa,pb,qa,qb                                            "
" UNITS      : kg, m, s                                                  "
"-----------------------------------------------------------------------"
INITIAL
      VARIABLE  t          $" independant variable : time     [s]       "
      CONSTANT  pi      = 3.1415926
      "----------------------commanded position               [m]       "
      CONSTANT  xcom    = 0.055
      "----------------------initial conditions for output: xo          "
      CONSTANT  xoic    = 0.0
      CONSTANT  xodic   = 0.0
      "----------------------initial condition servovalve     [m]       "
      CONSTANT  yic     = 0.0
      "----------------------initial condition for current  [A]         "
                iic     = yic/ks
      "----------------------maximum actuator-half-stroke     [m]       "
      "                      (LAT 5/89 EHS und MHS)                     "
      CONSTANT  xomx    = 0.055
      "----------------------minimum actuator-half-stroke     [m]       "
      CONSTANT  xomn    =-0.055
      "----------------------reduced mass of control surface [kg]       "
      "                      (ZKP III, Seitenruder)                     "
      CONSTANT  mr      = 315.0
      "----------------------damping ratio                    [N*s/m]   "
      CONSTANT  dr      = 0.0
      "----------------------piston area of actuator          [m**2]    "
      "                      (LAT 5/89 EHS und MHS)                     "
      CONSTANT  a       = 1549.e-6
      "----------------------volume of an actuator chamber with         "
      "                      actuator in mid position         [m**3]    "
                v0      = a*(xomx+0.002)
      "                      from total half-stroke: 0.057 m            "
      "----------------------supply pressure (converted units) [bar]    "
      CONSTANT  xp0     = 206.
      p0      = xp0*1.0e5    $" supply pressure               [N/m**2]  "
      "----------------------return pressure (converted units) [bar]    "
      CONSTANT  xpr     = 3.5
      pr      = xpr*1.0e5    $" return pressure               [N/m**2]  "
      "----------------------relative aircraft speed  vrel = v/vc       "
      CONSTANT  vrel    = 0.5
      "----------------------maximum actuator force           [N]       "
      fmx     = a*(p0 - pr)
      "----------------------gain of controller  k = i/xi   [A/m]       "
      CONSTANT  k       = 0.1454545
      "                      for  Imax = 8 mA   (ABEX 410)              "
      "----------------------gain of servo valve  ks = y/i  [m/A]       "
      CONSTANT  ks      = 0.0875
      "                      for  ymax = 0.7 mm (ABEX 410)              "
      "----------------------Servoventildaten  ABEX 410-----------------"
      "----------------------maximum half-stroke: servovalve    [m]     "
      CONSTANT  ymx     = 0.7e-3
      "----------------------minimum half-stroke: servovalve    [m]     "
      CONSTANT  ymn     =-0.7e-3
      "----------------------diameter servovalve piston         [m]     "
      CONSTANT  d       = 7.0e-3
      "----------------------servovalve:underlap,outflow,left   [m]     "
      CONSTANT  y01a    = 0.0
      "----------------------servovalve:underlap,inflow,left    [m]     "
      CONSTANT  y01e    = 0.0
      "----------------------servovalve:underdlap,inflow,right  [m]     "
      CONSTANT  y02e    = 0.0
      "----------------------servovalve:underlap,outflow,right  [m]     "
      CONSTANT  y02a    = 0.0
```

```
      "----------------------rad. clearance: piston-bore(servovalve)[m]"
      CONSTANT  dlr    = 2.0e-6
      "----------------------loss coefficient (turb. flow)  [ - ]       "
      CONSTANT  al     = 0.6
      "----------------------SQRT(crit. Reynolds Number)    [ - ]       "
      CONSTANT  sqrecr = 5.0
      "----------------------properties of Skydrol:--------------------"
      "----------------------density Skydrol LD              [kg/m**3]   "
      CONSTANT  ro     = 980.0
      "----------------------modulus of compression Skydrol             "
      "                                                     [N/m**2]    "
      CONSTANT  e      = 1.2e9
      "----------------------kinematic viscosiy Skydrol LD [m**2/s]     "
      CONSTANT  nu     = 1.4e-5
      "----------------------calculation of inital pressure and flow --"
      s1h     = 0.5*(y01e + yic + ABS(y01e + yic))
      s2h     = 0.5*(y02e - yic + ABS(y02e - yic))
      s3h     = 0.5*(y01a - yic + ABS(y01a - yic))
      s4h     = 0.5*(y02a + yic + ABS(y02a + yic))
      s1      = SQRT(s1h**2 + dlr**2)
      s2      = SQRT(s2h**2 + dlr**2)
      s3      = SQRT(s3h**2 + dlr**2)
      s4      = SQRT(s4h**2 + dlr**2)
      paic    = p0*1.0/(1. + (s3/s1)**2)
      pbic    = p0*1.0/(1. + (s4/s2)**2)
      a1      = pi*(d + dlr)*s1
      a2      = pi*(d + dlr)*s2
      a3      = pi*(d + dlr)*s3
      a4      = pi*(d + dlr)*s4
      q1      = a1*al*SQRT(2.0/ro*ABS(p0-paic))*SIGN(1.0,p0-paic)
      q2      = a2*al*SQRT(2.0/ro*ABS(p0-pbic))*SIGN(1.0,p0-pbic)
      q3      = a3*al*SQRT(2.0/ro*ABS(paic))*SIGN(1.0,paic)
      q4      = a4*al*SQRT(2.0/ro*ABS(pbic))*SIGN(1.0,pbic)
END $" of INITIAL section "
DYNAMIC
      CINTERVAL  CINT = 0.005
DERIVATIVE
      ALGORITHM  IALG = 4
      "----------------------MAXT should be less than 1.0e-4            "
      MAXTERVAL  MAXT = 1.0e-4
      "----------------------Frequency Analysis                         "
      CONSTANT  f = 10.0, fi = 0.0
      xi      = xcom*sin(2*pi*f*t + fi)
      "----------------------Ramp Input                                 "
"     xi      = BOUND(0.0,xcom,4*xomx*RAMP(0.0))                        "
      "----------------------Step Input                                 "
"     xi      = xcom*STEP(0.0)                                          "
      ia      = k*(xi-xo)
      y       = BOUND(ymn,ymx,ks*REALPL(0.005,i,iic))
     PROCEDURAL(s1,s2,s3,s4,cd1,cd2,cd3,cd4 = y)
      s1h     = 0.5*(y01e + y + ABS(y01e + y))
      s2h     = 0.5*(y02e - y + ABS(y02e - y))
      s3h     = 0.5*(y01a - y + ABS(y01a - y))
      s4h     = 0.5*(y02a + y + ABS(y02a + y))
      s1      = SQRT(s1h**2 + dlr**2)
      s2      = SQRT(s2h**2 + dlr**2)
      s3      = SQRT(s3h**2 + dlr**2)
      s4      = SQRT(s4h**2 + dlr**2)
      dh1     = 2.0*s1
      dh2     = 2.0*s2
      dh3     = 2.0*s3
      dh4     = 2.0*s4
      a1      = pi*(d + dlr)*s1
      a2      = pi*(d + dlr)*s2
      a3      = pi*(d + dlr)*s3
      a4      = pi*(d + dlr)*s4
      sqre1   = SQRT(ABS(q1)/a1*dh1/nu)
      sqre2   = SQRT(ABS(q2)/a2*dh2/nu)
      sqre3   = SQRT(ABS(q3)/a3*dh3/nu)
      sqre4   = SQRT(ABS(q4)/a4*dh4/nu)
      cd1     = -0.5*(-al/sqrecr*sqre1+al+ABS(-al/sqrecr*sqre1+al)) + al
      cd2     = -0.5*(-al/sqrecr*sqre2+al+ABS(-al/sqrecr*sqre2+al)) + al
      cd3     = -0.5*(-al/sqrecr*sqre3+al+ABS(-al/sqrecr*sqre3+al)) + al
      cd4     = -0.5*(-al/sqrecr*sqre4+al+ABS(-al/sqrecr*sqre4+al)) + al
     END $" of PROCEDURAL section"
      q1      = a1*cd1*SQRT(2.0/ro*ABS(p0 - pa))*SIGN(1.0,p0 - pa)
      q2      = a2*cd2*SQRT(2.0/ro*ABS(p0 - pb))*SIGN(1.0,p0 - pb)
      q3      = a3*cd3*SQRT(2.0/ro*ABS(pa))*SIGN(1.0,pa - pr)
      q4      = a4*cd4*SQRT(2.0/ro*ABS(pb))*SIGN(1.0,pb - pr)
      qa      = q1 - q3
```

```
      qb      = q2 - q4
      va      = v0 + a*xo
      vb      = v0 - a*xo
     PROCEDURAL(pa,pb,dlp = va,vb,xod)
      pad     = e/va*(qa - a*xod)
      pbd     = e/vb*(qb + a*xod)
      pa      = LIMINT(pad,paic,0.0,220.e5)
      pb      = LIMINT(pbd,pbic,0.0,220.e5)
     END $" of PROCEDURAL section"
      fp      = a*(pa-pb)
      fd      = dr*xod
      faero   = xo*fmx/xomx*vrel**2
      xodd    = 1.0/mr*(fp - fd - faero)
      xod     = INTEG(xodd,xodic)
      xo      = LIMINT(xod,xoic,xomn,xomx)
END $" of DERIVATIVE section "
DISCRETE sample
      "----------------------check surface position every 0.0125 s     "
      "                      (inner control loop)                      "
      INTERVAL  tsamp  = 0.0125
      CONSTANT  dlt    = 0.03
      il      = k*(xi - xo)
      SCHEDULE  delay .AT. t + dlt
END $" of DISCRETE section "
DISCRETE delay
      "----------------------delay output due to relais and solenoid   "
      i       = il
END $" of DISCRETE section "
      in      = xi/xomx
      out     = xo/xomx
      CONSTANT  tstp   = 0.99
      TERMT(t .GT. tstp)
END $" of DYNAMIC section "
TERMINAL
END $" of TERMINAL section"
END $" of PROGRAM "
```

```
      qb       = q2 - q4
      va       = v0 + a*xo
      vb       = v0 - a*xo
     PROCEDURAL(pa,pb,dlp = va,vb,xod)
      pad      = e/va*(qa - a*xod)
      pbd      = e/vb*(qb + a*xod)
      pa       = LIMINT(pad,paic,0.0,220.e5)
      pb       = LIMINT(pbd,pbic,0.0,220.e5)
     END $" of PROCEDURAL section"
      fp       = a*(pa-pb)
      fd       = dr*xod
      faero    = xo*fmx/xomx*vrel**2
      xodd     = 1.0/mr*(fp - fd - faero)
      xod      = INTEG(xodd,xodic)
      xo       = LIMINT(xod,xoic,xomn,xomx)
END $" of DERIVATIVE section "
DISCRETE sample
      "----------------------check surface position every 0.0125 s     "
      "                       (inner control loop)                     "
      INTERVAL  tsamp  = 0.0125
      CONSTANT  dlt    = 0.03
      il       = k*(xi - xo)
      SCHEDULE  delay .AT. t + dlt
END $" of DISCRETE section "
DISCRETE delay
      "----------------------delay output due to relais and solenoid   "
      i        = il
END $" of DISCRETE section "
      in       = xi/xomx
      out      = xo/xomx
      CONSTANT  tstp   = 0.99
      TERMT(t .GT. tstp)
END $" of DYNAMIC section "
TERMINAL
END $" of TERMINAL section"
END $" of PROGRAM "
```

```
PROGRAM PHASE AND GAIN

      "-----------------------COMPUTE PHASE AND GAIN OF A GIVEN "
      " TRANSFER FUNCTION BY INTEGRATING OVER A COMPLETE CYCLE. "
      " CONTINUE TO INTEGRATE UNTIL PHASE CHANGE FROM CYCLE TO CYCLE "
      " IS LESS THAN SOME PRESET MINIMUM "

      CINTERVAL      CINT = 1.0
      NSTEPS         NSTP = 1

      CONSTANT       RMN = 1.0E-30     , RMX = 1.0E30

INITIAL

      "-----------------------SET FIRST FREQUENCY AND PHASE "
      W       = WMX
      FI      = 0.0
      "-----------------------SET PREVIOUS "
      PP      = 1.0
      QP      = 1.0
      PDGP    = RMX
      "-----------------------INITIALISE PLOT VARIABLES "
      PHASE   = 0.0
      GAIN    = 0.0
      OMEGA   = W

      VARIABLE  t         $" independant variable : time    [s]          "
      CONSTANT  pi     = 3.1415926
      "----------------------commanded position            [m]          "
      CONSTANT  XMAG   = 0.055
      "----------------------initial conditions for output: xo          "
      CONSTANT  xoic   = 0.0
      CONSTANT  xodic  = 0.0
      "----------------------initial condition servovalve   [m]          "
      CONSTANT  yic    = 0.0
      "----------------------initial condition for current [A]          "
                iic    = yic/ks
      "----------------------maximum actuator-half-stroke  [m]          "
      "                       (LAT 5/89 EHS und MHS)                    "
      CONSTANT  xomx   = 0.055
      "----------------------minimum actuator-half-stroke  [m]          "
      CONSTANT  xomn   =-0.055
      "----------------------reduced mass of control surface [kg]       "
      "                       (ZKP III, Seitenruder)                    "
      CONSTANT  mr     = 315.0
      "----------------------damping ratio                 [N*s/m]      "
      CONSTANT  dr     = 0.0
      "----------------------piston area of actuator       [m**2]       "
      "                       (LAT 5/89 EHS und MHS)                    "
      CONSTANT  a      = 1549.e-6
      "----------------------volume of an actuator chamber with         "
      "                      actuator in mid position      [m**3]       "
                v0     = a*(xomx+0.002)
      "                      from total half-stroke: 0.057 m            "
      "----------------------supply pressure (converted units) [bar]    "
      CONSTANT  xp0    = 206.
      p0      = xp0*1.0e5    $" supply pressure             [N/m**2]     "
      "----------------------return pressure (converted units) [bar]    "
      CONSTANT  xpr    = 3.5
      pr      = xpr*1.0e5    $" return pressure             [N/m**2]     "
      "----------------------relative aircraft speed  vrel = v/vc       "
      CONSTANT  vrel   = 0.5
      "----------------------maximum actuator force        [N]          "
      fmx     = a*(p0 - pr)
      "----------------------gain of controller  k = i/xi  [A/m]        "
      CONSTANT  k      = 0.1454545
      "                      for  Imax = 8 mA   (ABEX 410)              "
      "----------------------gain of servo valve  ks = y/i [m/A]        "
      CONSTANT  ks     = 0.0875
      "                      for  ymax = 0.7 mm (ABEX 410)              "
      "----------------------Servoventildaten  ABEX 410----------------"
      "----------------------maximum half-stroke: servovalve   [m]      "
      CONSTANT  ymx    = 0.7e-3
      "----------------------minimum half-stroke: servovalve   [m]      "
      CONSTANT  ymn    =-0.7e-3
      "----------------------diameter servovalve piston        [m]      "
      CONSTANT  d      = 7.0e-3
      "----------------------servovalve:underlap,outflow,left  [m]      "
      CONSTANT  y01a   = 0.0
      "----------------------servovalve:underlap,inflow,left   [m]      "
```

```
      CONSTANT  y01e   = 0.0
      "----------------------servovalve:underdlap,inflow,right [m]     "
      CONSTANT  y02e   = 0.0
      "----------------------servovalve:underlap,outflow,right [m]     "
      CONSTANT  y02a   = 0.0
      "----------------------rad. clearance: piston-bore(servovalve)[m]"
      CONSTANT  dlr    = 2.0e-6
      "----------------------loss coefficient (turb. flow) [ - ]       "
      CONSTANT  al     = 0.6
      "----------------------SQRT(crit. Reynolds Number)   [ - ]       "
      CONSTANT  sqrecr = 5.0
      "----------------------properties of Skydrol:--------------------"
      "----------------------density Skydrol LD          [kg/m**3]    "
      CONSTANT  ro     = 980.0
      "----------------------modulus of compression Skydrol            "
      "                                                     [N/m**2]    "
      CONSTANT  e      = 1.2e9
      "----------------------kinematic viscosiy Skydrol LD [m**2/s]    "
      CONSTANT  nu     = 1.4e-5
      "----------------------calculation of inital pressure and flow --"
      s1h     = 0.5*(y01e + yic + ABS(y01e + yic))
      s2h     = 0.5*(y02e - yic + ABS(y02e - yic))
      s3h     = 0.5*(y01a - yic + ABS(y01a - yic))
      s4h     = 0.5*(y02a + yic + ABS(y02a + yic))
      s1      = SQRT(s1h**2 + dlr**2)
      s2      = SQRT(s2h**2 + dlr**2)
      s3      = SQRT(s3h**2 + dlr**2)
      s4      = SQRT(s4h**2 + dlr**2)
      paic    = p0*1.0/(1. + (s3/s1)**2)
      pbic    = p0*1.0/(1. + (s4/s2)**2)
      a1      = pi*(d + dlr)*s1
      a2      = pi*(d + dlr)*s2
      a3      = pi*(d + dlr)*s3
      a4      = pi*(d + dlr)*s4
      q1      = a1*al*SQRT(2.0/ro*ABS(p0-paic))*SIGN(1.0,p0-paic)
      q2      = a2*al*SQRT(2.0/ro*ABS(p0-pbic))*SIGN(1.0,p0-pbic)
      q3      = a3*al*SQRT(2.0/ro*ABS(paic))*SIGN(1.0,paic)
      q4      = a4*al*SQRT(2.0/ro*ABS(pbic))*SIGN(1.0,pbic)


END $" OF INITIAL "

DISCRETE

      INTERVAL       PERIOD = 0.0 $" INDICATE PERIOD WILL BE CALCULATED"
      CONSTANT       RADDEG = 57.3
      CONSTANT       EPDG = 0.1        , EPM = 1.0E-7
      CONSTANT       KW = 0.8          , TSTP = 10000.0

     PROCEDURAL
      "-----------------------CHANGE IN IN-PHASE AND QUADRATURE INTEG "
      "                       RALS OVER LAST CYCLE "
      DLP     = P - PP
      DLQ     = Q - QP
      "-----------------------IF RELATIVE CHANGE TOO SMALL FOR MACH ACC"
      TERMT((DLP**2 + DLQ**2)/(P**2 + Q**2 + RMN) .LT. EPM**2)
      "-----------------------SAVE NEW INTEGRALS AS PREVIOUS "
      PP      = P
      QP      = Q
      "-----------------------CALCULATE NEW PHASE AND GAIN "
      PDGN    = ATAN2(DLQ, DLP + RMN)*RADDEG
      GDBN    = (DLP**2 + DLQ**2)*(W/(2*PI*XMAG))**2
      "-----------------------IF CHANGE IN PHASE NOT SMALL ENOUGH YET "
      IF(ABS(PDGN - PDGP) .GT. EPDG) GO TO SKIP1
      "-----------------------IGNORE RESULTS UNTIL AFTER SETTLING TIME "
      IF(T .LT. TSETTL) GO TO SKIP1
      "-----------------------TERMINATE ON FREQUENCY SWEEP "
      TERMT(W .LE. WMN)
      "-----------------------SAVE VALUE IN SEPARATE NAME FOR PLOTTING "
      PHASE   = PDGN
      GAIN    = GDBN
      OMEGA   = W
      "-----------------------ADVANCE FREQUENCY GEOMETRICALLY "
      W       = AMAX1(WMN, KW*W)
      "-----------------------CALCULATE NEW PHASE FOR CONTINUITY "
      "                       OF FORCING FUNCTION AT NEW FREQUENCY "
      FI      = FI + T*(OMEGA - W)
      "-----------------------ENSURE PREVIOUS PHASE SET TO FORCE AT "
      "                       LEAST TWO CYCLES "
      PDGN    = RMX
```

```
      "-----------------------FORCE A DATA LOGGING ACTION "
      CALL LOGD(.FALSE.)
     SKIP1..CONTINUE
      "-----------------------RESET PREVIOUS PHASE FOR NEXT TIME "
      PDGP    = PDGN
      "-----------------------RECALCULATE NEW PERIOD AND STEP SIZE "
      PERIOD  = 2.0*PI/W
      MAXTC   = AMIN1(PERIOD/NSTPMN, MAXTXZ)
     END $" OF PROCEDURAL "
      TERMT(T .GT. TSTP)

END $" OF DISCRETE "

DERIVATIVE CONTIN

      MAXTERVAL      MAXTC  = 1.0e-4
      CONSTANT       MAXTXZ = 5.0e-4        , NSTPMN = 10.0
      CONSTANT       WMN    = 1.0           , WMX    = 1000.0
      CONSTANT       TSETTL = 0.1

      XI      = XMAG*SIN(W*T + FI)

      "-----------------------DEFINE MODEL "
      ALGORITHM  IALG = 4
      NSTEPS     NSTP = 1
      i       = k*(xi-xo)
      y       = BOUND(ymn,ymx,ks*REALPL(0.005,i,iic))
     PROCEDURAL(s1,s2,s3,s4,cd1,cd2,cd3,cd4 = y)
      s1h     = 0.5*(y01e + y + ABS(y01e + y))
      s2h     = 0.5*(y02e - y + ABS(y02e - y))
      s3h     = 0.5*(y01a - y + ABS(y01a - y))
      s4h     = 0.5*(y02a + y + ABS(y02a + y))
      s1      = SQRT(s1h**2 + dlr**2)
      s2      = SQRT(s2h**2 + dlr**2)
      s3      = SQRT(s3h**2 + dlr**2)
      s4      = SQRT(s4h**2 + dlr**2)
      dh1     = 2.0*s1
      dh2     = 2.0*s2
      dh3     = 2.0*s3
      dh4     = 2.0*s4
      a1      = pi*(d + dlr)*s1
      a2      = pi*(d + dlr)*s2
      a3      = pi*(d + dlr)*s3
      a4      = pi*(d + dlr)*s4
      sqre1   = SQRT(ABS(q1)/a1*dh1/nu)
      sqre2   = SQRT(ABS(q2)/a2*dh2/nu)
      sqre3   = SQRT(ABS(q3)/a3*dh3/nu)
      sqre4   = SQRT(ABS(q4)/a4*dh4/nu)
      cd1     = -0.5*(-al/sqrecr*sqre1+al+ABS(-al/sqrecr*sqre1+al)) + al
      cd2     = -0.5*(-al/sqrecr*sqre2+al+ABS(-al/sqrecr*sqre2+al)) + al
      cd3     = -0.5*(-al/sqrecr*sqre3+al+ABS(-al/sqrecr*sqre3+al)) + al
      cd4     = -0.5*(-al/sqrecr*sqre4+al+ABS(-al/sqrecr*sqre4+al)) + al
     END $" of PROCEDURAL section"
      q1      = a1*cd1*SQRT(2.0/ro*ABS(p0 - pa))*SIGN(1.0,p0 - pa)
      q2      = a2*cd2*SQRT(2.0/ro*ABS(p0 - pb))*SIGN(1.0,p0 - pb)
      q3      = a3*cd3*SQRT(2.0/ro*ABS(pa))*SIGN(1.0,pa - pr)
      q4      = a4*cd4*SQRT(2.0/ro*ABS(pb))*SIGN(1.0,pb - pr)
      qa      = q1 - q3
      qb      = q2 - q4
      va      = v0 + a*xo
      vb      = v0 - a*xo
     PROCEDURAL(pa,pb,dlp = va,vb,xod)
      pad     = e/va*(qa - a*xod)
      pbd     = e/vb*(qb + a*xod)
      pa      = LIMINT(pad,paic,0.0,220.e5)
      pb      = LIMINT(pbd,pbic,0.0,220.e5)
     END $" of PROCEDURAL section"
      fp      = a*(pa-pb)
      fd      = dr*xod
      faero   = xo*fmx/xomx*vrel**2
      xodd    = 1.0/mr*(fp - fd - faero)
      xod     = INTEG(xodd,xodic)
      xo      = LIMINT(xod,xoic,xomn,xomx)

      "-----------------------INTEGRATE FOR IN-PHASE AND QUADRATURE COM"
      P       = INTEG(XO*SIN(W*T + FI), 0.0)
      Q       = INTEG(XO*COS(W*T + FI), 0.0)

END $" OF CONTINUOUS SECTION "
END $" OF PROGRAM "
```