AIRCRAFT DESIGN AND SYSTEMS GROUP (AERO)

Turbofan Specific Fuel Consumption, Size, and Mass from Correlated Engine Parameters

Mohamed Oussama Hammami

1. Examiner: Prof. Dr.-Ing. Dieter Scholz, MSME
2. Examiner: Dr.-Ing. Daniel Nerger

Presentation of the Master Thesis

Hamburg, 08.10.2021

HAW HAMBURG

Table of Contents

I. Introduction

Conceptual Design Phase

- Sketching of Possible Configurations exp. Wing length, engine size …

Preliminary Design Phase

- Addressing of the Best Compromises in Terms of Weight and Costs
- Information Gathering

Detailed Design Phase

- Deals with the Fabrication Aspects of the Aircraft to be Manufactured

Goal:

- **Studying the Linearity of SFC, Mass and Engine Size**
- **Correlation between Engine Parameters**
- **Creation of Simplified and More Extended SFC Models with High Accuracy**

II. Calculation and Analysis

1. Calculation and Extraction of C_a and C_b for the Available Engines

$$C_t = C_a \cdot V + C_b \quad \Rightarrow \quad C_a\ ?\ C_b\ ?$$

C_t : SFC at cruise phase [kg/(Ns)]

C_a: SFC Factor [kg/(Nm)]

C_b: SFC Factor [kg/(Ns)]

V : Speed [m/s]

1. Calculation and Extraction of C_a and C_b for the Available Engines

- C_b :

  On ground at $V = 0$:

$$C_0 = \underbrace{C_a \cdot V}_{=0} + C_b \Rightarrow C_0 = C_b$$

C_0: SFC on ground

- C_a :

$$V = a\,M = a_0 \sqrt{\frac{T_h}{T_0}}\,M$$

M : Mach Number

T_h: Cruise Temperature [K]

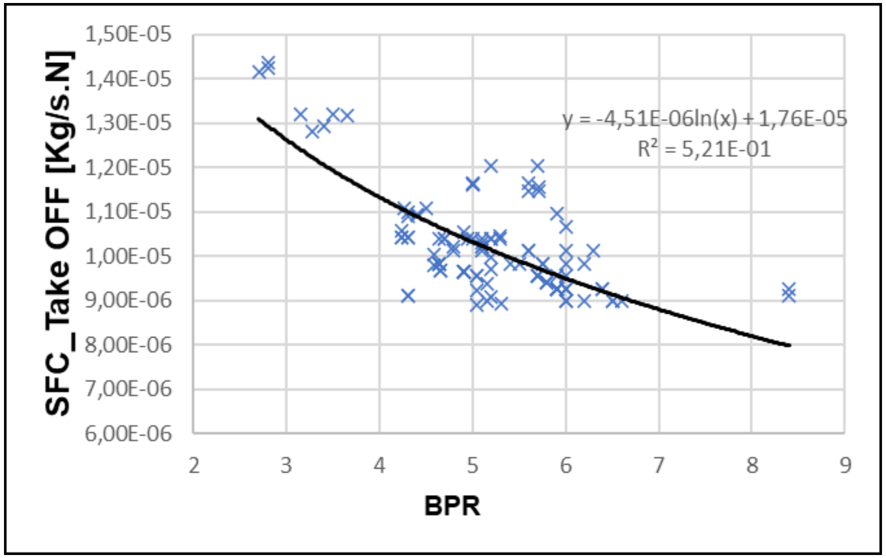
T_0: Temperature at sea lvl [K]

a_0: Speed of sound [m/s]

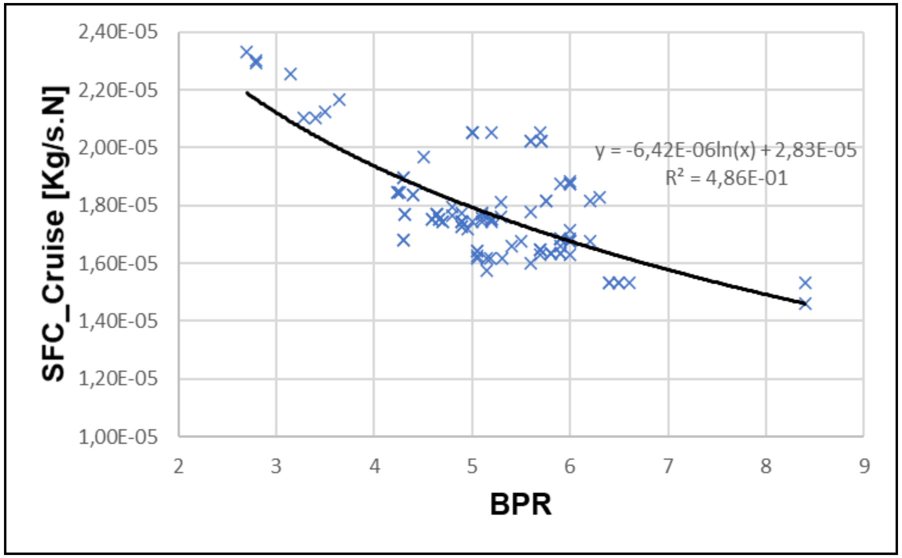
$$C_a = \frac{C_t - C_0}{a_0 \sqrt{\frac{T_h}{T_0}}\,M}$$

2. Analysis of the Variation of Engine Design Parameters with Thrust, BPR, and Engine Geometric Parameters

Take-off and Cruise SFC Variation with BPR



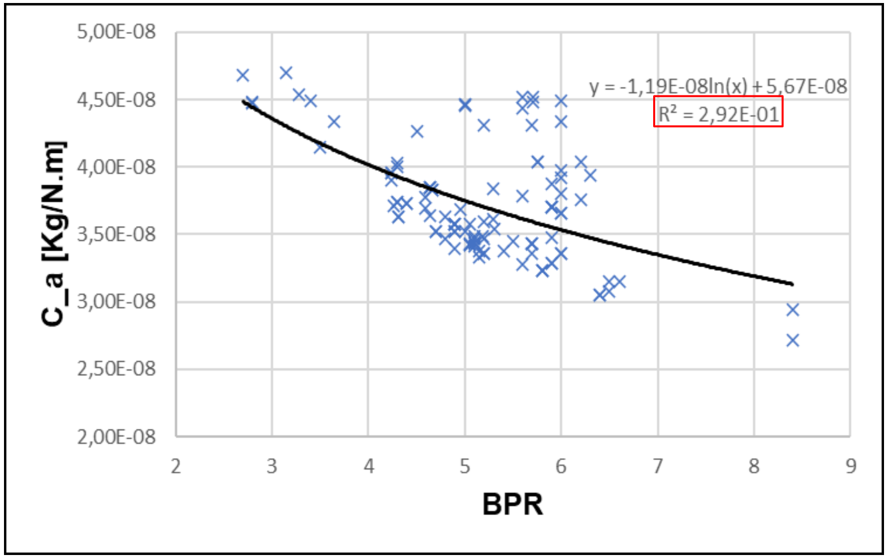
Bypass ratio effect on take-off specific fuel consumption



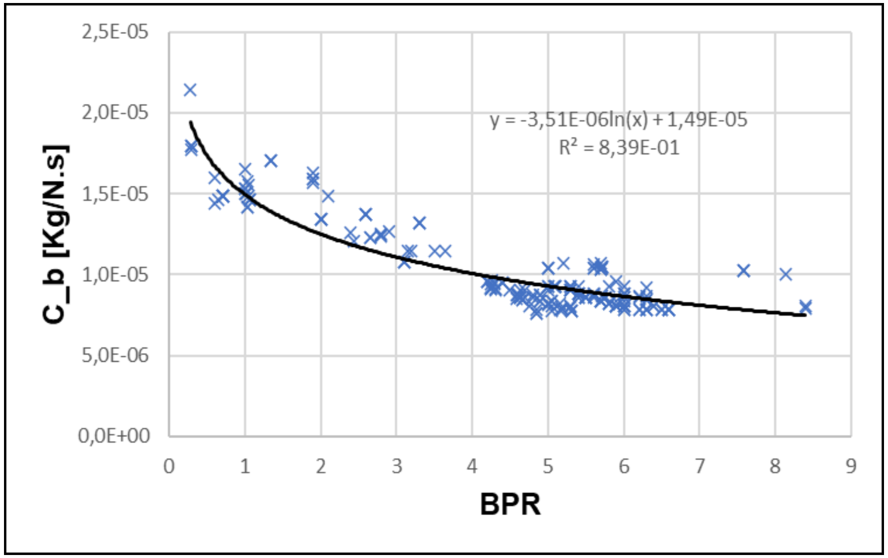
Bypass ratio effect on cruise specific fuel consumption

2. Analysis of the Variation of Engine Design Parameters with Thrust, BPR, and Engine Geometric Parameters

C_a and C_b Variation with BPR



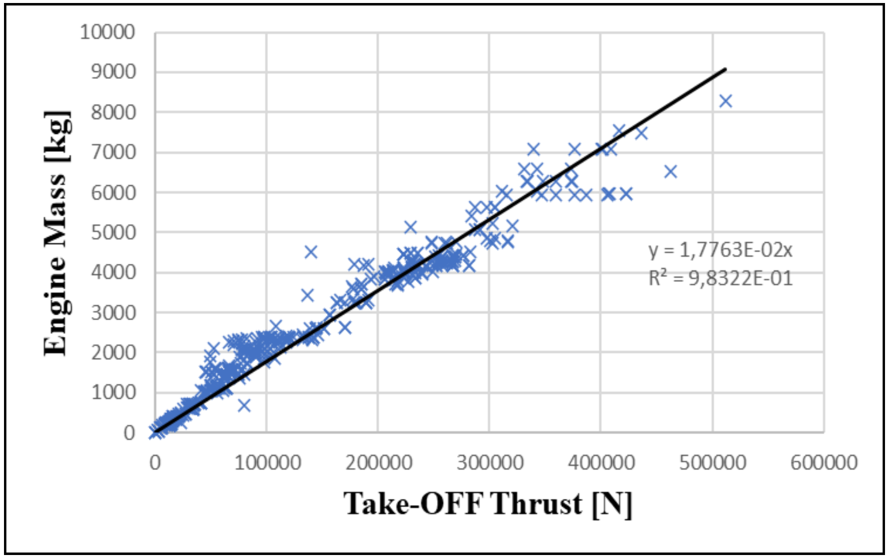
Variation of C_a with Bypass ratio



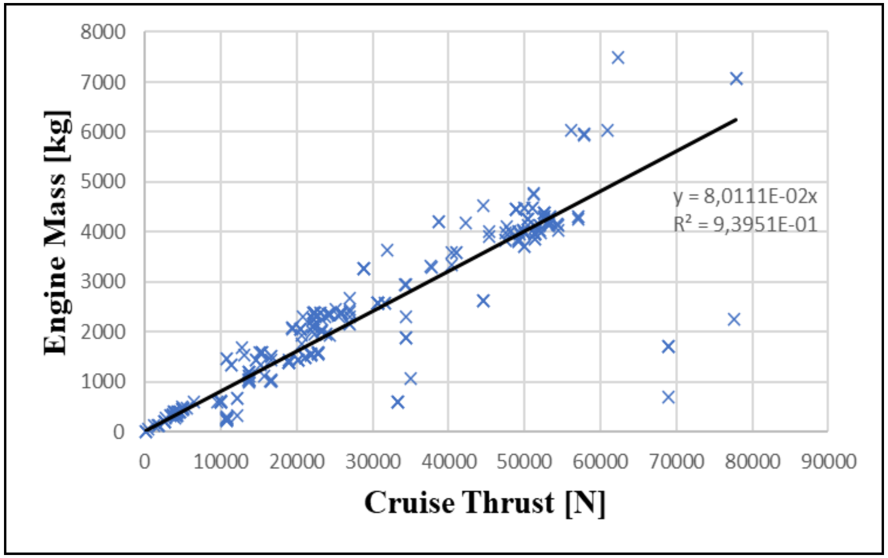
Variation of C_b with Bypass ratio

2. Analysis of the Variation of Engine Design Parameters with Thrust, BPR, and Engine Geometric Parameters

Engine Mass Variation with Take-off and Cruise Thrust



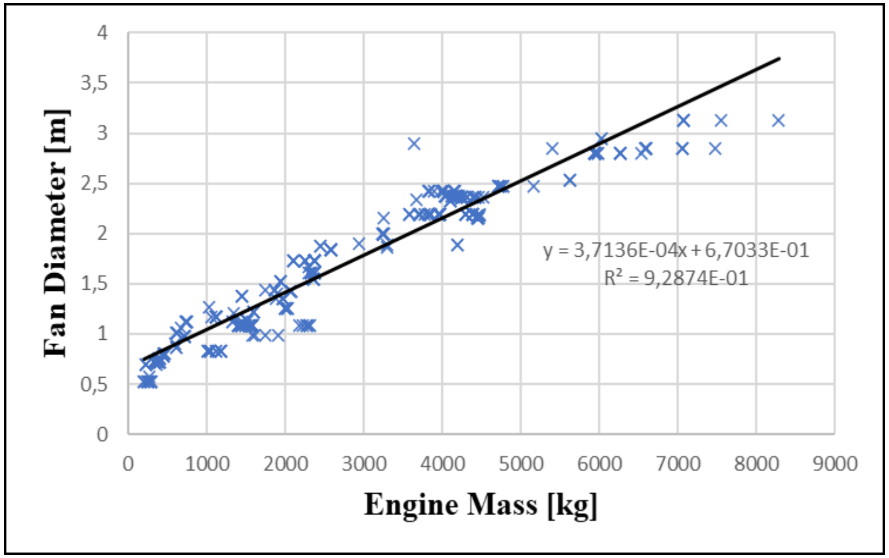
Engine Mass as function of Take-off Thrust



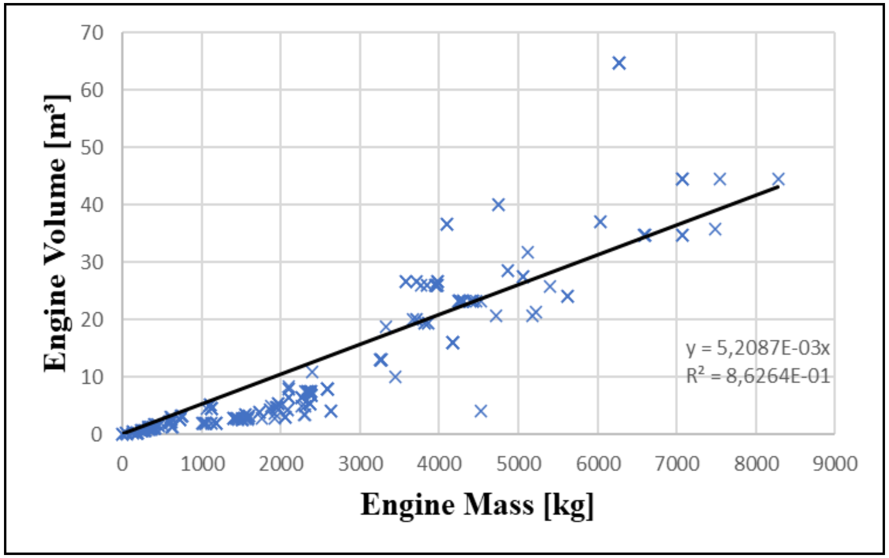
Engine Mass as function of Cruise Thrust

2. Analysis of the Variation of Engine Design Parameters with Thrust, BPR, and Engine Geometric Parameters

Engine Mass Variation with Engine Geometric Parameters



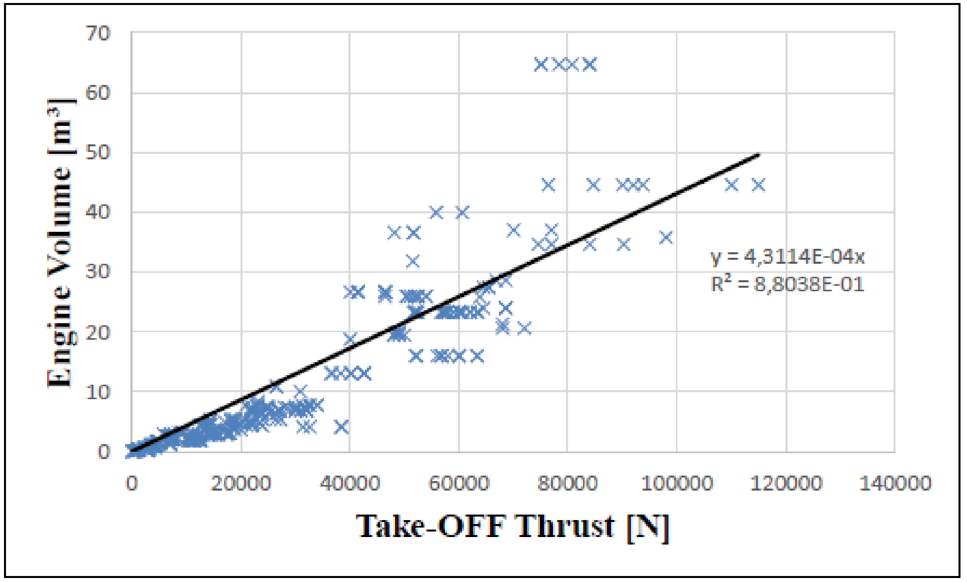
Variation of Engine Mass with Fan Diameter



Variation of Engine Mass with Engine Volume

2. Analysis of the Variation of Engine Design Parameters with Thrust, BPR, and Engine Geometric Parameters

Thrust Variation with Engine Geometric Parameters



Variation of Take-off Thrust with Engine Volume

HAW HAMBURG

III. Establishment of New SFC Models

Parameters	Units
Thrust	N
SFC	kg/Ns
C_a	kg/Nm
C_b	kg/Ns
Engine Mass	kg
Cruise Altitude	m
Engine Length	m
Engine Diameter	m
Engine Volume	m^3
Temperature	K
Speed	m/s

1. SFC Models from Linear Regression

First model:

$$SFC_{Cruise} = 1{,}5 \cdot 10^{-5} + 1{,}6 \cdot 10^{-11} \cdot F_{Cruise} - 4{,}2 \cdot 10^{-10} \cdot h + 2{,}97 \cdot 10^{-5} \cdot M - 6{,}86 \cdot 10^{-7} \cdot \lambda - 1{,}3 \cdot 10^{-9} \cdot m$$

Parameters	Coefficients	P-Values	R^2
Intercept	$1{,}5 \cdot 10^{-5}$	$1{,}5 \cdot 10^{-9}$	67,2%
Thrust [N]	$1{,}6 \cdot 10^{-11}$	0,06	
Cruise Altitude [m]	$-4{,}2 \cdot 10^{-10}$	$3{,}5 \cdot 10^{-28}$	
Mach Number	$2{,}97 \cdot 10^{-5}$	$1{,}06 \cdot 10^{-15}$	
BPR	$-6{,}86 \cdot 10^{-7}$	$5{,}08 \cdot 10^{-19}$	
Mass [kg]	$-1{,}3 \cdot 10^{-9}$	$1{,}1 \cdot 10^{-17}$	

1. SFC Models from Linear Regression

Second model:

$$SFC_{Cruise} = 1{,}48 \cdot 10^{-5} - 4{,}13 \cdot 10^{-10} \cdot h + 2{,}96 \cdot 10^{-5} \cdot M - 6{,}82 \cdot 10^{-7} \cdot \lambda - 1{,}11 \cdot 10^{-9} \cdot m$$

Parameters	Coefficients	P-Values	R^2	Error
Intercept Cruise Altitude [m] Mach Number BPR Mass [kg]	$1{,}48 \cdot 10^{-5}$ $-4{,}13 \cdot 10^{-10}$ $2{,}96 \cdot 10^{-5}$ $-6{,}82 \cdot 10^{-7}$ $-1{,}11 \cdot 10^{-9}$	$2{,}44 \cdot 10^{-9}$ $1{,}65 \cdot 10^{-27}$ $1{,}62 \cdot 10^{-15}$ $9{,}28 \cdot 10^{-19}$ $1{,}9 \cdot 10^{-22}$	67%	7,86%

1. SFC Models from Linear Regression

Third model:

$$SFC_{Cruise} = 2{,}207 \cdot 10^{-5} + 1{,}59 \cdot 10^{-11} \cdot F_{Cruise} - 4{,}94 \cdot 10^{-10} \cdot h + 1{,}02 \cdot 10^{-5} \cdot M - 7{,}97 \cdot 10^{-7} \cdot \lambda - 5{,}7 \cdot 10^{-10} \cdot m - 8{,}83 \cdot 10^{-8} \cdot \varepsilon_c$$

Parameters	Coefficients	P-Values	R^2	Error
Intercept Thrust [N] Cruise Altitude [m] Mach Number BPR Mass [kg] OPR	$2{,}207 \cdot 10^{-5}$ $1{,}59 \cdot 10^{-11}$ $-4{,}94 \cdot 10^{-10}$ $1{,}02 \cdot 10^{-5}$ $-7{,}97 \cdot 10^{-7}$ $-5{,}7 \cdot 10^{-10}$ $-8{,}83 \cdot 10^{-8}$	$4{,}34 \cdot 10^{-16}$ 0,0002 0,0003 0,015 $2{,}68 \cdot 10^{-21}$ 0,00013 0,027	70%	5,28%

2. SFC Models from Minimum Mean Square Error

$$y = a \cdot x^b$$

Models for C_a	R^2	Err [%]
$C_a = 3{,}96 \cdot 10^{-8} \cdot C_0^{5{,}3 \cdot 10^{-8}}$	0,54	8,84
$C_a = 9{,}594 \cdot 10^{-8} \cdot \lambda^{-0{,}6}$	0,3	9,88
$C_a = 9{,}5 \cdot 10^{-1} \cdot \lambda^{0{,}9} \cdot C_0^{1{,}6}$	0,07	12,04
$C_a = 9{,}5 \cdot 10^{-1} \cdot T_c^{0{,}2} \cdot C_0^{1{,}7}$	0,04	13,42
$C_a = 9{,}3 \cdot 10^{-1} \cdot \lambda^{0{,}9} \cdot T_c^{0{,}26} \cdot C_0^{1{,}8}$	0,12	57,2
$C_a = 5 \cdot 10^{8} \cdot T_0^{-0{,}6} \cdot \lambda^{0{,}77} \cdot T_c^{-0{,}4} \cdot C_0^{2{,}5}$	0,43	78,7

Models for C_b	R^2	Err [%]
$C_b = 7{,}46 \cdot 10^{-5} \cdot T_0^{-0{,}2}$	0,65	11,37
$C_b = 5{,}47 \cdot 10^{-5} \cdot T_C^{-0{,}2}$	0,51	12,18
$C_b = 1{,}65 \cdot 10^{-5} \cdot \lambda^{-0{,}4}$	0,64	5,81
$C_b = 9{,}94 \cdot 10^{-6} \cdot \lambda^{8{,}37 \cdot 10^{-6}} \cdot T_C^{1{,}25 \cdot 10^{-5}} \cdot T_0^{2{,}15 \cdot 10^{-5}}$	0,58	15,63
$C_b = 1{,}06 \cdot 10^{-5} \cdot \lambda^{-0{,}12} \cdot T_C^{-0{,}7} \cdot T_0^{-0{,}8} \cdot C_0^{0{,}1}$	0,6	9,23

2. SFC Models from Minimum Mean Square Error

$$C_t = 3{,}96 \cdot 10^{-8} \cdot C_0{}^{5{,}3\cdot 10^{-3}} \cdot V + 1{,}65 \cdot 10^{-5} \cdot \lambda^{-0{,}4}$$

Since

$$C_b = C_0 = 1{,}65 \cdot 10^{-5} \cdot \lambda^{-0{,}4}$$

So

$$C_t = 3{,}735 \cdot 10^{-8} \cdot \lambda^{-2{,}12\cdot 10^{-3}} \cdot V + 1{,}65 \cdot 10^{-5} \cdot \lambda^{-0{,}4}$$

Error
6,16%

IV. Singular Value Decomposition

- Useful for Modeling Components and Subsystems.
- Gives Possibility to Create Models with Few Parameters as Input and the Design Attributes as Output.

1. Singular Value Decomposition with Excel

6 Input parameters

CF6-50A

	Rel error	CF6-50A	Estimate	Adjusted	Result	Average								SVD variabl	w-diagonal	residual
Thrust (Cruise) [N]	0,01%	50042,00	50046,33	4,70	0,34	4,36	0,295	-0,060	0,057	-0,007	0,006	-0,001	0,000	0,93	5,42	0,48
OPR_ Sea lvl	0,00%	28,60	28,60	1,46	0,07	1,38	0,107	0,040	-0,034	-0,041	0,005	-0,001	0,000	-0,39	2,23	0,20
Cruise Altitude [m]	0,00%	10668,00	10668,00	4,03	0,00	4,03	-0,009	0,023	0,008	-0,001	0,005	0,015	0,000	0,68	0,99	0,11
Mach Number	0,00%	0,80	0,80	-0,10	0,00	-0,10	0,002	0,003	0,001	0,001	-0,001	0,000	0,007	-0,28	0,55	0,03
BPR+1	0,00%	5,40	5,40	0,73	0,05	0,68	0,125	0,173	0,013	0,013	0,004	-0,002	0,000	0,21	0,28	0,02
Dry weight [Kg]	0,00%	3719,00	3719,00	3,57	0,28	3,29	0,330	-0,028	-0,048	0,016	-0,006	0,002	0,000	0,78	0,18	0,01
SFC (Cruise) [kg/s.N]	7,34%	1,7985E-05	1,6664E-05	-4,78	-0,04	-4,74	-0,035	-0,037	-0,026	0,014	0,022	-0,002	0,000	-0,13	0,08	0,00
	7,35%															

$$Err = 7{,}35\%$$

SVD Err	Linear Regression Err
7,35%	5,25%

1. Singular Value Decomposition with Excel

9 Input parameters

	Rel error	CF6-50A	Estimate	Adjusted	Result	Average											SVD variable	w-diagonal	residual
Takeoff Thrust [N]	0,00%	215000,00	214989,80	5,33	0,34	4,99	0,364	-0,018	0,029	-0,005	0,008	-0,007	0,016	-0,001	0,002	0,000	0,96	7,50	0,49
Thrust (Cruise) [N]	0,00%	50042,00	50040,57	4,70	0,34	4,36	0,293	-0,059	-0,068	-0,019	0,002	-0,006	-0,004	-0,004	0,000	0,000	-0,49	2,31	0,17
OPR_Sea lvl	0,00%	28,60	28,60	1,46	0,07	1,39	0,106	0,038	0,030	-0,029	0,028	0,004	-0,010	-0,003	-0,003	0,000	-0,59	1,06	0,14
Mach Number	0,00%	0,80	0,80	-0,10	0,00	-0,10	0,002	0,003	-0,001	0,001	-0,001	0,000	0,001	0,000	-0,001	-0,006	0,63	0,79	0,10
BPR+1	0,00%	5,40	5,40	0,73	0,05	0,68	0,119	0,167	-0,022	0,006	-0,005	0,009	0,003	-0,007	0,002	0,000	0,75	0,50	0,08
Dry weight [Kg]	0,00%	3719,00	3719,02	3,57	0,28	3,29	0,333	-0,021	0,031	-0,010	-0,024	0,013	-0,009	0,003	0,002	0,000	0,23	0,41	0,06
Fan Diameter [m]	0,00%	2,19	2,19	0,34	0,17	0,17	0,182	0,034	0,003	0,013	-0,009	-0,008	0,001	0,002	-0,011	0,001	0,37	0,27	0,05
Length [m]	0,01%	4,65	4,65	0,67	0,20	0,47	0,127	-0,058	-0,009	0,042	0,015	0,022	0,000	-0,002	-0,001	0,000	-0,47	0,22	0,02
Width/Diameter [m]	0,00%	2,34	2,34	0,37	0,15	0,22	0,167	0,041	0,000	0,034	0,009	-0,018	-0,009	0,006	0,004	0,000	-1,65	0,14	0,02
SFC (Cruise) [kg/s.N]	4,60%	1,7985E-05	1,7158E-05	-4,77	-0,02	-4,74	-0,032	-0,035	0,028	0,017	-0,008	-0,010	-0,005	-0,016	0,000	0,000	0,54	0,07	0,01
	4,63%																		

$$Err = 4{,}63\%$$

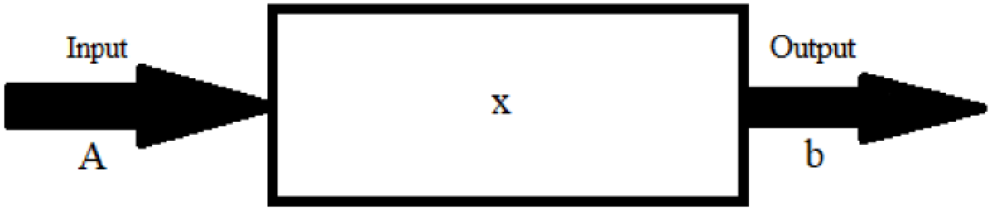
Inputs	Error
5	11%
6	7,35%
9	4,63%

HAW HAMBURG

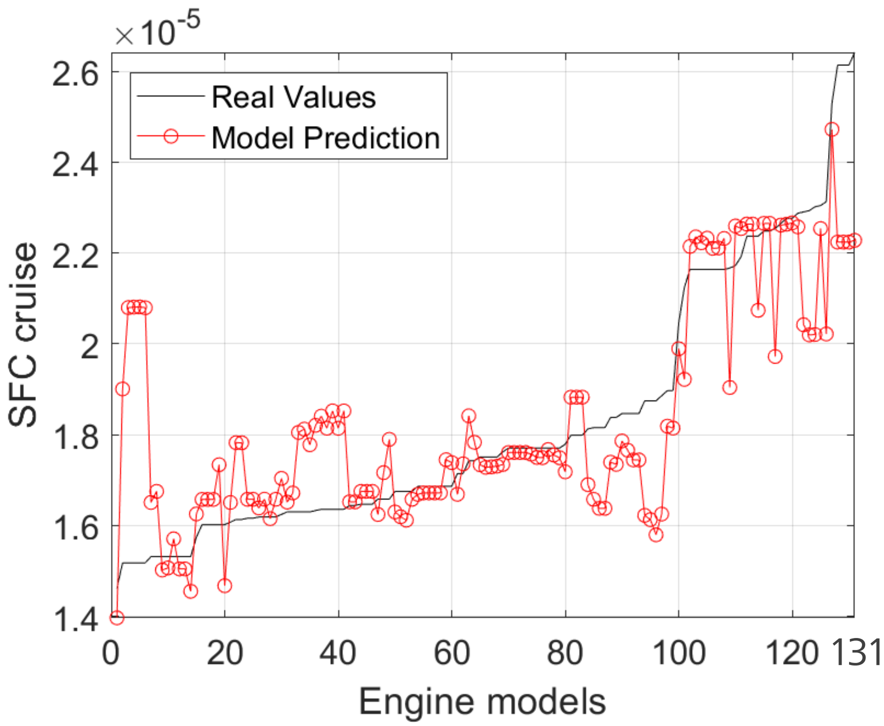
IV. Singular Value Decomposition

2. Singular Value Decomposition with MATLAB

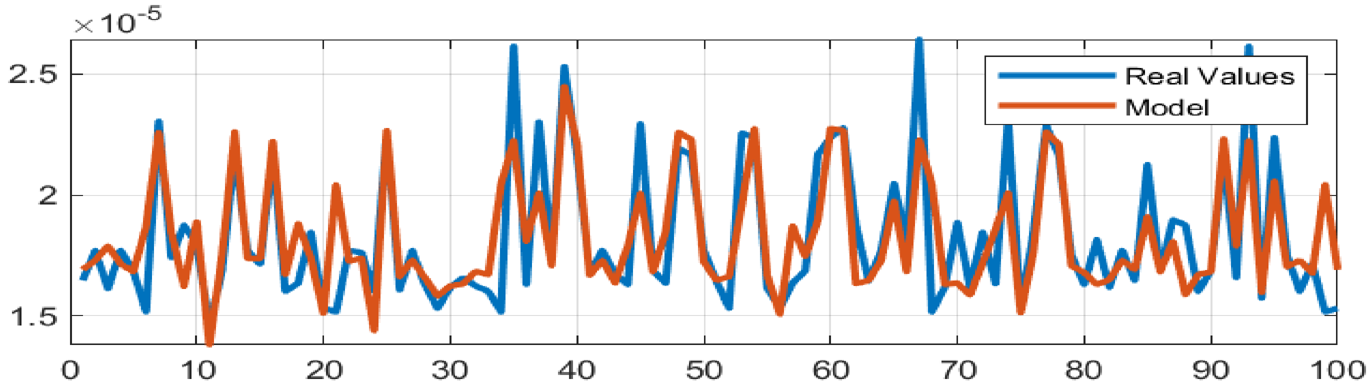
$$b = A \cdot x$$

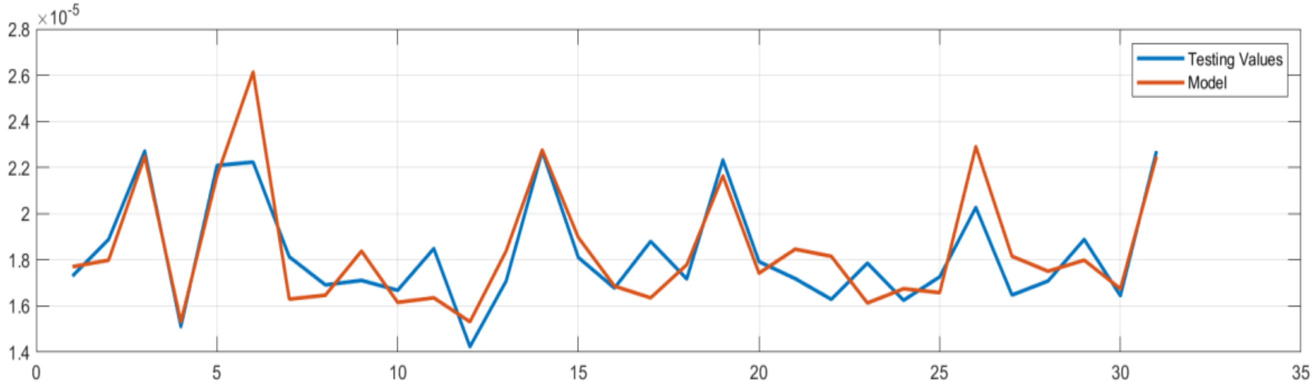


2. Singular Value Decomposition with MATLAB



2. Singular Value Decomposition with MATLAB





V. Conclusion

Accomplishments:

- Use of Simple Mathematical Methods to obtain high accurate models
  - -Linear Regression
  - -Minimum Mean Square Error
  - -Singular Value Decomposition

- Development of Simple and Precise Calculation Models capable of competing with complex equations.

  Exp. Torenbeek Model (Gas generator power function)

V. Conclusion

Perspectives:

- Classify and assess whether the SVD is sufficiently precise for specific applications
- Use of Updated data
- Inclusion of more flight conditions and phases during the calculation

HAW HAMBURG

Contact

info@ProfScholz.de

http://library.ProfScholz.de

Quote this document:

HAMMAMI, Mohamed Oussama, 2021. *Turbofan Specific Fuel Consumption, Size, and Mass from Correlated Engine Parameters.* Presentation of the Master Thesis. Aircraft Design and Systems Group (AERO), HAW Hamburg, 2021-10-08. Available from: https://nbn-resolving.org/urn:nbn:de:gbv:18302-aero2021-09-15.018.