Bachelor Thesis

From CAS to EAS –
Calculating and Plotting the
Compressibility Correction Chart

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Abstract

Purpose – Relatively cumbersome is the conversion between calibrated airspeed (CAS) and equivalent airspeed (EAS), because it involves the calculation of incompressible flow. Equations are quite long. If calculations on the computer are required, a conversion with these equations is necessary. In contrast, this report uses the equations to calculate and construct the CAS to EAS Compressibility Correction Chart. In this way, the result can be read quickly from the chart.

Methodology – In Excel, compressibility correction is achieved through equations from flight mechanics. The correction is calculated with two distinct functions, one based on Mach number and the other on pressure altitude. These functions are graphed individually and then integrated to produce the Compressibility Correction Chart.

Findings – The Compressibility Correction Chart was successfully created as a 2-D graph. Upon comparison with other correction charts, the determined correction for CAS showed no variation, proving the accuracy of the findings.

Research Limitations – Due to a limitation in Excel, which allows for 255 series for plotting, the range of input parameters had to be adjusted accordingly. The iterations of altitude span 1000 ft intervals, while those for Mach Number span 0.05 intervals.

Practical Implications – Pilots can easily use the Compressibility Correction Chart for a quick and highly accurate conversion between CAS and EAS.

Originality – CAS-EAS Compressibility Correction Charts are also available from other sources. This report presents a creation of the 2-D Correction Chart using Excel as spreadsheet.

DEPARTMENT OF AUTOMOTIVE AND AERONAUTICAL ENGINEERING

From CAS to EAS – Calculating and Plotting the Compressibility Correction Chart

Task for a *Bachelor Thesis*

Background

Various speed definitions exist in aviation: indicated airspeed (IAS, V_I), calibrated airspeed (CAS, V_C), equivalent airspeed (EAS, V_E), true airspeed (TAS, V), and ground speed (GS, V_G). Equations exist to get from one speed to the other. In the direction as given above it is from the "wrong speed to the true speed" in the opposite direction the conversion is from the "true speed to the wrong speed". Relatively cumbersome is the conversion between calibrated airspeed (CAS) and equivalent airspeed (EAS), because this involves the calculation of incompressible flow, and equations are quite long. If calculations on the computer are required, conversions with equations are necessary. However, if quick calculations with a pocket calculator are done, it is good and fast to read the difference $\Delta V_C = V_C - V_E$ as a function of CAS and altitude or CAS and Mach number or Mach number and altitude from a graph. As defined, ΔV_C is always positive and $V_E = V_C - \Delta V_C$ is always smaller than V_C. For CAS up to 100 kt the difference can be neglected for most practical cases. The difference can also be neglected for CAS up to 250 kt, if the altitude is below 10000 ft. In contrast, for Mach 1 and 30000 ft, EAS is almost 30 kt less than CAS.

Task

Charts for $\Delta V_C = V_C - V_E$ are available. Nevertheless, we want to produce a Compressibility Correction Chart (CCC) ourselves. The following sub-tasks should be considered:

- Derive the equation to calculate relative pressure in the troposphere and the stratosphere of the International Standard Atmosphere (ISA).
- Derive the equation to calculate the compressibility correction.
- Explain how to calculate all parameters for a CCC with a spreadsheet.
- Write a user guide for your spreadsheet.
- Compare your results with other publications on the topic.
- Compare your exact results with rules of thumb.

The report has to be written in English based on German or international standards on report writing.

Detailed advise for the project:

The FM-Script from Trevor Young has all necessary equations. It is supplemented by Unterlagen zur Vorlesung Flugmechanik 1. Task is to produce a plot of the Compressibility Correction Chart. $\Delta V_C = V_C - V_E$. V_C is the input value (x axis). V_E is calculated from [1.4-20] (from **page 25**) as function of V_C . Relative pressure, $\delta = f(h)$ from "1.2.3 Pressure and Density in the Standard Atmosphere". Altitude, h is taken as parameter producing the various curves in the Compressibility Correction Chart. See also Example 1.3 in the FM-Script from Trevor Young and consider $\Delta V_C = V_C - V_E$. Plotting can be done with a spreadsheet.

More difficult to produce is the Compressibility Correction Chart with Mach number, M as parameter producing the various curves. This is the way forward: ΔV_C is calculated with [1.4-20] (from **page 24**) as function of M. Relative pressure, $\delta = f(h)$. V_C as value for the x-axis is obtained from $V_C = \Delta V_C + V_E$ and $V_E = M\, a_0\, \delta^{1/2}$ [1.4-19]. Values for the x-axis and the y-axis are calculated, stored and subsequently used for the plot.

The report should give an introduction into the topic similar to the section "Calibrated Airspeed" from FM-Script by Trevor Young. It should show the derivation of the equation used to produce the plot. A literature review should point to other publications, in which the production of the Compressibility Correction Chart is explained. One such example is Walter Bislin. Please note also the contribution of Dennis Lucht and his check of the rule of thumb (ROT) based on two equations:

$V = 6\ \mathrm{FL}/10 + V_C + T_T$ (in kt, FL: Flight Level, T_T in °C) and

$V = V_C + 2\%$ of $h/1000$ ft (valid only for low level and low speed).

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List of Symbols

A	Cross Sectional Area
a	Speed of Sound
F	Force
g	Gravitational Constant
H	Pressure Altitude
p	Air Pressure
k	Recovery Factor
L	Lapse Rate
M	Mach Number
q	Dynamic Pressure
R	Gas Constant
V	True Airspeed
V_C	Calibrated Airspeed
ΔV_C	Compressibility Correction
V_E	Equivalent Airspeed

Greek Symbols

δ	Relative Pressure
Δ	Difference
$\in$	Relative Error
γ	Ratio of Specific Heats of Air
ρ	Air Density
σ	Relative Density
θ	Temperature Ratio

Subscripts

0	Standard Condition
$*$	Tropopause Conditions
ROT	Rule of Thumb
T	Total

List of Abbreviations

ASI	Airspeed Indicator
CAS	Calibrated Airspeed
CTRL	Control key
EAS	Equivalent Airspeed
FL	Flight Level
IAS	Indicated Airspeed
ICAO	International Civil Aviation Organization
ISA	International Standard Atmosphere
ROT	Rule of Thumb
TAS	True Airspeed
VSI	Vertical Speed Indicator

1 Introduction

1.1 Motivation

Pilots must understand and determine the speed of the plane through the air using the built-in Airspeed indicator equipment in the airplane. In the case that the airspeed indicator does not function, pilots must know how to calculate and determine the aircraft speed hands-on using tools such as the Compressibility Correction Chart. Therefore, it is significant to fully acknowledge the fundamentals and derivations behind the Compressibility Correction Chart. This is the reason for producing a plot of the Compressibility Correction Chart in Excel where the derivation of the equation is highly detailed. In this case we will primarily focus calculating and plotting the Compressibility Correction Chart from Calibrated Airspeeds to Equivalent Airspeeds.

1.2 Title Terminology

Airspeed
The term *Airspeed* "is defined as the aircraft speed *"*measured against the speed of the air though it is moving" (CUPA 2024).

This thesis aims to explore a variety of airspeeds including Indicated Airspeed, Calibrated Airspeed, Equivalent Airspeed, and True Airspeed.

Altitude
This term *Altitude* is the "vertical distance of an object measured from mean sea level*"* (SKYbrary 2024b).

The compressibility correction chart primarily focuses on using the pressure altitude as a key parameter.

ISA
The term is *ISA* is known as the International Standard Atmosphere (ISA) where it "is a standard which to compare the actual atmosphere at any point and time*"* (SKYbrary 2024c).

The ISA will provide valuable information of pressure, density, and temperature relative to the change in altitude.

ICAO
This term *ICAO* is the International Civil Aviation Organization that "provides means for coordinating, prioritizing and managing the development of a state's air transport system" (United Nations 2024).

The ICAO standard atmosphere is the standard for the atmosphere from sea level via the troposphere into the stratosphere.

Mach
The *Mach* number is the “the ratio between the true air speed (TAS) and the local speed of sound (LSS)*”* (SKYbrary 2024d)

The Mach number is crucial for the operation of airplanes at high speeds.

1.3 Objectives

The objective of this thesis is to plot the CAS to EAS Compressibility Correction Chart in Excel. It is important to provide an in-depth theoretical review of the fundamentals and derivations to calculate the compressibility correction.

1.4 Literature Review

The fundamentals and all equations of the compressibility correction chart are based on “*Flight Mechanics: Chapter 1”* manuscript by Young 2001 and the publishment of “*Performance of Jet Transport Airplane*” by Young 2017, accompany with lecture notes “*Flight Mechanics 1”* and “*Flugmechanik – Flugleistung und statische Stabilität der Längsbewegung”* at the Hamburg University of Applied Sciences by Scholz 2022. Other sources used briefly and can be found in the References section of this paper.

1.5 Structure of Work

The structure of the thesis works as follows:

Chapter 2	covers the derivation of equations essential to understanding flight mechanics, while also providing explanations for the fundamental concepts used in the Compressibility Correction Chart.
Chapter 3	provides the creation of the Compressibility Correction Chart, detailing its formation through the functions of Mach Number and Pressure Altitude.
Chapter 4	presents an instructive user manual, detailed the operational techniques and structure comprehension necessary to interact with the tab embedded within the Excel Sheet.

Chapter 5	provides a comparison of the results of this Compressibility Correction Chart with other similar charts.
Chapter 6	provides a summary of the work.
Chapter 7	states any future recommendations for the work.
Appendix A	provides the VBA Macro used to select odd columns in the Excel Worksheet.

2 Derivation of Equations

In this chapter the theoretical review required to understand the Compressibility Correction Chart will be discussed. This is solely based on an introduction to flight mechanics including topics such as Compressible airflow, International Standard Atmosphere, and flight speeds.

2.1 Incompressible and Compressible Flows

2.1.1 Incompressible Flows

Incompressible flow is the state in which density is constant through space and time, where viscosity does not cause any changes in the flow density

$$\nabla \cdot V = 0 \quad . \tag{2.1}$$

This implies the principle of conversation of mass for a fluid flow. By applying Newton's second law while considering the net force of pressure, density, and velocity in the flow direction then (Young 2018)

$$\Sigma F = pA - (p + \mathrm{d}p)A = m\frac{dV}{dt} \quad . \tag{2.2}$$

Thus, the cross-sectional area and velocity across two points on a streamline will also remain constant. Integrating (2.2) results with derivatives of density becoming zero, demonstrating the flow is incompressible

$$p + \frac{1}{2}\rho V^2 = \text{constant} \quad . \tag{2.3}$$

Equation (2.3) represents Bernoulli's equation for incompressible flow which effectively illustrates the conservation of energy along a streamline. As the surrounding air molecules flow around an aircraft, Bernoulli’s principle is modified to accommodate compressible air flow using Mach numbers.

2.1.2 Mach Number

When an aircraft travels at speeds less than 250 kt, the density of the surrounding air remains stable. However, as the aircraft accelerates beyond speeds of 250 kt, the aircraft's energy release compresses with the air, resulting in a density alternation. This compressibility effect produces small isentropic disturbances in the flow that alter the lift and drag of an aircraft (Hall 2021). To determine the compressibility effects, the role of Mach number is introduced

$$M = \frac{V}{a}. \tag{2.4}$$

The Mach number represents the ratio of the aircraft's true air speed to the local speed of sound. It is important to note that (2.4) appears as a scaling parameter to many practical applications to express incompressibility and compressible flows.

For instance, The Bernoulli equation from (2.3) can now be expressed in terms of Mach number instead of true airspeed for high speeds where the dynamic pressure

$$q = \frac{1}{2}\rho V^2 \tag{2.5}$$

is generalized as

$$q = \frac{1}{2}\rho_0 \sigma V^2 \tag{2.6}$$

with velocity

$$V = M a_0 \sqrt{\theta} \tag{2.7}$$

therefore

$$q = \frac{1}{2}\rho_0 \sigma M^2 a_0^2 \theta\ \ . \tag{2.8}$$

Using Equation (2.8), Bernoulli equation for incompressible flow can now be used for high-speed Mach number as a measurement of velocity (Young 2001).

Mach numbers are correlated with various airspeed regimes as (Aeronautics 2024):
Subsonic – Mach numbers below 0.75
Transonic – Mach numbers from 0.75 to 1.20
Supersonic – Mach numbers from 1.20 to 5.00
Hypersonic – Mach numbers above 5.00

In this compressibility correction chart, the focus will be on using Subsonic and Transonic airspeed regimes, along with their correlated Mach numbers. It is crucial to note that anything above Mach 1 is considered critical as that is the threshold where an aircraft may enter supersonic flight, for which most commercial wings and airfoils are not designed to withstand.

2.1.3 Compressible Flow

The airfoil of an aircraft produces significant lift by the compression of air. As illustrated in Figure 2.1, higher pressure is being created below the airfoil than it is above it.

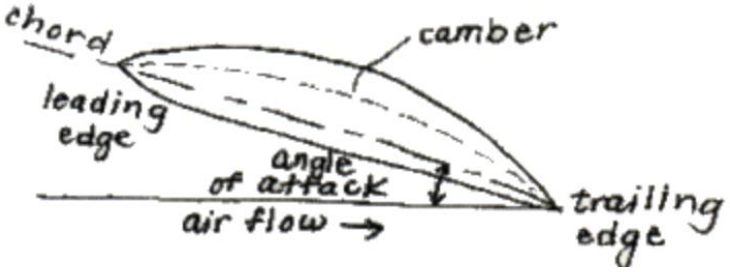


Figure 2.1 Airflow of an Airplane Wing (Nakamura 1999).

The magnitude of lift coefficient will vary depending on the angles of attack with respect to chord line. As the angle of attack rises, the life coefficient typically increases almost linearly. However, for most airfoils, an angle of attack of 17° results in a loss of lift, commonly referred to as wing stalling, due to the separation airflow from the upper surface of the airfoil (SKYbrary 2024a).

The airflow dynamics over the airfoil surface experiences diverse pressure distribution that accelerates the airflow. Under specific circumstances, this acceleration can prompt the airflow to transition from subsonic to sonic air speeds. In the event that an aircraft’s upper surface experiences sonic airflow over an area of the maximum chamber, compressibility effects are apparent, forming shock waves, increased drag, and alterations in stability, as featured in Figure 2.2. As mentioned prior, commercial flights tend to operate efficiently as they approach their respective critical Mach number, which varies depending on the aircraft.

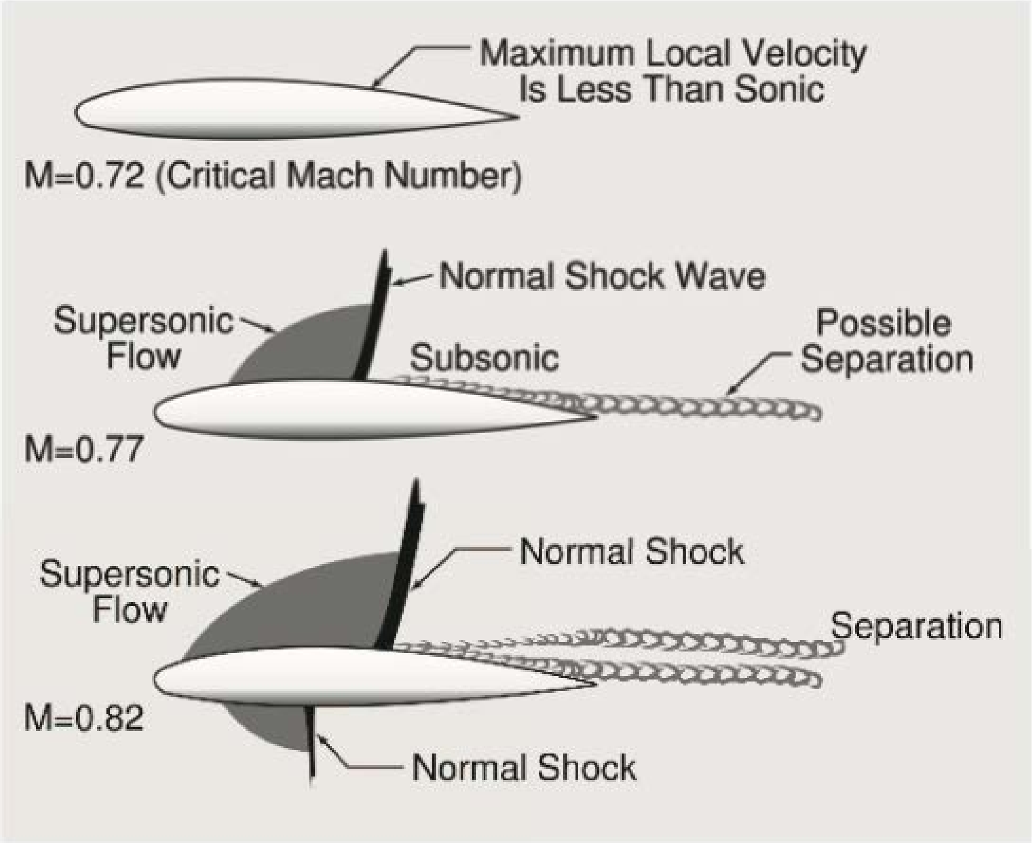


Figure 2.2 Transonic Flow Patterns with Critical Mach (Arnedo 2024)

At higher altitudes, compressibility effects become of concern due to the potential for structural failures in dynamic pressure, leading to loss of aircraft control. Beyond a threshold typically above "5-10 percent" of the critical Mach number, the impact of compressibility significantly induces flutter in dynamic pressure, potentially leading to drag divergence, characterized by a rapid increase of drag on the airfoil (Arnedo 2024). As critical Mach number is associated with the maximum operating calibrated airspeed at high altitudes, pilots must be able to measure the airspeed within flight to prevent any degradation in aircraft control.

In every aircraft, the pitot static system is implemented as an essential tool to measure the pressure differences between static (ambient) air pressure to the total air pressure. The pitot tube is typically mounted on the wing or nose of the aircraft facing the direction of the incoming airflow. Ram air pressure is directed into the pitot tube via the central port, where it is slowed down before progressing through the system. The central port will therefore give the reading for total pressure. The static port is positioned perpendicular to the fuselage's airflow direction to prevent dynamic pressure from affecting the airflow. As the airflow accelerates by the nose of the tube, it will effectively restore to its original speed upon reaching the static port, allowing for an accurate measurement of static air pressure. Mechanical instruments are installed within the system through small pressure lines that will transmit air pressure illustrating the measurements on the airspeed indicator (ASI) and vertical speed indicator (VSI) panels as in Figure 2.3.

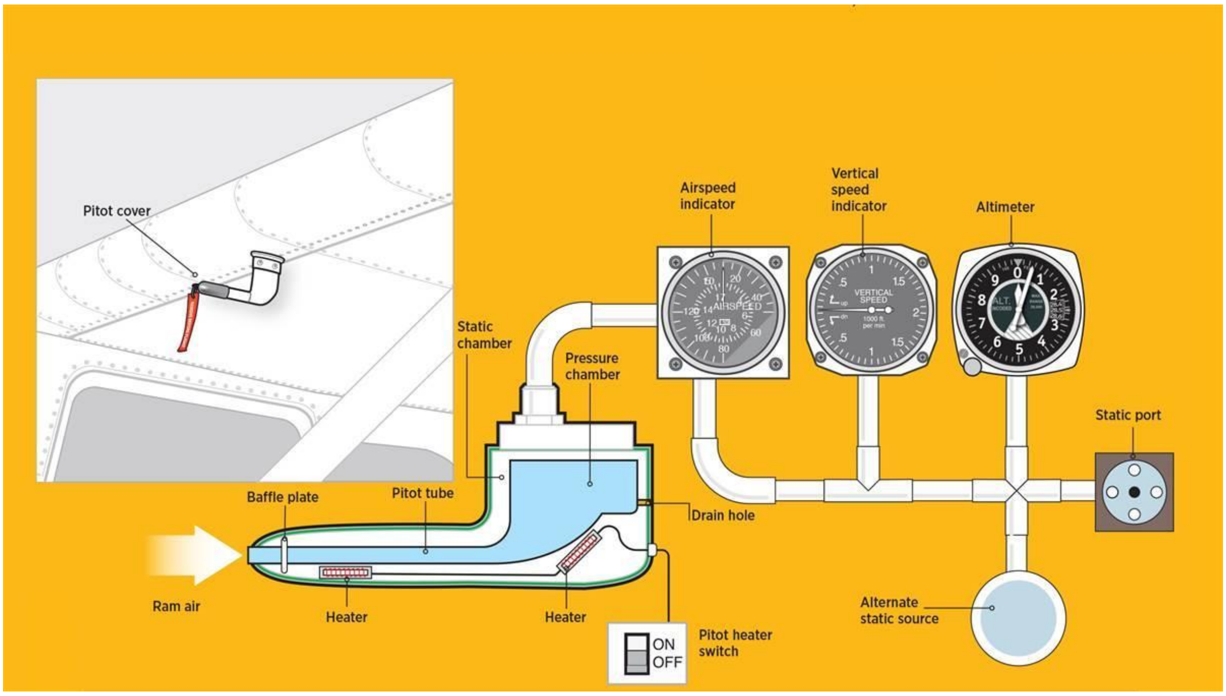


Figure 2.3 Pitot-Static System (Pilot Institute 2022)

For speeds under Mach 0.3, the effects of compressibility are neglected, thus the differential pressure gauge of the ASI will display the equivalent airspeed as

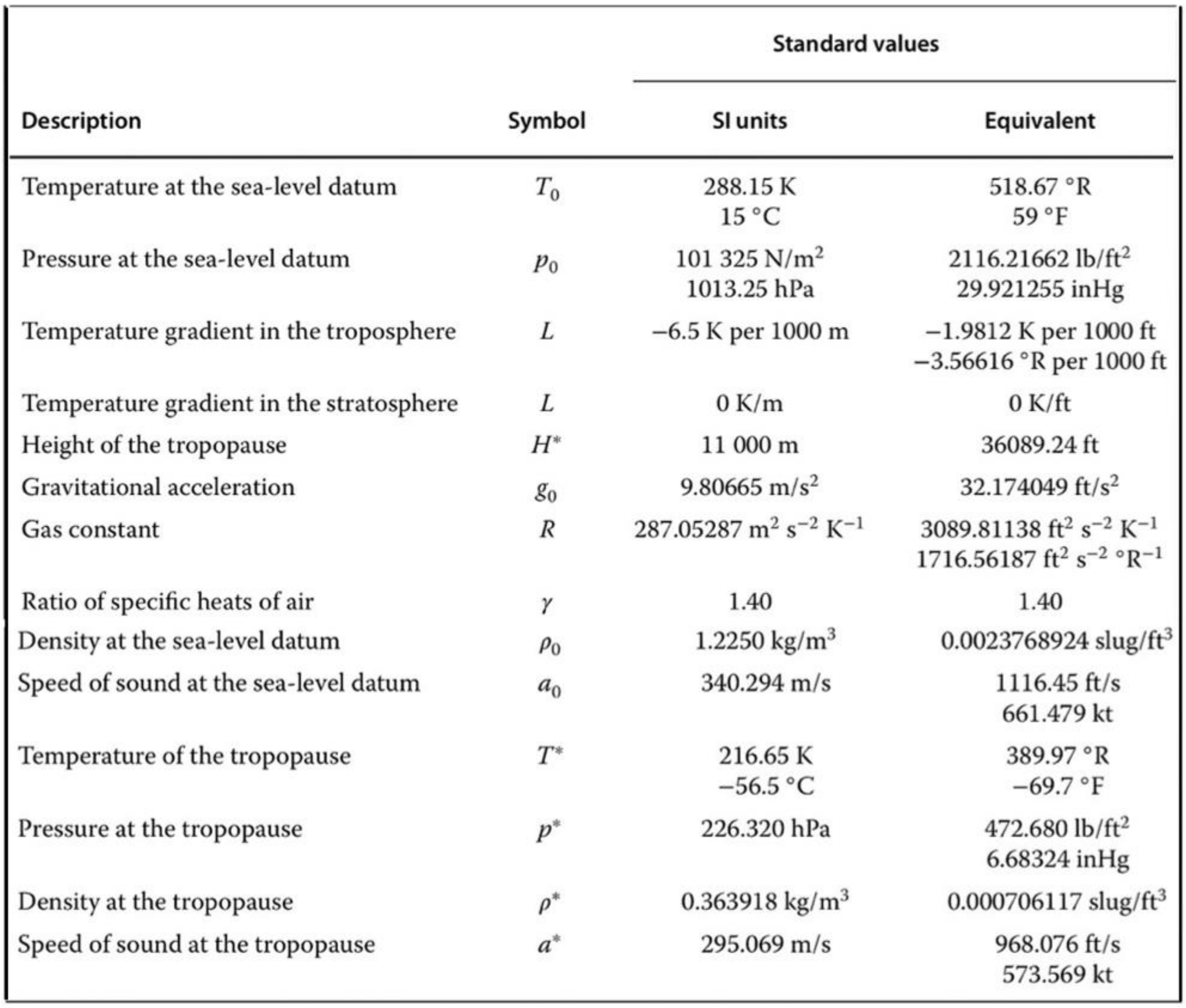
$$(\frac{\gamma}{\gamma - 1})\,(\frac{p}{p}) + \frac{1}{2}\rho V^2 = \text{constant} \quad . \tag{2.9}$$

For speeds above Mach 0.3, the ASI reading will require a Mach number calibration for high airspeed due to the effect of compressibility. This calibration only intakes altitudes at sea level, any other heights above will need a compressibility correction as discussed in Section 2.4.

2.2 Relative Pressure in the ISA

Aviation's performance operation and analysis follow with the International Civil Aviation Organization Standard Atmosphere (ICAO) which is integrated with the International Standard Atmosphere (ISA). The ISA gives the standard values of density, temperature, and pressure over a range of altitudes. Standard values of the ISA are given in Table 2.1.

Table 2.1 Standard Values of ISA (Young 2018)



		Standard values	
Description	**Symbol**	**SI units**	**Equivalent**
Temperature at the sea-level datum	T_0	288.15 K 15 °C	518.67 °R 59 °F
Pressure at the sea-level datum	p_0	101 325 N/m^2 1013.25 hPa	2116.21662 lb/ft^2 29.921255 inHg
Temperature gradient in the troposphere	L	−6.5 K per 1000 m	−1.9812 K per 1000 ft −3.56616 °R per 1000 ft
Temperature gradient in the stratosphere	L	0 K/m	0 K/ft
Height of the tropopause	H^*	11 000 m	36089.24 ft
Gravitational acceleration	g_0	9.80665 m/s^2	32.174049 ft/s^2
Gas constant	R	287.05287 m^2 s^{-2} K^{-1}	3089.81138 ft^2 s^{-2} K^{-1} 1716.56187 ft^2 s^{-2} °R^{-1}
Ratio of specific heats of air	γ	1.40	1.40
Density at the sea-level datum	ρ_0	1.2250 kg/m^3	0.0023768924 slug/ft^3
Speed of sound at the sea-level datum	a_0	340.294 m/s	1116.45 ft/s 661.479 kt
Temperature of the tropopause	T^*	216.65 K −56.5 °C	389.97 °R −69.7 °F
Pressure at the tropopause	p^*	226.320 hPa	472.680 lb/ft^2 6.68324 inHg
Density at the tropopause	ρ^*	0.363918 kg/m^3	0.000706117 slug/ft^3
Speed of sound at the tropopause	a^*	295.069 m/s	968.076 ft/s 573.569 kt

The fundamental parameters of standard sea level values in Table 2.1 are sufficient enough to determine the pressure and density at any height in the ISA. The relative air density, σ, relative pressure, δ and relative temperature, θ are configured as ratios of the ISA sea-level datum parameters.

$$\sigma = \frac{\rho}{\rho_0} \tag{2.10}$$

$$\delta = \frac{p}{p_0} \tag{2.11}$$

$$\theta = \frac{T}{T_0} \tag{2.12}$$

Using Mach number, total temperature ratio is defined as

$$\theta_T = \theta(1 + 0.2kM^2) \tag{2.13}$$

as well as total pressure ratio

$$\delta_T = \delta(1 + 0.2M^2)^{3.5} \quad . \tag{2.14}$$

After establishing sea-level pressure and temperature based on the standard atmosphere, the fluctuation in pressure within any attitude within the atmosphere is expressed as

$$\frac{dp}{dh} = -\rho g \quad . \tag{2.15}$$

Equation (2.15) defines the hydrostatic equation, which articulates the linear increase rate in pressure with geometric altitude, influenced by the density and gravitational acceleration of the atmosphere. The limitations of (2.15) lie from the assumption of the negligible variation of the gravitational term remains constant within altitude, whereas in reality, the gravitational acceleration reduces with increasing height (Roberts 1995). By combining the hydrostatic equation with the ideal gas law, a new height scale of geopotential height, H is introduced

$$\frac{dp}{p} = -\frac{g}{RT}\, dH \quad . \tag{2.16}$$

Equation (2.16) provides a practical rendition of the hydrostatic equation by accounting for the change in local acceleration across atmosphere altitudes. Earth has five major layers of atmosphere: troposphere, stratosphere, mesosphere, thermosphere, and exosphere. In this work, the revised hydrostatic equation can be evaluated though integration, focusing on two primary regions: the troposphere and stratosphere, each distinguished by their respective arbitrary heights H.

2.2.1 In the Troposphere

During standard operational procedures, commercial jet aircraft operate the uppermost layer of the troposphere, ascending towards geopotential altitude nearing 36089 ft. As these aircraft ascend through the troposphere, a reduction of the air temperature occurs, adhering with the lapse rate L (Scholz 2022). Temperature readings at any given altitude are calculated via

$$T = T_0 + LH \tag{2.17}$$

and simplified as

$$\theta = 1 + \frac{LH}{T_0} \quad . \tag{2.18}$$

By incorporating (2.17) into the revised (2.16) and integrating from the ISA sea level-datum to geopotential height, the following equation is derived as

$$\ln\left(\frac{p}{p_0}\right) = \ln\left[\frac{T_0 + LH}{T_0}\right]^{-g/RL} \quad . \tag{2.19}$$

Thus, creating the relative pressure in the troposphere

$$\delta = \frac{p}{p_0} = \left[1 + \frac{LH}{T_0}\right]^{-g/RL} \quad . \tag{2.21}$$

Similarly using the ideal of gas law, the relative density simplified to

$$\sigma = \frac{\rho}{\rho_0} = \left(\frac{T}{T_0}\right)^{-\frac{g}{RL}-1} \quad . \tag{2.22}$$

Thus, creating the relative density in the troposphere

$$\sigma = \left[1 + \frac{LH}{T_0}\right]^{-\frac{g}{RL}-1} \quad . \tag{2.23}$$

2.2.2 In the Stratosphere

In the Stratosphere, ranging from altitudes of 36089 ft to 65617 ft, an isothermal assumption is applied to facilitate integration procedures. This method accounts for the conditions specified at the tropopause as detailed in Table 2.2.

$$\begin{aligned} \theta^* &= 0.751865 \\ \delta^* &= 0.223361 \\ \sigma^* &= 0.297076 \end{aligned} \tag{2.24}$$

In similar manner, (2.16) from the arbitrary height of the Tropopause, H yields relative pressure within the Stratosphere

$$\frac{\delta}{\delta^*} = \frac{p}{p^*} = e^{\frac{-g}{RT^*}(H-H^*)} \quad . \tag{2.25}$$

According to the ideal gas law for isothermal conditions, the relative density of the Stratosphere results to:

$$\frac{\sigma}{\sigma^*} = \frac{\delta}{\delta^*} = e^{\frac{-g}{RT^*}(H-H^*)} \quad . \tag{2.26}$$

Table 2.2 provides a summary of the equations derived for the ISA.

Table 2.2 Equations of ISA as function of height (Young 2018)

Relative temperature	**Relative pressure**	**Relative density**
In the troposphere:		
$\theta = 1 + \frac{L}{T_0}H$	$\delta = \left[1 + \frac{LH}{T_0}\right]^{-g_0/RL}$ or $\delta = \theta^{5.25588}$	$\sigma = \left[1 + \frac{LH}{T_0}\right]^{(-g_0/RL)-1}$ or $\sigma = \theta^{4.25588}$
At the tropopause:		
$\theta^* = 0.751865$	$\delta^* = 0.223361$	$\sigma^* = 0.297076$
In the stratosphere:		
$\theta = \theta^*$	$\delta = \delta^* e^{(-g_0/RT^*)(H-H^*)}$	$\sigma = \sigma^* e^{(-g_0/RT^*)(H-H^*)}$

2.3 Equivalent Airspeed

Under the conditions of the ISA, an aircraft will achieve an Equivalent Airspeed (EAS), V_E, corresponding to the same incompressible dynamic pressure it generates at its true airspeed, regardless of altitude. This is written as follows

$$\frac{1}{2}\rho_0 V_E^2 = \frac{1}{2}\rho V^2 \tag{2.27}$$

simplifying it to

$$V_E = \sqrt{\sigma}V \quad . \tag{2.28}$$

Given its significance in aircraft analysis, Equivalent Airspeed can be effectively correlated with both the aircraft's Mach number and static pressure

$$V_E = \sqrt{\delta} a_0 M \quad . \qquad (2.29)$$

2.4 Compressibility Correction

For subsonic flight operation, the compressible isentropic flow is generalized through the utilization of the airspeed indicator (ASI), integrated within the pitot system. This instrument accurately measures the differential between stagnation pressure and static pressure.

$$(p_{t-p})_{imcompressible} = \frac{1}{2}\rho V^2 f(M)$$

$$f(M) = [1 + \frac{M^2}{4} + \frac{M^4}{40} \dots] \qquad (2.30)$$

Equation (2.30) calculates the parameters of density and Mach number, crucial for computing true airspeed while factoring in their respective altitudes. However, trying to integrate the ASI to accommodate different airspeed scales for across different pressure altitudes will present a complex challenge to integrate into the pitot system. Thus, if the ASI is calibrated to standard sea level conditions, then a simple correction error can be introduced.

This method of utilizing the ASI aligns with Bernoulli's principle of compressible airflow

$$(\frac{p_t}{p})_{compressible} = (1 + \frac{\gamma - 1}{2} M^2)^{\frac{\gamma}{\gamma - 1}} \quad . \qquad (2.31)$$

Equation (2.31) can be refined to conform to the standard calibrated equation through the selection of ISA sea level values and designating the resulting velocity as the Calibrated Airspeed, V_C

$$(p_t - p) = p_0 \{[1 + 0.2 (\frac{V_C}{a_0})^2]^{3.5} - 1\} \quad . \qquad (2.32)$$

It is essential to acknowledge that CAS will not always be consistently align to EAS at every Mach number at sea level, as it depicted in (2.32). Thus, as altitude increases, a difference between CAS and EAS becomes evident

$$(p_t - p) = p\{[1 + 0.2M^2]^{3.5} - 1\} = p_0 \{[1 + 0.2 (\frac{V_C}{a_0})^2]^{3.5} - 1\} \qquad (2.33)$$

$$[1 + 0.2\,(\frac{V_C}{a_0})^2\,]^{3.5} \;=\; \delta\{[1 + 0.2M^2]^{3.5} - 1\} + 1 \;\; .$$

Resulting in the Calibrated Airspeed

$$V_C = a_0\sqrt{5\,\{[\delta\{(1 + 0.2M^2)^{3.5} - 1\} + 1]^{\frac{1}{3.5}} - 1\}} \tag{2.34}$$

and the Equivalent Airspeed

$$V_E = V\sqrt{\sigma} = Ma_0\sqrt{\delta} \;\; . \tag{2.35}$$

As the ASI incorporates the function f(M), airspeeds exceeding Mach 0.3 will result in the calibrated airspeed readings surpassing the equivalent airspeed due to the increased to pressure altitude. Consequently, the compressibility correction, ΔV_C, is expressed as

$$\Delta V_C = V_C - V_E$$

$$\Delta V_C = a_0\,\{\sqrt{5\,\{[\delta\{(1 + 0.2M^2)^{3.5} - 1\} + 1]^{\frac{1}{3.5}} - 1\}} - M\sqrt{\delta}.\} \;\; . \tag{2.36}$$

The expression of Mach number in terms of CAS can be conveyed as

$$M = \frac{V}{a} = \sqrt{5\,\{\,[\frac{1}{\delta}\{[1 + 0.2\,(\frac{V_C}{a_0})^2\,]^{3.5} - 1\} + 1]^{\frac{1}{3.5}} - 1\}} \;\; . \tag{2.37}$$

This linear correlation between Mach number and CAS facilitates the development of the relations between EAS and CAS

$$V_E = a_0\sqrt{5\delta\,\{\,[\frac{1}{\delta}\{[1 + 0.2\,(\frac{V_C}{a_0})^2\,]^{3.5} - 1\} + 1]^{\frac{1}{3.5}} - 1\}} \;\; . \tag{2.38}$$

3 Compressibility Correction Chart

To construct the Compressibility Correction Chart, it is essential to develop two separate plots within Excel, each dedicated to a specific parameter: one plot featuring Mach Number and the other with pressure altitude. By combining these plots, the various curves generate the Compressibility Chart.

3.1 Mach Number as a Parameter

In producing the Compressibility Correction Chart based on Mach number, it is necessary to utilize (2.36), as it provides the compressibility correction, ΔV_C, in terms of both Mach number, M, and relative pressure, δ.

Relative pressure computations are contingent upon the atmospheric conditions experienced by the aircraft. At altitudes below 36084 ft, the troposphere relative pressure equation is applied, while altitude surpassing this threshold employ the stratosphere equation, as detailed in Table 2.2. Importantly, relative pressure values remain consistent across all Mach number parameters.

Once relative pressure calculations are completed, along with the Mach number values, (2.36) is used to determine the compressibility correction, thereby yielding the y-values for the plot.

Subsequently, (2.36) is utilized to calculate the calibrated airspeed, with the equivalent airspeed expressed in terms of Mach number as in (2.35). These steps provide the x-axis values for the plot. An example of the calculation is demonstrated.

Mach number, $M = 0.60$
Altitude, $H = 40000$ ft
All standard values of the ISA are given in Table 2.1:

$$\delta = \delta^{*} e^{-k_b \cdot (H - H_T)}$$

since H is higher than 36084 ft

$$\delta = 0.185$$

$$\Delta V_C = a_0 \left\{ \sqrt{5\{[\delta\{(1+0.2M^2)^{3.5} - 1\} + 1]^{\frac{1}{3.5}} - 1\}} - M\sqrt{\delta}. \right\}$$

$$\Delta V_C = (661.48)\{\sqrt{5\{[(0.185)\{(1+0.2(0.60)^2)^{3.5}-1\}+1]^{\frac{1}{3.5}}-1\}}$$

$$- 0.60\sqrt{0.185}\}.$$

$$= 6.196 \text{ kt}$$

$$\Delta V_C = V_C - Ma_0\sqrt{\delta}$$

$$V_C = 176.904 \text{ kt}$$

Please be advised that the calculations were manually conducted using the equations from (Scholz 2019), with consideration for values up to the thousandth decimal place. However, the Excel version provides enhanced accuracy, as it comprehensively accounts for all decimal points in the computation process.

It is crucial to acknowledge that the compressibility correction fluctuates with each Mach number, resulting in distinct calibrated airspeed values. Hence, it is essential to systematically iterate the process for each of Mach number variable for 0 to 1, with increments of 0.05, as illustrated in the Excels spreadsheet.

3.2 Pressure Altitude as Parameter

In producing the Compressibility Correction Chart with pressure altitude as a parameter, (2.38) is essential, as it establishes the relationship between EAS and CAS. Through (2.38), the values of the equivalent airspeed are calculated based on calibrated airspeed and relative pressure. Consequently, by (2.36), the compressibility correction is calculated, thereby determining in the y-values of this plot.

In this Excel spreadsheet, calibrated airspeed serves as the input variable ranging from 0 kt to 670 kt, representing the x-values of the plot. The computation of relative pressure follows the same principles outlined in Section 3.1.

An example of the calculation is demonstrated:

H = 20000 ft
V_C = 174 kt
All standard values of the ISA are given in Table 2.1

$$\delta = \frac{p}{p_0} = (1 - k_a \cdot H)^{5.25588}$$

since H is less than 36084 ft

$$\delta = 0.459$$

$$V_E = a_0\sqrt{5\left\{\left[\delta\left\{(1 + 0.2M^2)^{3.5} - 1\right\} + 1\right]^{\frac{1}{3.5}} - 1\right\}}$$

$$= 172.311 \text{ kt}$$

$$\Delta V_C = V_E - V_C$$

$$\Delta V_C = 1.69 \text{ kt} \ .$$

Please be advised that the calculations were manually conducted using the equations from (Scholz 2019), with consideration for values up to the thousandth decimal place. However, the Excel version provides enhanced accuracy, as it comprehensively accounts for all decimal points in the computation process.

It is crucial to emphasize that the calculation of relative pressure is contingent upon the pressure altitude, which spans from 1000 ft to 65000 ft in increments of 1000 ft. Notably, for each altitude, the calculated equivalent airspeed values, and consequently, compressibility correction values will vary, as outlined in the Excel sheet.

3.3 Production of the Chart

Both charts generated from Mach number and Pressure altitude as parameters follow the same consistent structure: the x-axis represents calibrated airspeed values ranging from 100 kt to 540 kt in increments of 20 kt, while y-values spans the Compressibility Correction values from 0 kt to 32 kt in increments of 2 kt. In the Mach number parameter chart, red lines are utilized, while blue lines denote pressure altitude. To enhance visual clutter, only Mach numbers from 0.60 and onwards are displayed.

Combing both charts in one single plot yields the Compressibility Correction Chart illustrated in Figure 4.1.

4 Users Guide for Compressibility Correction Chart

Each chart within the Excel spreadsheets maintains consistent axis parameters: Calibrated airspeed (x-axis) spans from 100 to 540 kt in increments of 20, while Compressibility Correction (y-axis) values range from 0 to 32 kt in increments of 2. The input values for Pressure altitude and Mach number can be adjusted according to the user's preference, although such adjustments may necessitate modifications to the Compressibility Correction Chart produced in Tab 1. The same follows the Pressure altitude and Calibrated airspeed input values and resulting chart within Tab 2. The operational functionality of the tabs is detailed in this chapter.

It is important to understand that Excel has a data series limitation of 255. Considering these limitations the following structure was made:

In simplifying the plotting process for Tab 2, Mach numbers M are iterated in increments of 0.05, with Altitude H represented in 1000 ft intervals. In contrast to other plots detailed in Chapter 5, where Mach number iterations are as fine as 0.001, this would result in over 1000 data series, exceeding Excel's capacity. While both versions yield the exact same compressibility chart, having more data series could potentially enhance the creation of an interactive chart, allowing users to hover over parameters as Mach values with precision down to 0.001 intervals. However, our objective in this work is to recreate the Compressibility Correction Chart in Excel without resorting to interactive modes, hence we opt for the simpler approach outlined in this chapter.

4.1 Tab 1: Compressibility Correction Chart

This tab represents the culminations of our findings, presenting the Compressibility Correction Chart from CAS to EAS. Within this chart, the effects of compressibility correction as functions of Mach number (illustrated by red curves) and pressure altitude (depicted as the blue curves) are displayed. To facilitate readability, labels were incorporated corresponding to the values of pressure altitude and Mach number. It serves as near perfect adaptations of the CAS-EAS Compressibility Correction Chart outlined in the Flight Mechanics script by Trevor Young. Figure 4.1 illustrates the Compressibility Correction Chart.

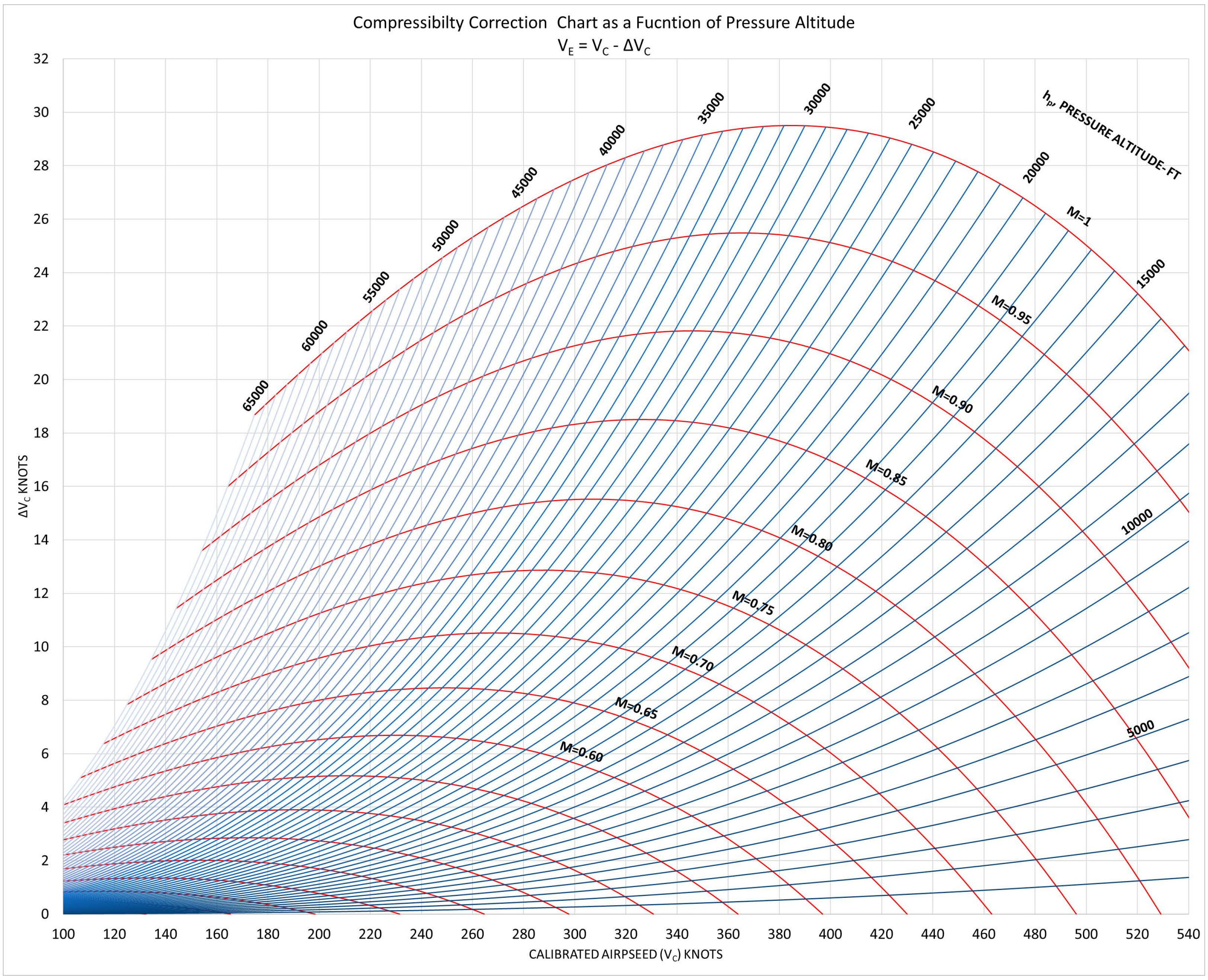
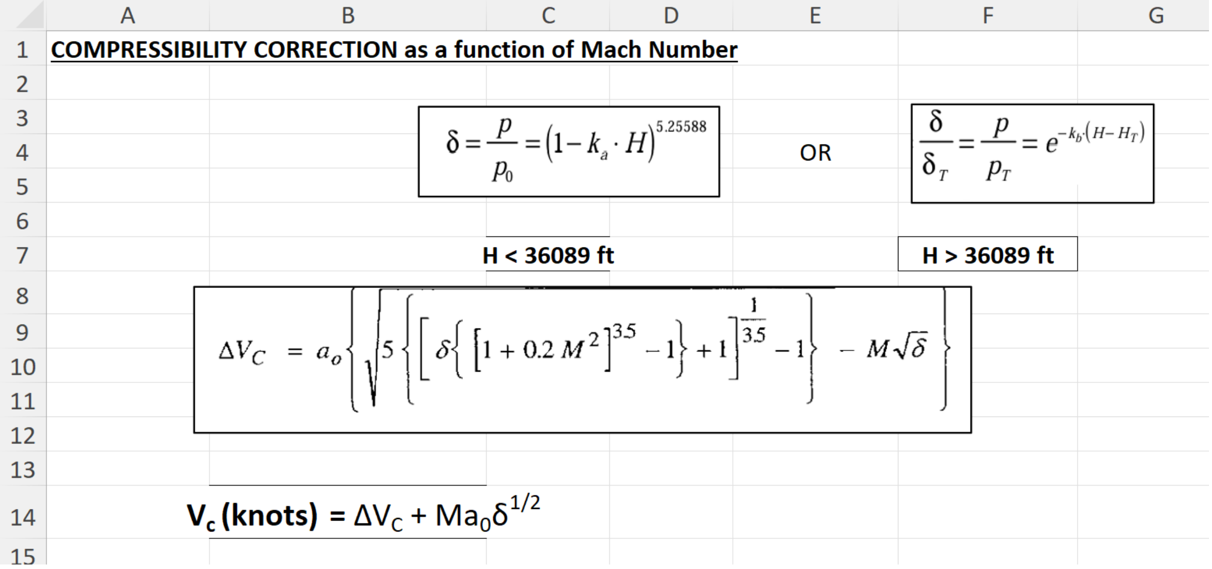


Figure 4.1 CAS to EAS Compressibility Correction Chart- $V_E = V_C - \Delta V_C$

Interpreting the chart is straightforward, requiring only three parameters from: Mach number, calibrated airspeed, pressure altitude, or Compressibility Correction. By estimating the positions of these parameters, all relevant factors can be calculated. While the chart is a 2-D representation of the Compressibility Correction Chart, its potential for interactivity is highly recommended for future work.

4.2 Tab 2: Compressibility Correction, Function of Mach Number

Building upon the principles discussed in Section 3.1, Tab 2 outlines the required equations for deriving the Compressibility Correction as a function of Mach number.



COMPRESSIBILITY CORRECTION as a function of Mach Number

$$\delta = \frac{p}{p_0} = (1 - k_a \cdot H)^{5.25588}$$

OR

$$\frac{\delta}{\delta_T} = \frac{p}{p_T} = e^{-k_b (H - H_T)}$$

H < 36089 ft

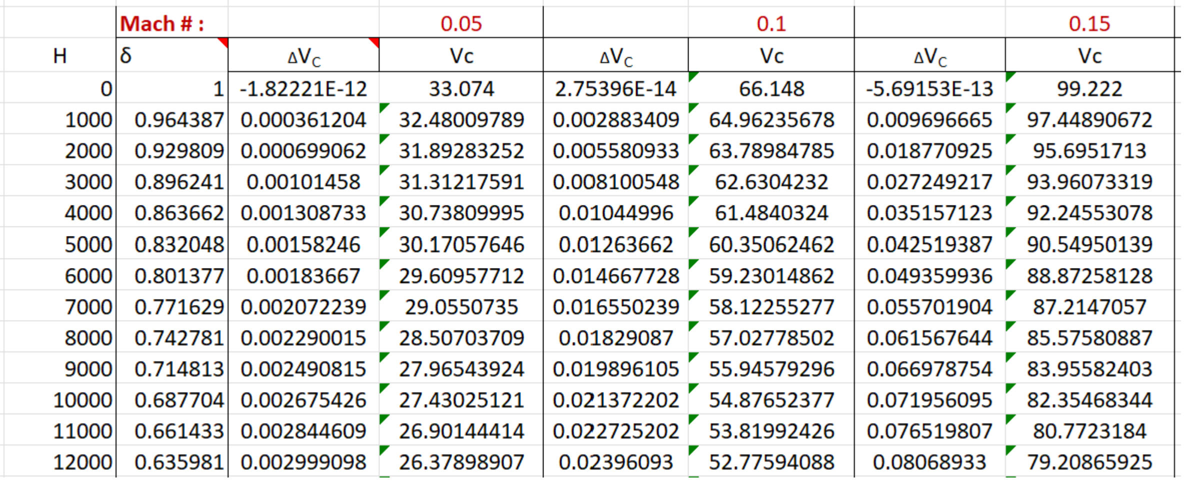
H > 36089 ft

$$\Delta V_C = a_o \left\{ \sqrt{5 \left\{ \left[\delta \left\{ \left[1 + 0.2 M^2 \right]^{3.5} - 1 \right\} + 1 \right]^{\frac{1}{3.5}} - 1 \right\}} - M\sqrt{\delta} \right\}$$

$$\mathbf{V_c\,(knots) = \Delta V_C + M a_0 \delta^{1/2}}$$

Figure 4.2 Equations necessary for Tab 2

Input parameters include Pressure altitude *H*, ranges from sea level to 65000 ft with increments of 1000 ft, and Mach number *M*, emphasized in red font, ranging from 0.05 to 1.00 in increments of 0.05.



	Mach # :		0.05		0.1		0.15
H	δ	ΔV_C	Vc	ΔV_C	Vc	ΔV_C	Vc
0	1	-1.82221E-12	33.074	2.75396E-14	66.148	-5.69153E-13	99.222
1000	0.964387	0.000361204	32.48009789	0.002883409	64.96235678	0.009696665	97.44890672
2000	0.929809	0.000699062	31.89283252	0.005580933	63.78984785	0.018770925	95.6951713
3000	0.896241	0.00101458	31.31217591	0.008100548	62.6304232	0.027249217	93.96073319
4000	0.863662	0.001308733	30.73809995	0.01044996	61.4840324	0.035157123	92.24553078
5000	0.832048	0.00158246	30.17057646	0.01263662	60.35062462	0.042519387	90.54950139
6000	0.801377	0.00183667	29.60957712	0.014667728	59.23014862	0.049359936	88.87258128
7000	0.771629	0.002072239	29.0550735	0.016550239	58.12255277	0.055701904	87.2147057
8000	0.742781	0.002290015	28.50703709	0.01829087	57.02778502	0.061567644	85.57580887
9000	0.714813	0.002490815	27.96543924	0.019896105	55.94579296	0.066978754	83.95582403
10000	0.687704	0.002675426	27.43025121	0.021372202	54.87652377	0.071956095	82.35468344
11000	0.661433	0.002844609	26.90144414	0.022725202	53.81992426	0.076519807	80.7723184
12000	0.635981	0.002999098	26.37898907	0.02396093	52.77594088	0.08068933	79.20865925

Figure 4.3 Format of Tab 2

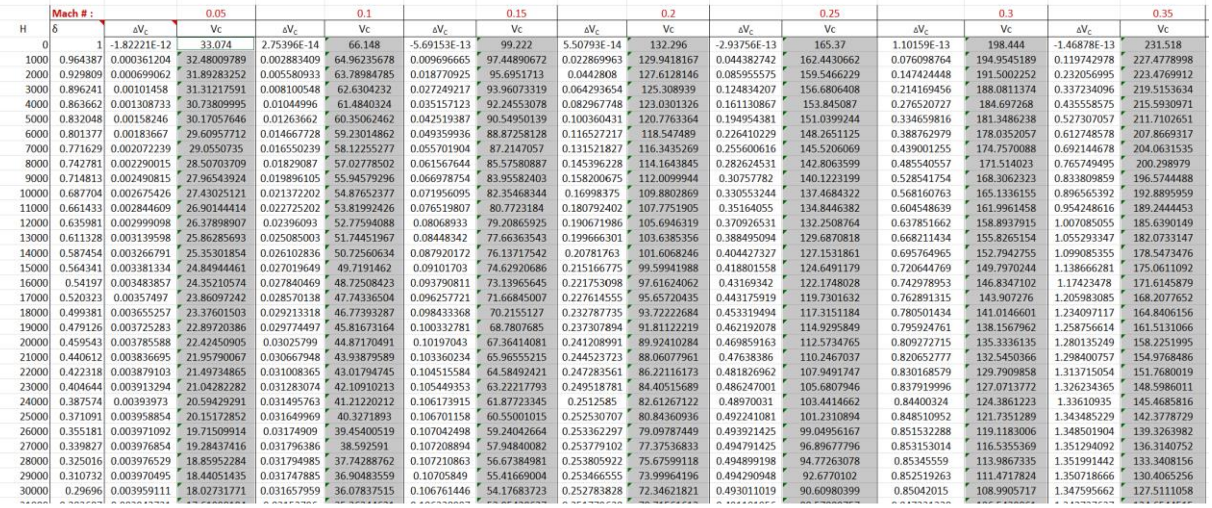
For the determination of relative pressure *δ*, the corresponding excel cell formula is incorporated with an *IF ()* function, a sample from *H* = 1000 ft is generalized as:

=IF(C19<36089, (1-k_a__ft*$C19)^5.25588,0.223361*EXP(-k_b_ft*($C19-H_T__ft)))

This syntax specifies that IF (the value of *H* in cells C19(-C84) is less than 36,089, then calculate using the Troposphere relative pressure equation; otherwise, use the Stratosphere Equation.

Following the calculation of relative pressure, the compressibility correction (ΔV_C) and calibrated airspeed (V_C) can be determined. Thereby, the process can be repeated for each of Mach number variable for 0.05 to 1.00, with increments of 0.05, as illustrated in the Excels spreadsheet in Figure 4.3.

To plot the Calibrated Airspeed and Compressibility Correction Values, begin by ensuring to only select the entire table starting from the corresponding values of M = 0.5 to M = 1.00 Exclude any variables as well as the pressure altitude (H) and relative pressure column (δ). A VBA macro, crafted by Alexander Trifuntov and detailed in Appendix A, is embedded within the worksheet. This macro functions to isolate every alternate even column within the selected range, thereby creating a new range comprising only the V_C values, as demonstrated briefly in Figure 4.4. These isolated values serve as the x- axis values.



	Mach # :		0.05		0.1		0.15		0.2		0.25		0.3		0.35
H	δ	ΔVc	Vc	ΔVc	Vc	ΔVc	Vc	ΔVc	Vc	ΔVc	Vc	ΔVc	Vc	ΔVc	Vc
0	1	-1.82221E-12	33.074	2.75396E-14	66.148	-5.69153E-13	99.222	5.50793E-14	132.296	-2.93756E-13	165.37	1.10159E-13	198.444	-1.46878E-13	231.518
1000	0.964387	0.000361204	32.48009789	0.002883409	64.96235678	0.009696665	97.44890672	0.022869963	129.9418167	0.044382742	162.4430662	0.076098764	194.9545189	0.119742978	227.4778998
2000	0.929809	0.000699062	31.89283252	0.005580933	63.78984785	0.018770925	95.6951713	0.0442808	127.6128146	0.085955575	159.5466229	0.147424448	191.5002252	0.232056995	223.4769912
3000	0.896241	0.00101458	31.31217591	0.008100548	62.6304232	0.027249217	93.96073319	0.064293654	125.308939	0.124834207	156.6806408	0.214169456	188.0811374	0.337234096	219.5153634
4000	0.863662	0.001308733	30.73809995	0.01044996	61.4840324	0.035157123	92.24553078	0.082967748	123.0301326	0.161130867	153.845087	0.276520727	184.697268	0.435558575	215.5930971
5000	0.832048	0.00158246	30.17057646	0.01263662	60.35062462	0.042519387	90.54950139	0.100360431	120.7763364	0.194954381	151.0399244	0.334659816	181.3486238	0.527307057	211.7102651
6000	0.801377	0.00183667	29.60957712	0.014667728	59.23014862	0.049359936	88.87258128	0.116527217	118.547489	0.226410229	148.2651125	0.388762979	178.0352057	0.612748578	207.8669317
7000	0.771629	0.002072239	29.0550735	0.016550239	58.12255277	0.055701904	87.2147057	0.131521827	116.3435269	0.255600616	145.5206069	0.439001255	174.7570088	0.692144678	204.0631535
8000	0.742781	0.002290015	28.50703709	0.01829087	57.02778502	0.061567644	85.57580887	0.145396228	114.1643845	0.282624531	142.8063599	0.485540557	171.514023	0.765749495	200.298979
9000	0.714813	0.002490815	27.96543924	0.019896105	55.94579296	0.066978754	83.95582403	0.158200675	112.0099944	0.30757782	140.1223199	0.528541754	168.3062323	0.833809859	196.5744488
10000	0.687704	0.002675426	27.43025121	0.021372202	54.87652377	0.071956095	82.35468344	0.16998375	109.8802869	0.330553244	137.4684322	0.568160763	165.1336155	0.896565392	192.8895959
11000	0.661433	0.002844609	26.90144414	0.022725202	53.81992426	0.076519807	80.7723184	0.180792402	107.7751905	0.35164055	134.8446382	0.604548639	161.9961458	0.954248616	189.2444453
12000	0.635981	0.002999098	26.37898907	0.02396093	52.77594088	0.08068933	79.20865925	0.190671986	105.6946319	0.370926531	132.2508764	0.637851662	158.8937915	1.007085055	185.6390149
13000	0.611328	0.003139598	25.86285693	0.025085003	51.74451967	0.08448342	77.66363543	0.199666301	103.6385356	0.388495094	129.6870818	0.668211434	155.8265154	1.055293347	182.0733147
14000	0.587454	0.003266791	25.35301854	0.026102836	50.72560634	0.087920172	76.13717542	0.20781763	101.6068246	0.404427327	127.1531861	0.695764965	152.7942755	1.099085355	178.5473476
15000	0.564341	0.003381334	24.84944461	0.027019649	49.7191462	0.09101703	74.62920686	0.215166775	99.59941988	0.418801558	124.6491179	0.720644769	149.7970244	1.138666281	175.0611092
16000	0.54197	0.003483857	24.35210574	0.027840469	48.72508423	0.093790811	73.13965645	0.221753098	97.61624062	0.43169342	122.1748028	0.742978953	146.8347102	1.17423478	171.6145879
17000	0.520323	0.00357497	23.86097242	0.028570138	47.74336504	0.096257721	71.66845007	0.227614555	95.65720435	0.443175919	119.7301632	0.762891315	143.907276	1.205983085	168.2077652
18000	0.499381	0.003655257	23.37601503	0.029213318	46.77393287	0.098433368	70.2155127	0.232787735	93.72222684	0.453319494	117.3151184	0.780501434	141.0146601	1.234097117	164.8406156
19000	0.479126	0.003725283	22.89720386	0.029774497	45.81673164	0.100332781	68.7807685	0.237307894	91.81122219	0.462192078	114.9295849	0.795924761	138.1567962	1.258756614	161.5131066
20000	0.459543	0.003785588	22.42450905	0.03025799	44.87170491	0.10197043	67.36414081	0.241208991	89.92410284	0.469859163	112.5734765	0.809272715	135.3336135	1.280135249	158.2251995
21000	0.440612	0.003836695	21.95790067	0.030667948	43.93879589	0.103360234	65.96555215	0.244523723	88.06077961	0.47638386	110.2467037	0.820652777	132.5450366	1.298400757	154.9768486
22000	0.422318	0.003879103	21.49734865	0.031008365	43.01794745	0.104515584	64.58492421	0.247283561	86.22116173	0.481826962	107.9491747	0.830168579	129.7909858	1.313715054	151.7680019
23000	0.404644	0.003913294	21.04282282	0.031283074	42.10910213	0.105449353	63.22217793	0.249518781	84.40515689	0.486247001	105.6807946	0.837919996	127.0713772	1.326234365	148.5986011
24000	0.387574	0.00393973	20.59429291	0.031495763	41.21220212	0.106173915	61.87723345	0.2512585	82.61267122	0.48970031	103.4414662	0.84400324	124.3861223	1.33610935	145.4685816
25000	0.371091	0.003958854	20.15172852	0.031649969	40.3271893	0.106701158	60.55001015	0.252530707	80.84360936	0.492241081	101.2310894	0.848510952	121.7351289	1.343485229	142.3778729
26000	0.355181	0.003971092	19.71509914	0.03174909	39.45400519	0.107042498	59.24042664	0.253362297	79.09787449	0.493921425	99.04956167	0.851532288	119.1183006	1.348501904	139.3263982
27000	0.339827	0.003976854	19.28437416	0.031796386	38.592591	0.107208894	57.94840082	0.253779102	77.37536833	0.494791425	96.89677796	0.853153014	116.5355369	1.351294092	136.3140752
28000	0.325016	0.003976529	18.85952284	0.031794985	37.74288762	0.107210863	56.67384981	0.253805922	75.67599118	0.494899198	94.77263078	0.85345559	113.9867335	1.351991442	133.3408156
29000	0.310732	0.003970495	18.44051435	0.031747885	36.90483559	0.10705849	55.41669004	0.253466555	73.99964196	0.494290948	92.6770102	0.852519263	111.4717824	1.350718666	130.4065256
30000	0.29696	0.003959111	18.02731771	0.031657959	36.07837515	0.106761446	54.17683723	0.252783828	72.34621821	0.493011019	90.60980399	0.85042015	108.9905717	1.347595662	127.5111058

Figure 4.4: ΔV_C values selected using the VBA Macro

The user will proceed by manually selecting each Compressibility Correction (ΔV_C) column corresponding to its Mach number, while holding the CTRL button to maintain the pre-selected x-values. These Compressibility Correction data points are the y-values of the plot. By opting for a scatter plot, it seamlessly generates the Compressibility Correction Chart as a function of M, from 0.05 to 1.00 (Figure 4.5).

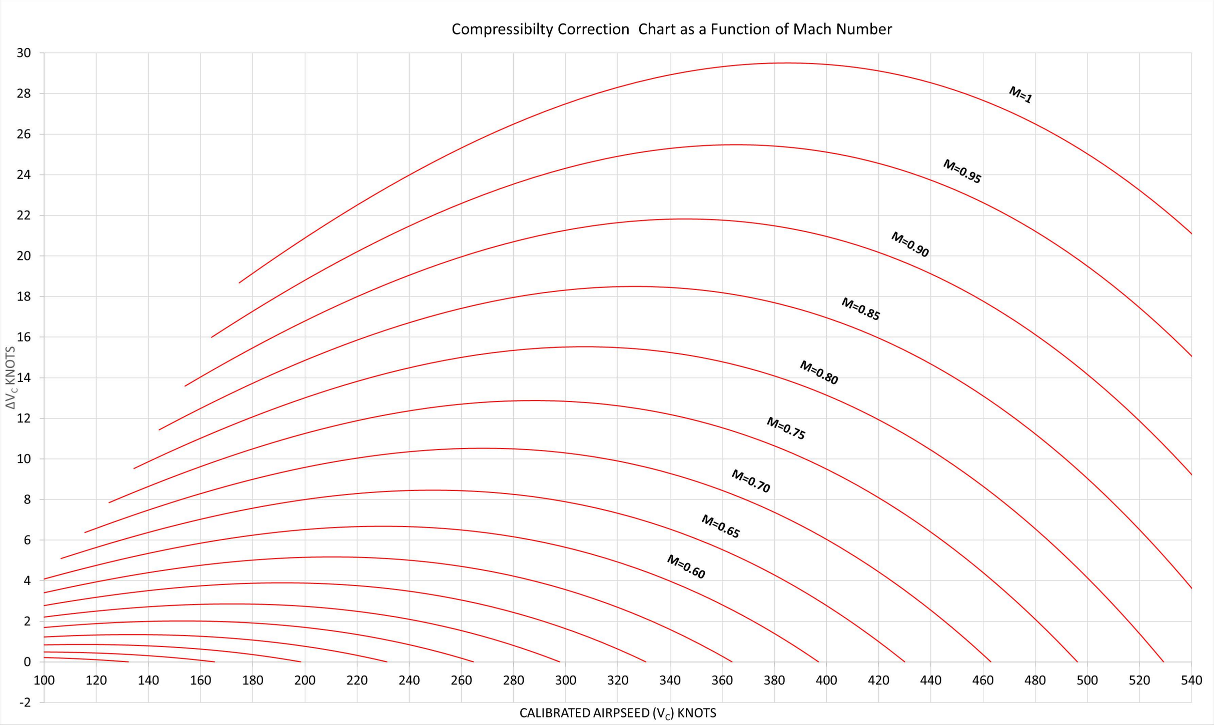
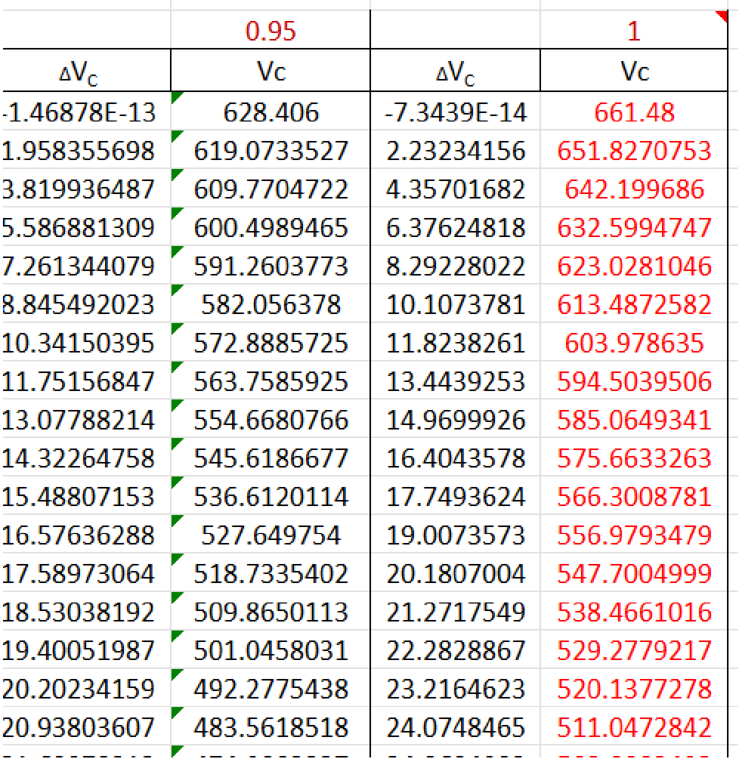


Figure 4.5 Compressibility Correction Chart as a function of Mach mumber- $V_E = V_C - \Delta V_C$



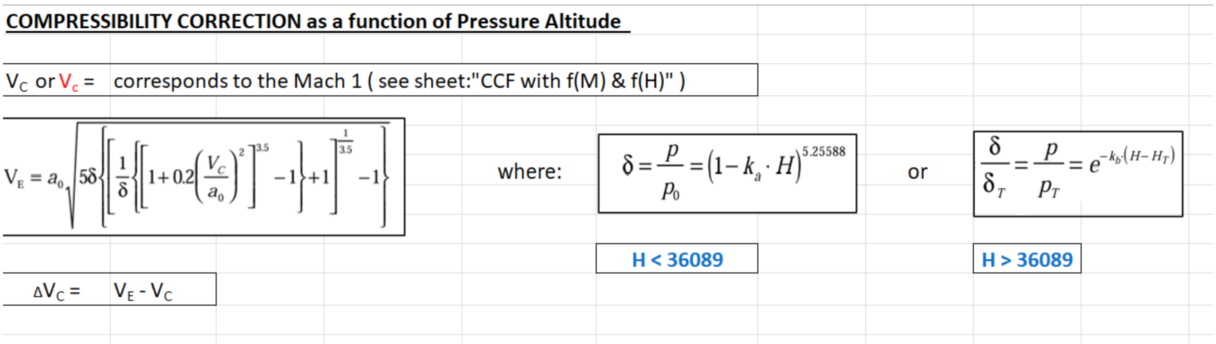
	0.95		1
ΔV$_C$	Vc	ΔV$_C$	Vc
-1.46878E-13	628.406	-7.3439E-14	661.48
1.958355698	619.0733527	2.23234156	651.8270753
3.819936487	609.7704722	4.35701682	642.199686
5.586881309	600.4989465	6.37624818	632.5994747
7.261344079	591.2603773	8.29228022	623.0281046
8.845492023	582.056378	10.1073781	613.4872582
10.34150395	572.8885725	11.8238261	603.978635
11.75156847	563.7585925	13.4439253	594.5039506
13.07788214	554.6680766	14.9699926	585.0649341
14.32264758	545.6186677	16.4043578	575.6633263
15.48807153	536.6120114	17.7493624	566.3008781
16.57636288	527.649754	19.0073573	556.9793479
17.58973064	518.7335402	20.1807004	547.7004999
18.53038192	509.8650113	21.2717549	538.4661016
19.40051987	501.0458031	22.2828867	529.2779217
20.20234159	492.2775438	23.2164623	520.1377278
20.93803607	483.5618518	24.0748465	511.0472842

Figure 4.6 Mach 1 V_C values essential for Tab 3

It is imperative to recognize that for $M = 1$, the calibrated airspeed values are highlighted in red font as shown in Figure 4.5. These values hold significant importance as they constitute a vital component of the Calibrated Airspeed parameters for Tab 3. As demonstrated in the next section, these values will help produce the ΔV_C values, which aligns with the final $M = 1$ curve and does not intersect it. With some adjustments to the data series, the pressure altitudes curves can be modified to avoid intersecting the Mach 1 curve. It is essential to remember that intersecting Mach 1 results in supersonic airspeeds, which are beyond the scope of this thesis.

4.3 Tab 3: Compressibility Correction, Function of Pressure Altitude

Building upon the foundational concepts outlined in Section 3.2, Tab 3 integrates the same equations and principles (Figure 4.7).



COMPRESSIBILITY CORRECTION as a function of Pressure Altitude

V_C or V_c = corresponds to the Mach 1 (see sheet:"CCF with f(M) & f(H)")

$$V_E = a_0\sqrt{5\delta\left\{\left[\frac{1}{\delta}\left\{\left[1+0.2\left(\frac{V_C}{a_0}\right)^2\right]^{3.5}-1\right\}+1\right]^{\frac{1}{3.5}}-1\right\}}$$

where:

$$\delta=\frac{p}{p_0}=\left(1-k_a\cdot H\right)^{5.25588}$$

H < 36089

or

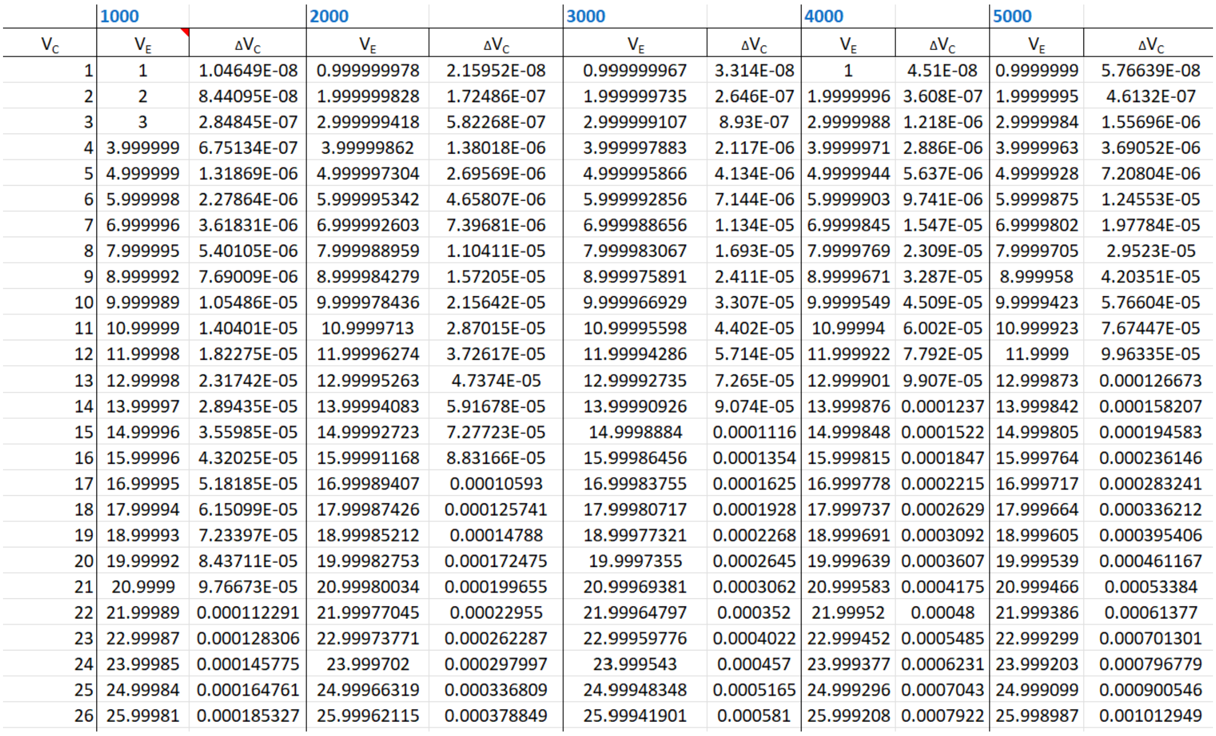
$$\frac{\delta}{\delta_T}=\frac{p}{p_T}=e^{-k_b\cdot\left(H-H_T\right)}$$

H > 36089

$\Delta V_C = V_E - V_C$

Figure 4.7 Equations necessary for Tab 3

Calibrated airspeed (V_C) and Pressure Altitude (H), distinguished in blue font, serve as the input parameters. It is worth nothing that the calibrated airspeed values in red font correspond to the constraints of the Mach = 1.00 curve, a topic furthered elaborated upon this section. Calibrated airspeed values range from 0 to 670 kt in increments of 1, while pressure altitude ranges from 1000 ft to 65000 ft in increments of 1000 and Calibrated Airspeed values in increments of 1 kt. An example of this is shown in Figure 4.8.



	1000		2000		3000		4000		5000	
V_C	V_E	ΔV_C	V_E	ΔV_C	V_E	ΔV_C	V_E	ΔV_C	V_E	ΔV_C
1	1	1.04649E-08	0.999999978	2.15952E-08	0.999999967	3.314E-08	1	4.51E-08	0.9999999	5.76639E-08
2	2	8.44095E-08	1.999999828	1.72486E-07	1.999999735	2.646E-07	1.9999996	3.608E-07	1.9999995	4.6132E-07
3	3	2.84845E-07	2.999999418	5.82268E-07	2.999999107	8.93E-07	2.9999988	1.218E-06	2.9999984	1.55696E-06
4	3.999999	6.75134E-07	3.99999862	1.38018E-06	3.999997883	2.117E-06	3.9999971	2.886E-06	3.9999963	3.69052E-06
5	4.999999	1.31869E-06	4.999997304	2.69569E-06	4.999995866	4.134E-06	4.9999944	5.637E-06	4.9999928	7.20804E-06
6	5.999998	2.27864E-06	5.999995342	4.65807E-06	5.999992856	7.144E-06	5.9999903	9.741E-06	5.9999875	1.24553E-05
7	6.999996	3.61831E-06	6.999992603	7.39681E-06	6.999988656	1.134E-05	6.9999845	1.547E-05	6.9999802	1.97784E-05
8	7.999995	5.40105E-06	7.999988959	1.10411E-05	7.999983067	1.693E-05	7.9999769	2.309E-05	7.9999705	2.9523E-05
9	8.999992	7.69009E-06	8.999984279	1.57205E-05	8.999975891	2.411E-05	8.9999671	3.287E-05	8.999958	4.20351E-05
10	9.999989	1.05486E-05	9.999978436	2.15642E-05	9.999966929	3.307E-05	9.9999549	4.509E-05	9.9999423	5.76604E-05
11	10.99999	1.40401E-05	10.9999713	2.87015E-05	10.99995598	4.402E-05	10.99994	6.002E-05	10.999923	7.67447E-05
12	11.99998	1.82275E-05	11.99996274	3.72617E-05	11.99994286	5.714E-05	11.999922	7.792E-05	11.9999	9.96335E-05
13	12.99998	2.31742E-05	12.99995263	4.7374E-05	12.99992735	7.265E-05	12.999901	9.907E-05	12.999873	0.000126673
14	13.99997	2.89435E-05	13.99994083	5.91678E-05	13.99990926	9.074E-05	13.999876	0.0001237	13.999842	0.000158207
15	14.99996	3.55985E-05	14.99992723	7.27723E-05	14.9998884	0.0001116	14.999848	0.0001522	14.999805	0.000194583
16	15.99996	4.32025E-05	15.99991168	8.83166E-05	15.99986456	0.0001354	15.999815	0.0001847	15.999764	0.000236146
17	16.99995	5.18185E-05	16.99989407	0.00010593	16.99983755	0.0001625	16.999778	0.0002215	16.999717	0.000283241
18	17.99994	6.15099E-05	17.99987426	0.000125741	17.99980717	0.0001928	17.999737	0.0002629	17.999664	0.000336212
19	18.99993	7.23397E-05	18.99985212	0.00014788	18.99977321	0.0002268	18.999691	0.0003092	18.999605	0.000395406
20	19.99992	8.43711E-05	19.99982753	0.000172475	19.9997355	0.0002645	19.999639	0.0003607	19.999539	0.000461167
21	20.9999	9.76673E-05	20.99980034	0.000199655	20.99969381	0.0003062	20.999583	0.0004175	20.999466	0.00053384
22	21.99989	0.000112291	21.99977045	0.00022955	21.99964797	0.000352	21.99952	0.00048	21.999386	0.00061377
23	22.99987	0.000128306	22.99973771	0.000262287	22.99959776	0.0004022	22.999452	0.0005485	22.999299	0.000701301
24	23.99985	0.000145775	23.999702	0.000297997	23.999543	0.000457	23.999377	0.0006231	23.999203	0.000796779
25	24.99984	0.000164761	24.99966319	0.000336809	24.99948348	0.0005165	24.999296	0.0007043	24.999099	0.000900546
26	25.99981	0.000185327	25.99962115	0.000378849	25.99941901	0.000581	25.999208	0.0007922	25.998987	0.001012949

Figure 4.8 Format of Tab 3

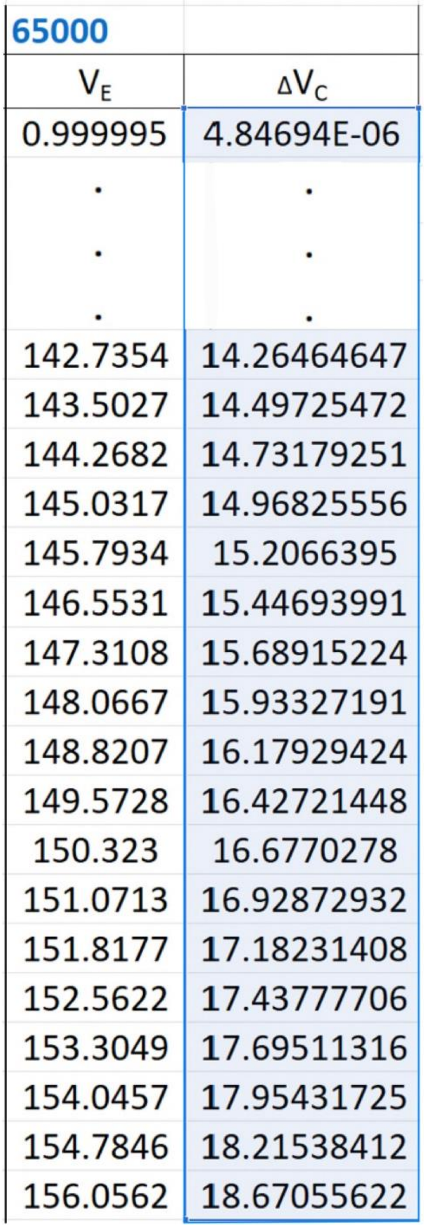
The calibrated airspeed values are the x-axis values for the plot, ensuring consistency throughout the calculation of Equivalent Airspeed (V_E) and Compressibility Correction (ΔV_C) for every indicated pressure altitude level.

The procedure of calculating relative pressure mirrors the IF () syntax used in Tab 2 . However, in Tab 3, this calculation is integrated within the formula of the Equivalent Airspeed cells that is incorporated within the Equivalent Airspeed cell columns. Thus, an example from D16 cell demonstrates the process as follows:

```
=IF(D$13<36089,a_0_kt*SQRT(5*(1-6.8756*10^(-6)*D$13)^(5.25588)*((1/(1-6.8756*10^(-6)*D$13)^(5.25588)*((1+0.2*($C16/a_0_kt)^2)^3.5-1)+1)^(1/3.5)-1)),a_0_kt*SQRT(5*0.223361*EXP(-k_b_ft*(D$13-H_T__ft))*((1/(0.223361*EXP(-k_b_ft*(D$13-H_T__ft)))*((1+0.2*($C16/a_0_kt)^2)^3.5-1)+1)^(1/3.5)-1)))
```

Subsequently, ΔV_C values can be calculated with regard to the arbitrary pressure altitude, constituting the y-axis values of the plot.

To plot the Calibrated airspeed and Compressibility Correction Values, begin by selecting the entire table starting from the first data of V_E and ΔV_C from H = 1000 ft to H = 650000 ft. Utilize the VBA macro to isolate the ΔV_C values. Simultaneously, while holding down CTRL key, select the V_C values in column C and generate a scatter plot. At the moment, if the Compressibility Correction chart as a function of Mach number from Tab 2 were plotted into the recently created chart in Tab 3, all the pressure altitude curves would interest the $M = 1.00$ curve.



65000	
V_E	ΔV_C
0.999995	4.84694E-06
.	.
.	.
.	.
142.7354	14.26464647
143.5027	14.49725472
144.2682	14.73179251
145.0317	14.96825556
145.7934	15.2066395
146.5531	15.44693991
147.3108	15.68915224
148.0667	15.93327191
148.8207	16.17929424
149.5728	16.42721448
150.323	16.6770278
151.0713	16.92872932
151.8177	17.18231408
152.5622	17.43777706
153.3049	17.69511316
154.0457	17.95431725
154.7846	18.21538412
156.0562	18.67055622

Figure 4.9 ΔV_C Series selected for Plotting

Given that the correlated calibrated airspeed values, highlighted in red font, correspond to Mach 1, and were utilized for the calculations in this tab, users are only required to adjust the constraints of the compressibility correction data, ΔV_C , for each distinct altitude in the plot series. An example is illustrated in Figure 4.9, where, for $H = 65000$ ft, the y-values depicted in the plot are restricted to the corresponding Compressibility Correction value of, $\Delta V_C = 18.67056$ kt, which is obtained from the calculated $V_C = 174.7268$ kt from Tab 2. Thus, generating the Compressibility Chart as a function of pressure altitude in Figure 4.10.

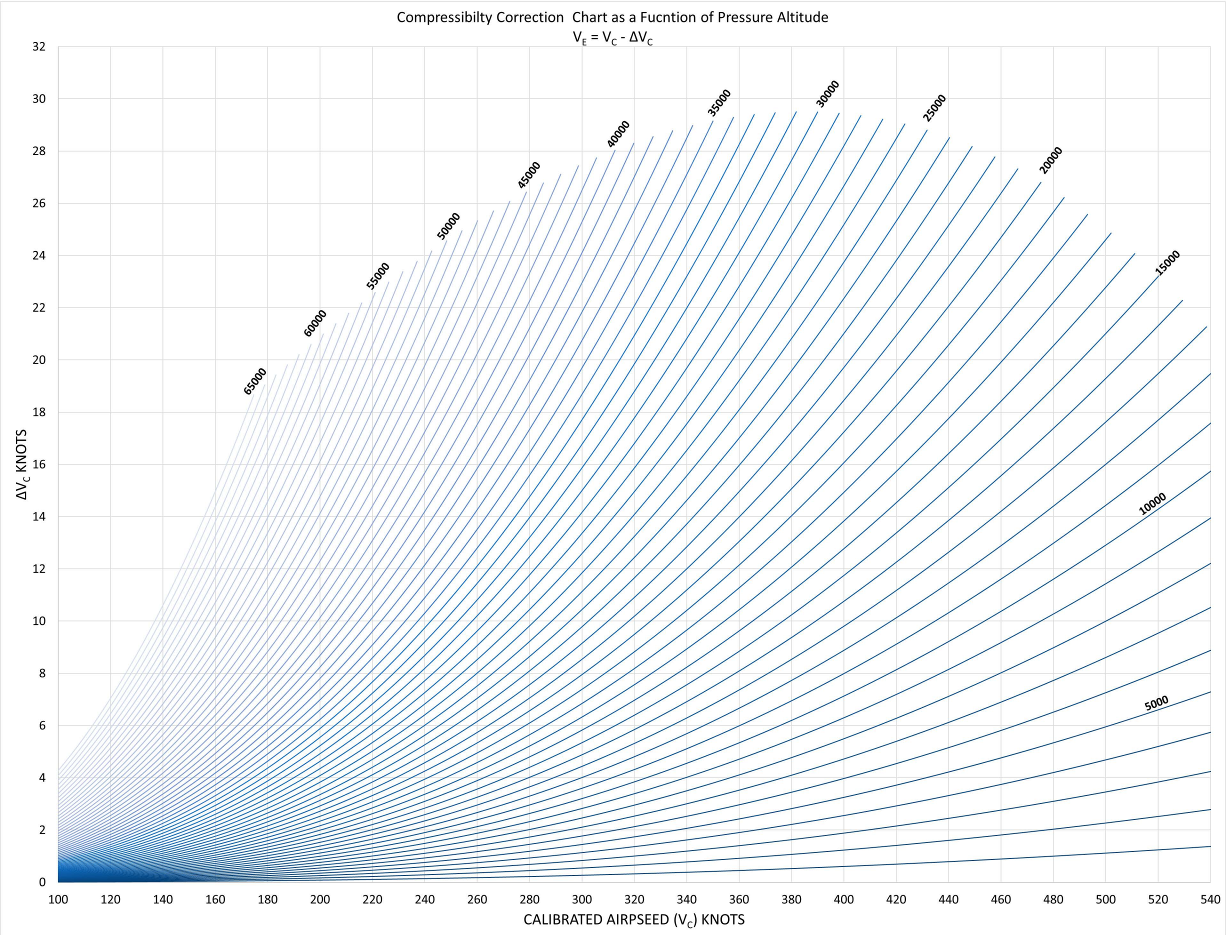
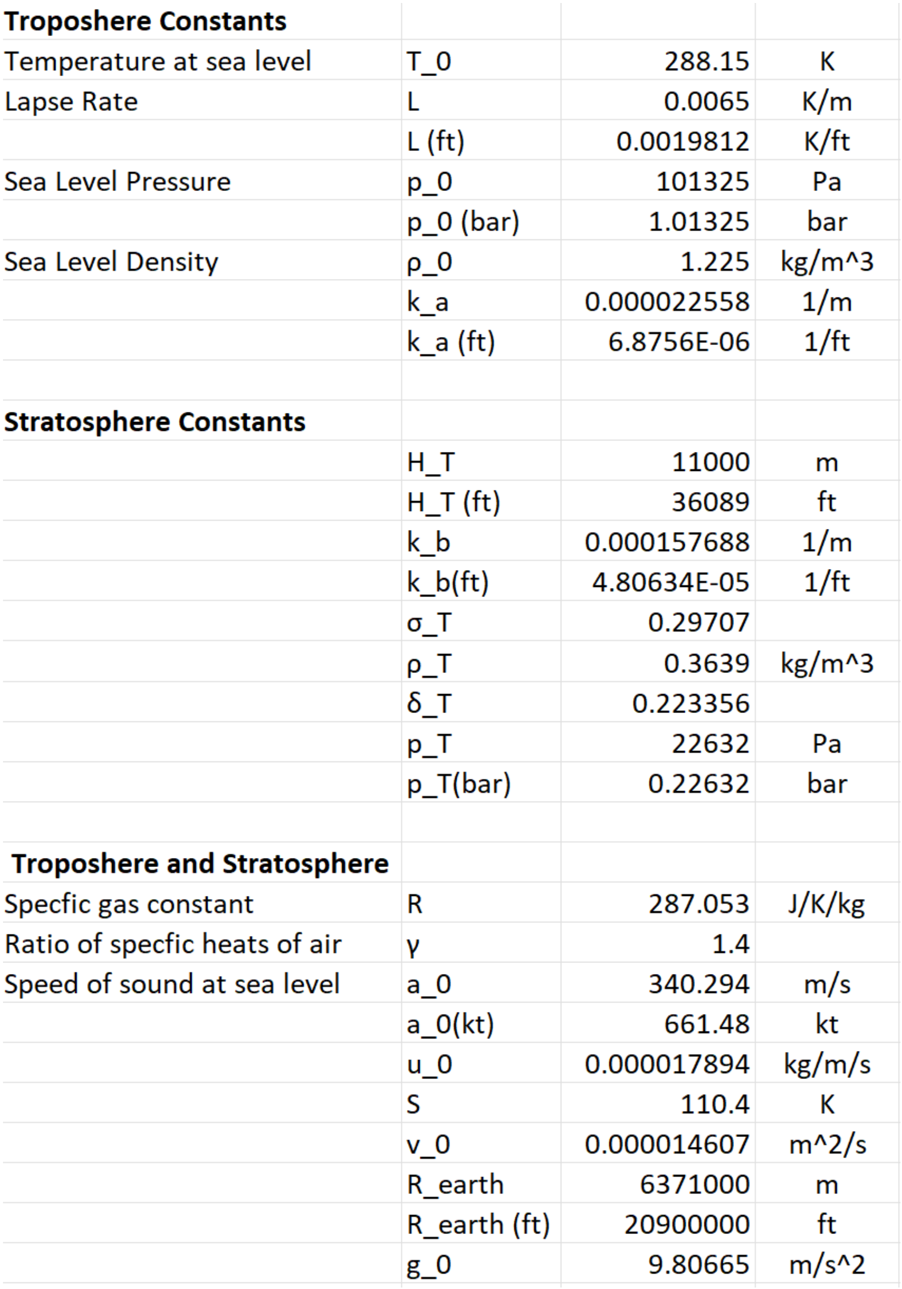


Figure 4.10 Compressibility Chart as a function of pressure altitude- $V_E = V_C$ - ΔV_C

It is important to note that any values exceeding Mach 1.0 are classified as supersonic. As previously discussed, commercial aircraft can encounter phenomena associated with supersonic flight, potentially leading to stability loss and various control malfunctions.

4.4 Tab 4: Constants Tab

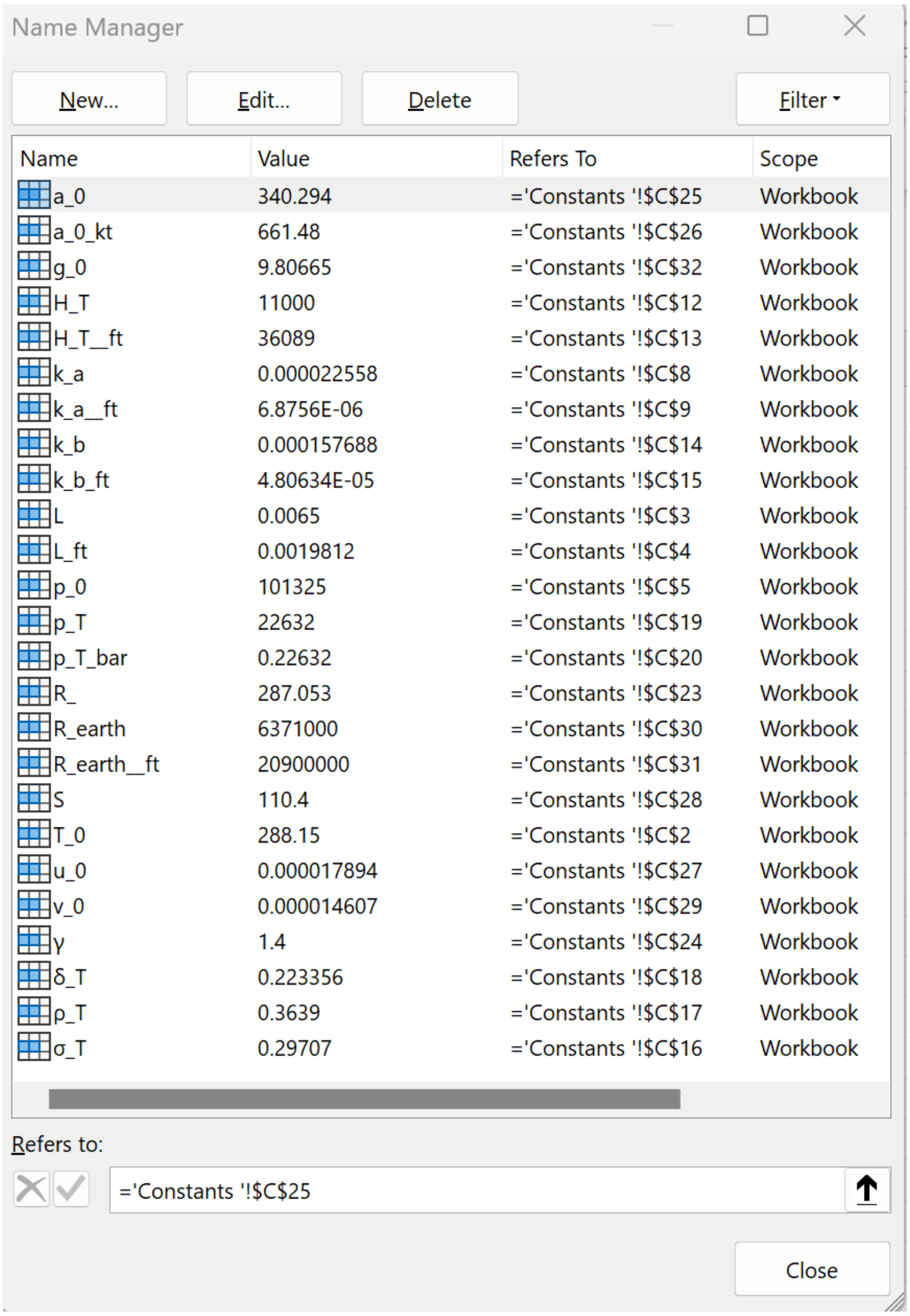
The constant tab depicts all necessary International Standard Atmosphere standard values used for the calculations (Figure 4.11).



Troposhere Constants			
Temperature at sea level	T_0	288.15	K
Lapse Rate	L	0.0065	K/m
	L (ft)	0.0019812	K/ft
Sea Level Pressure	p_0	101325	Pa
	p_0 (bar)	1.01325	bar
Sea Level Density	ρ_0	1.225	kg/m^3
	k_a	0.000022558	1/m
	k_a (ft)	6.8756E-06	1/ft
Stratosphere Constants			
	H_T	11000	m
	H_T (ft)	36089	ft
	k_b	0.000157688	1/m
	k_b(ft)	4.80634E-05	1/ft
	σ_T	0.29707	
	ρ_T	0.3639	kg/m^3
	δ_T	0.223356	
	p_T	22632	Pa
	p_T(bar)	0.22632	bar
Troposphere and Stratosphere			
Specfic gas constant	R	287.053	J/K/kg
Ratio of specfic heats of air	γ	1.4	
Speed of sound at sea level	a_0	340.294	m/s
	a_0(kt)	661.48	kt
	u_0	0.000017894	kg/m/s
	S	110.4	K
	v_0	0.000014607	m^2/s
	R_earth	6371000	m
	R_earth (ft)	20900000	ft
	g_0	9.80665	m/s^2

Figure 4.11 ISA Standard Values in the Constants tab

The Excel NAME MANGER will substitute formulas, which in this case are the description of the ISA sea level values, for their corresponding standard value (Firgure 4.12).



Name	Value	Refers To	Scope
a_0	340.294	='Constants '!$C$25	Workbook
a_0_kt	661.48	='Constants '!$C$26	Workbook
g_0	9.80665	='Constants '!$C$32	Workbook
H_T	11000	='Constants '!$C$12	Workbook
H_T__ft	36089	='Constants '!$C$13	Workbook
k_a	0.000022558	='Constants '!$C$8	Workbook
k_a__ft	6.8756E-06	='Constants '!$C$9	Workbook
k_b	0.000157688	='Constants '!$C$14	Workbook
k_b_ft	4.80634E-05	='Constants '!$C$15	Workbook
L	0.0065	='Constants '!$C$3	Workbook
L_ft	0.0019812	='Constants '!$C$4	Workbook
p_0	101325	='Constants '!$C$5	Workbook
p_T	22632	='Constants '!$C$19	Workbook
p_T_bar	0.22632	='Constants '!$C$20	Workbook
R_	287.053	='Constants '!$C$23	Workbook
R_earth	6371000	='Constants '!$C$30	Workbook
R_earth__ft	20900000	='Constants '!$C$31	Workbook
S	110.4	='Constants '!$C$28	Workbook
T_0	288.15	='Constants '!$C$2	Workbook
u_0	0.000017894	='Constants '!$C$27	Workbook
v_0	0.000014607	='Constants '!$C$29	Workbook
γ	1.4	='Constants '!$C$24	Workbook
δ_T	0.223356	='Constants '!$C$18	Workbook
ρ_T	0.3639	='Constants '!$C$17	Workbook
σ_T	0.29707	='Constants '!$C$16	Workbook

Refers to: ='Constants '!$C$25

Figure 4.12 Name Manager dialog box for the Worksheet

5 Comparison with Other Results

5.1 Walter Bislin: Interactive Compressibility Chart

Walter Bislin Compressibility Chart follows a mathematical model between TAS to CAS and TAS to EAS (Figure 5.1 and Figure 5.2).

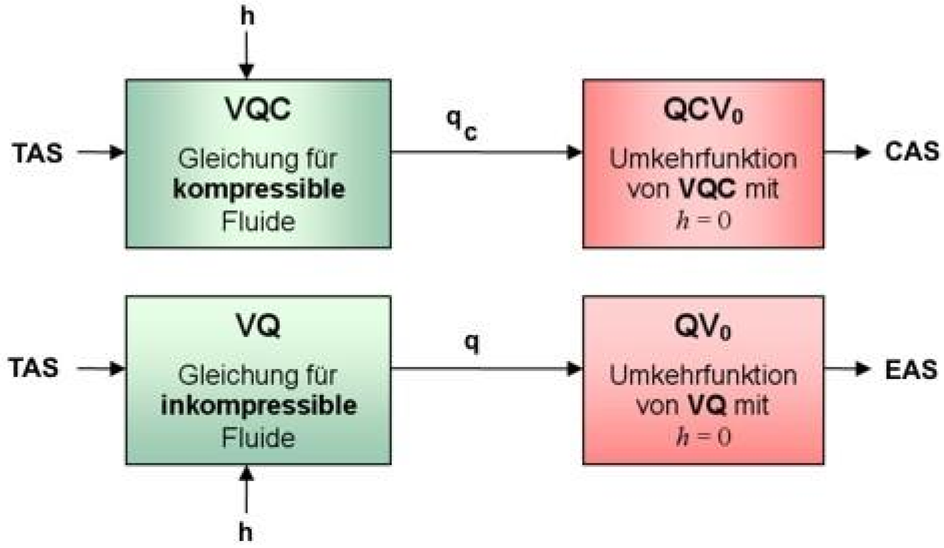


Figure 5.1 Different Airspeed Mathematical model (Bislin 2016)

$$\boxed{\mathrm{CAS} = \mathbf{QCV}(\,\mathbf{VQC}(\,\mathrm{TAS}, h\,), 0\,)}$$

$$\boxed{\mathrm{EAS} = \mathbf{QV}(\,\mathbf{VQ}(\,\mathrm{TAS}, h\,), 0\,)}$$

$$\boxed{\mathrm{Vc} = \mathrm{EAS} - \mathrm{CAS}}$$

Figure 5.2 Summary of formulas used (Bislin 2016)

For a thorough explanation of these equations, refer to Walter Bislin's work (Bislin 2016), which clarifies the full derivations. Notice that the correction, **Vc is defined as a negative value!** This is in contrast to the definiton used by Trevor Young and throughout this thesis.

Bislin's methodology follows the same fundamental equations used in this work; however, a notable deviation between Bislin's approach and the chart presented in this work lies in his utilization of True Airspeed (TAS). Bislin's approach is distinguished by his derivation of CAS, where it functions as a variable dependent on TAS and altitude, contrasting with the method used in this study where CAS serves as an input value in the Excel worksheet.

True airspeed, in this Bislin's text, is expressed as a function of Mach number from 0 to 1 for various altitudes different height as the equation listed below.

$$\mathrm{TAS} = \mathrm{Ma} \cdot a = \mathrm{Ma} \cdot \sqrt{\kappa \cdot R_\mathrm{S} \cdot T(h)} = \mathrm{Ma} \cdot \sqrt{\kappa \cdot R_\mathrm{S} \cdot \left(T_\mathrm{ref} + \alpha_i \cdot (h - h_\mathrm{ref})\right)}$$

Additionally, unlike Excel's limitations of 255 data series, Bislins' compressibility chart transcends these constraints. This is evident when navigating through his interactive chart (Figure 5.3), where Mach numbers are iterated in increments of 0.001, and pressures are presented in 100 ft intervals. This implies that a TAS value must have been computed for each of the 1000 ft individual Mach numbers, along with calculating the corresponding parameters as depicted in Figure 5.2. It is highly possible that Bislin utilized more sophisticated programming languages such as JavaScript or Python to effectively manage larger datasets while also implementing the interactive mode.

As previously stated, Mach numbers are in iterations of 0.001, while the pressure altitude is presented in 100 ft intervals. CAS, EAS, and TAS are in increments of 1 kt. This precision is readily apparent through the interactive chart display, where hovering over data points reveals the intricacies of the calculations as shown in Figure 5.3.

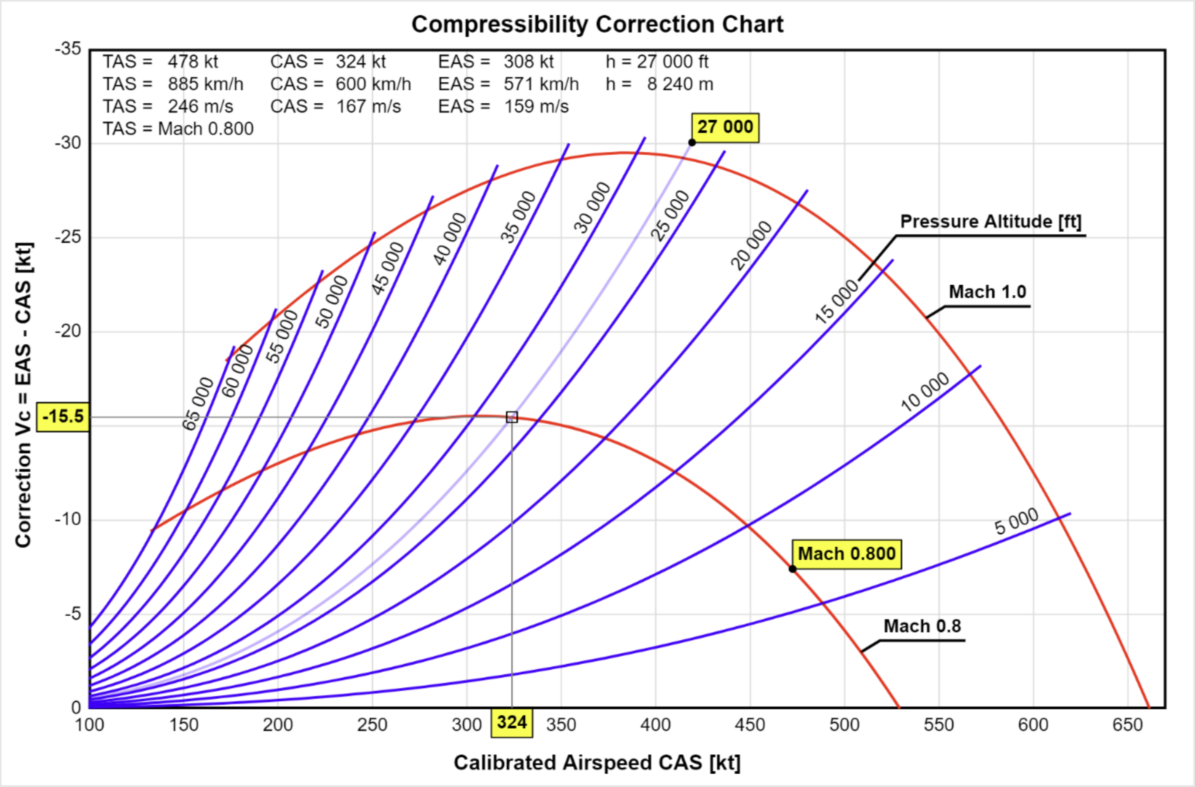


Figure 5.3 Interactive CAS-EAS Compressibility Correction Chart (Bislin 2019)

Bislin Compressibility Chart in Figure 5.3 results in:
CAS = 324 kt
H = 27000 ft
Mach number = 0.800
ΔV_C = 15.5 kt

Comparing it this works closest results (which are highlighted yellow):
From Tab 3:
At H = 27000 ft and M = 0.8:
ΔV_C = 15.45275426 kt ≈ 15.5 kt (Cell: AI46)
ΔV_C percent difference from Bislins ΔV_C: 0%

From Tab 4:
At CAS = 373.7564 kt and H = 21000 ft
ΔV_C = 15.46045056 kt ≈ 15.5 kt (Cell: BD338)
ΔV_C Percent difference from Bislins ΔV_C: 0%

When approximating the ΔV_C to the tenths place, following the methodology displayed in Figure 5.3, the percent differences in ΔV_C are 0%. Despite Bislins approach of deriving the CAS values through TAS, as opposed to having them as input values as done in this work, both methodologies adhere to the same fundamental equations. Consequently, it can confidently be asserted that the CAS-EAS Compressibility Chart developed in this work demonstrates a high level of accuracy.

5.2 Dennis Lucht: Accuracy of Rules of Thumb

Dennis Lucht's research aimed to validate the applicability of the rule of thumb for converting CAS to TAS within a range of viability. To effectively utilize the CAS-EAS Compressibility Chart for this purpose, a comprehensive range of values derived from the results of the rules of thumb is essential. Lucht primarily focused on the dynamic temperature differentials ΔT within the atmosphere as altitude increased, consistently emphasized throughout his study via the use of the ICAO table. In contrast, the Compressibility Correction Chart in this work overlooks variations of temperature.

The rules of thumb presented are

$$v_{ROT} = \frac{6 \cdot FL}{10} + v_c + T_t$$

which is for high altitudes and high velocities, and

$$v_{ROT_H} = v_c + 2\% \cdot \frac{H}{1000\,ft}$$

for low altitudes and low velocities. The calibrated airspeed, v_c, flight level, FL, absolute temperature, T_t, and the calculated velocity, v_{ROT} from the rule of thumb ROT are displayed.

To apply the rules of thumb effectively, Lucht ensures that these variables are derived with the same variables to use to build the Compressibility Correction Chart in flight mechanics. Summarizing the derivations: the parameters resulted in following equations

$$v = 661{,}48\sqrt{1 + \left(\frac{\Delta T - 0{,}0019812\,H}{288{,}15}\right)} \cdot \sqrt{5\left\{\left[\frac{1}{\delta}\left\{\left[1 + 0{,}2\left(\frac{v_c}{661{,}48}\right)^2\right]^{3{,}5} - 1\right\} + 1\right]^{\frac{1}{3{,}5}} - 1\right\}}$$

$$v_{ROT} = \frac{288{,}15\,H}{100\,(288{,}15 + \Delta T)} \cdot \frac{6}{10} + v_c + (288{,}15 - 1{,}9812 \cdot 10^{-3}\,\mathrm{H} + \Delta T) \cdot \left(1 + \left\{\left[\frac{1}{\delta}\left\{\left[1 + 0{,}2\left(\frac{v_c}{661{,}48}\right)^2\right]^{3{,}5} - 1\right\} + 1\right]^{\frac{1}{3{,}5}} - 1\right\}\right)$$

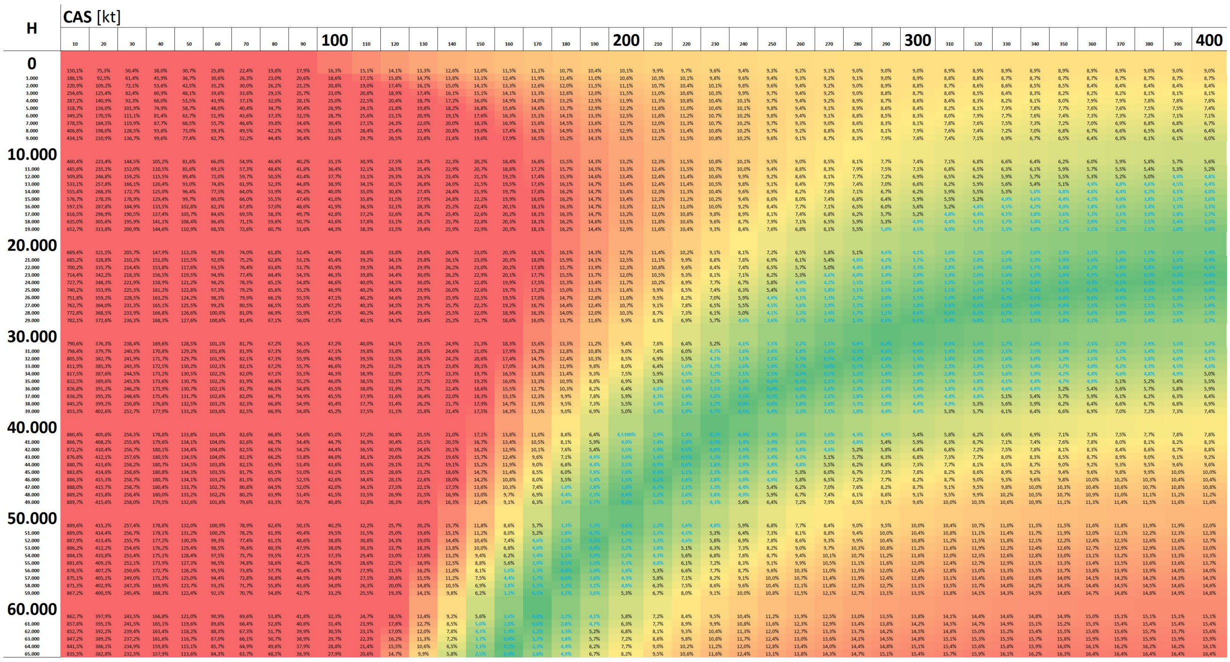
$$\delta = \left(1 - \frac{0{,}0019812\,H}{288{,}15 + \Delta T}\right)^{5{,}2558} .$$

Where v is the true airspeed from the flight mechanics equations, while v_{ROT} is the true airspeed from the high velocities and high altitude rule of thumb. Correct units need to be used: kt, ft, K.

Thus, the relative error can be calculated from

$$\in = \left|\frac{v - v_{ROT}}{v}\right| \cdot 100 \quad .$$

By incorporating the ISA table and the preceding equations into Excel, Lucht evaluates the viability of range of the rule of thumb cross various altitudes and calibrated airspeeds. Using conditional formatting in a color gradient from green to red, the application of the rule of thumb results in TAS values with a relative error of less than 5% (colored green) under cruising conditions as demonstrated in Figure 5.4. A similar analysis is conducted for lower velocity and altitude; for further details, please refer to Dennis Lucht's Work (Lucht 2019).



H	CAS [kt]									
	10	20	30	40	50	60	70	80	90	100
0	150,1%	75,3%	50,4%	38,0%	30,7%	25,8%	22,4%	19,8%	17,9%	16,3%
1.000	186,1%	92,5%	61,4%	45,9%	36,7%	30,6%	26,3%	23,0%	20,6%	18,6%
2.000	220,9%	109,2%	72,1%	53,6%	42,5%	35,2%	30,0%	26,2%	23,2%	20,8%
3.000	254,6%	125,4%	82,4%	60,9%	48,1%	39,6%	33,6%	29,1%	25,7%	23,0%
4.000	287,2%	140,9%	92,3%	68,0%	53,5%	43,9%	37,1%	32,0%	28,1%	25,0%
5.000	318,7%	156,0%	101,9%	74,9%	58,7%	48,0%	40,4%	34,7%	30,4%	26,9%
6.000	349,2%	170,5%	111,1%	81,4%	63,7%	51,9%	43,6%	37,3%	32,5%	28,7%
7.000	378,5%	184,5%	119,9%	87,7%	68,5%	55,7%	46,6%	39,8%	34,6%	30,4%
8.000	406,8%	198,0%	128,5%	93,8%	73,0%	59,3%	49,5%	42,2%	36,5%	32,1%
9.000	434,1%	210,9%	136,7%	99,6%	77,4%	62,7%	52,2%	44,4%	38,4%	33,6%
10.000	460,4%	223,4%	144,5%	105,2%	81,6%	66,0%	54,9%	46,6%	40,2%	35,1%
11.000	485,6%	235,3%	152,0%	110,5%	85,6%	69,1%	57,3%	48,6%	41,8%	36,4%
12.000	509,8%	246,8%	159,2%	115,5%	89,4%	72,0%	59,7%	50,5%	43,4%	37,7%
13.000	533,1%	257,8%	166,1%	120,4%	93,0%	74,8%	61,9%	52,3%	44,8%	38,9%
14.000	555,4%	268,3%	172,7%	125,0%	96,4%	77,5%	64,0%	53,9%	46,2%	40,0%
15.000	576,7%	278,3%	178,9%	129,4%	99,7%	80,0%	66,0%	55,5%	47,4%	41,0%
16.000	597,1%	287,8%	184,9%	133,5%	102,8%	82,3%	67,8%	57,0%	48,6%	41,9%
17.000	616,5%	296,9%	190,5%	137,4%	105,7%	84,6%	69,5%	58,3%	49,7%	42,8%
18.000	635,0%	305,6%	195,9%	141,1%	108,4%	86,6%	71,1%	59,6%	50,7%	43,6%
19.000	652,7%	313,8%	200,9%	144,6%	110,9%	88,5%	72,6%	60,7%	51,6%	44,3%
20.000	669,4%	321,5%	205,7%	147,9%	113,3%	90,3%	74,0%	61,8%	52,4%	44,9%
21.000	685,2%	328,8%	210,2%	151,0%	115,5%	92,0%	75,2%	62,8%	53,1%	45,4%
22.000	700,2%	335,7%	214,4%	153,8%	117,6%	93,5%	76,4%	63,6%	53,7%	45,9%
23.000	714,4%	342,2%	218,3%	156,5%	119,5%	94,9%	77,4%	64,4%	54,3%	46,3%
24.000	727,7%	348,3%	221,9%	158,9%	121,2%	96,2%	78,3%	65,1%	54,8%	46,6%
25.000	740,2%	353,9%	225,3%	161,2%	122,8%	97,3%	79,2%	65,6%	55,2%	46,9%
26.000	751,8%	359,2%	228,5%	163,2%	124,2%	98,3%	79,9%	66,1%	55,5%	47,1%
27.000	762,7%	364,0%	231,3%	165,1%	125,5%	99,2%	80,5%	66,5%	55,8%	47,2%
28.000	772,8%	368,5%	233,9%	166,8%	126,6%	100,0%	81,0%	66,9%	55,9%	47,3%
29.000	782,1%	372,6%	236,3%	168,3%	127,6%	100,6%	81,4%	67,1%	56,0%	47,3%
30.000	790,6%	376,3%	238,4%	169,6%	128,5%	101,1%	81,7%	67,2%	56,1%	47,2%
31.000	798,4%	379,7%	240,3%	170,8%	129,2%	101,6%	81,9%	67,3%	56,0%	47,1%
32.000	805,5%	382,7%	241,9%	171,7%	129,7%	101,9%	82,1%	67,3%	55,9%	46,9%
33.000	811,9%	385,3%	243,3%	172,5%	130,2%	102,1%	82,1%	67,2%	55,7%	46,6%
34.000	817,5%	387,6%	244,5%	173,2%	130,5%	102,2%	82,0%	67,1%	55,5%	46,3%
35.000	822,5%	389,6%	245,5%	173,6%	130,7%	102,2%	81,9%	66,8%	55,2%	46,0%
36.000	826,8%	391,2%	246,2%	173,9%	130,7%	102,1%	81,7%	66,5%	54,8%	45,5%
37.000	836,2%	395,3%	248,6%	175,4%	131,7%	102,6%	82,0%	66,7%	54,9%	45,5%
38.000	845,2%	399,2%	250,8%	176,8%	132,5%	103,2%	82,3%	66,8%	54,9%	45,4%
39.000	853,3%	402,6%	252,7%	177,9%	133,2%	103,6%	82,5%	66,9%	54,8%	45,2%
40.000	860,4%	405,6%	254,3%	178,8%	133,8%	103,8%	82,6%	66,8%	54,6%	45,0%
41.000	866,7%	408,2%	255,6%	179,6%	134,1%	104,0%	82,6%	66,7%	54,4%	44,7%
42.000	872,2%	410,4%	256,7%	180,1%	134,4%	104,0%	82,5%	66,5%	54,2%	44,4%
43.000	876,8%	412,1%	257,6%	180,5%	134,5%	104,0%	82,3%	66,2%	53,8%	44,0%
44.000	880,7%	413,6%	258,2%	180,7%	134,5%	103,8%	82,1%	65,9%	53,4%	43,6%
45.000	883,8%	414,6%	258,6%	180,8%	134,3%	103,5%	81,7%	65,5%	53,0%	43,1%
46.000	886,3%	415,3%	258,7%	180,7%	134,1%	103,2%	81,3%	65,0%	52,5%	42,6%
47.000	888,0%	415,7%	258,7%	180,4%	133,7%	102,7%	80,8%	64,5%	52,0%	42,0%
48.000	889,2%	415,8%	258,4%	180,0%	133,2%	102,2%	80,2%	63,9%	51,4%	41,5%
49.000	889,7%	415,6%	258,0%	179,5%	132,6%	101,6%	79,6%	63,3%	50,7%	40,8%
50.000	889,6%	415,2%	257,4%	178,8%	132,0%	100,9%	78,9%	62,6%	50,1%	40,2%
51.000	889,0%	414,4%	256,7%	178,1%	131,2%	100,2%	78,2%	61,9%	49,4%	39,5%
52.000	887,9%	413,4%	255,7%	177,2%	130,3%	99,3%	77,4%	61,1%	48,6%	38,8%
53.000	886,2%	412,2%	254,6%	176,2%	129,4%	98,5%	76,6%	60,3%	47,9%	38,0%
54.000	884,1%	410,8%	253,4%	175,1%	128,4%	97,5%	75,7%	59,5%	47,1%	37,3%
55.000	881,6%	409,1%	252,1%	173,9%	127,3%	96,5%	74,8%	58,6%	46,2%	36,5%
56.000	878,5%	407,2%	250,6%	172,7%	126,2%	95,5%	73,8%	57,7%	45,4%	35,7%
57.000	875,1%	405,1%	249,0%	171,3%	125,0%	94,4%	72,8%	56,8%	44,5%	34,8%
58.000	871,3%	402,9%	247,3%	169,9%	123,7%	93,3%	71,7%	55,8%	43,6%	34,0%
59.000	867,2%	400,5%	245,4%	168,3%	122,4%	92,1%	70,7%	54,8%	42,7%	33,2%
60.000	862,7%	397,9%	243,5%	166,8%	121,0%	90,9%	69,6%	53,8%	41,8%	32,3%
61.000	857,8%	395,1%	241,5%	165,1%	119,6%	89,6%	68,4%	52,8%	40,8%	31,4%
62.000	852,7%	392,2%	239,4%	163,4%	118,2%	88,3%	67,3%	51,7%	39,9%	30,5%
63.000	847,2%	389,2%	237,2%	161,6%	116,7%	87,0%	66,1%	50,7%	38,9%	29,7%
64.000	841,5%	386,1%	234,9%	159,8%	115,1%	85,7%	64,9%	49,6%	37,9%	28,8%
65.000	835,5%	382,8%	232,5%	157,9%	113,6%	84,3%	63,7%	48,5%	36,9%	27,9%

H	CAS [kt]									
	110	120	130	140	150	160	170	180	190	200
0	15,1%	14,1%	13,3%	12,6%	12,0%	11,5%	11,1%	10,7%	10,4%	10,1%
1.000	17,1%	15,8%	14,7%	13,8%	13,1%	12,4%	11,9%	11,4%	11,0%	10,6%
2.000	19,0%	17,4%	16,1%	15,0%	14,1%	13,3%	12,6%	12,0%	11,5%	11,1%
3.000	20,8%	18,9%	17,4%	16,1%	15,1%	14,1%	13,3%	12,6%	12,0%	11,5%
4.000	22,5%	20,4%	18,7%	17,2%	16,0%	14,9%	14,0%	13,2%	12,5%	11,9%
5.000	24,1%	21,8%	19,8%	18,2%	16,8%	15,6%	14,6%	13,7%	12,9%	12,2%
6.000	25,6%	23,1%	20,9%	19,1%	17,6%	16,3%	15,1%	14,1%	13,3%	12,5%
7.000	27,1%	24,3%	22,0%	20,0%	18,3%	16,9%	15,6%	14,5%	13,6%	12,7%
8.000	28,4%	25,4%	22,9%	20,8%	19,0%	17,4%	16,1%	14,9%	13,9%	12,9%
9.000	29,7%	26,5%	23,8%	21,6%	19,6%	17,9%	16,5%	15,2%	14,1%	13,1%
10.000	30,9%	27,5%	24,7%	22,3%	20,2%	18,4%	16,8%	15,5%	14,3%	13,2%
11.000	32,1%	28,5%	25,4%	22,9%	20,7%	18,8%	17,2%	15,7%	14,5%	13,3%
12.000	33,1%	29,3%	26,1%	23,4%	21,1%	19,2%	17,4%	15,9%	14,6%	13,4%
13.000	34,1%	30,1%	26,8%	24,0%	21,5%	19,5%	17,6%	16,1%	14,7%	13,4%
14.000	35,0%	30,8%	27,4%	24,4%	21,9%	19,7%	17,8%	16,2%	14,7%	13,4%
15.000	35,8%	31,5%	27,9%	24,8%	22,2%	19,9%	18,0%	16,2%	14,7%	13,4%
16.000	36,5%	32,1%	28,3%	25,2%	22,4%	20,1%	18,1%	16,3%	14,7%	13,3%
17.000	37,2%	32,6%	28,7%	25,4%	22,6%	20,2%	18,1%	16,3%	14,7%	13,2%
18.000	37,8%	33,1%	29,1%	25,7%	22,8%	20,3%	18,1%	16,2%	14,6%	13,1%
19.000	38,3%	33,5%	29,4%	25,9%	22,9%	20,3%	18,1%	16,2%	14,4%	12,9%
20.000	38,8%	33,8%	29,6%	26,0%	23,0%	20,3%	18,1%	16,1%	14,3%	12,7%
21.000	39,2%	34,1%	29,8%	26,1%	23,0%	20,3%	18,0%	15,9%	14,1%	12,5%
22.000	39,5%	34,3%	29,9%	26,2%	23,0%	20,2%	17,8%	15,7%	13,9%	12,3%
23.000	39,8%	34,4%	30,0%	26,2%	22,9%	20,1%	17,7%	15,5%	13,7%	12,0%
24.000	40,0%	34,5%	30,0%	26,1%	22,8%	19,9%	17,5%	15,3%	13,4%	11,7%
25.000	40,2%	34,6%	29,9%	26,0%	22,6%	19,7%	17,2%	15,0%	13,1%	11,4%
26.000	40,2%	34,6%	29,9%	25,9%	22,5%	19,5%	17,0%	14,7%	12,8%	11,0%
27.000	40,3%	34,5%	29,7%	25,7%	22,2%	19,2%	16,7%	14,4%	12,4%	10,7%
28.000	40,2%	34,4%	29,6%	25,5%	22,0%	18,9%	16,3%	14,0%	12,0%	10,3%
29.000	40,1%	34,3%	29,4%	25,2%	21,7%	18,6%	16,0%	13,7%	11,6%	9,9%
30.000	40,0%	34,1%	29,1%	24,9%	21,3%	18,3%	15,6%	13,3%	11,2%	9,4%
31.000	39,8%	33,8%	28,8%	24,6%	21,0%	17,9%	15,2%	12,8%	10,8%	9,0%
32.000	39,5%	33,5%	28,5%	24,2%	20,6%	17,4%	14,7%	12,4%	10,3%	8,5%
33.000	39,2%	33,2%	28,1%	23,8%	20,1%	17,0%	14,3%	11,9%	9,8%	8,0%
34.000	38,9%	32,8%	27,7%	23,3%	19,7%	16,5%	13,8%	11,4%	9,3%	7,5%
35.000	38,5%	32,3%	27,2%	22,9%	19,2%	16,0%	13,3%	10,9%	8,8%	6,9%
36.000	38,0%	31,9%	26,7%	22,4%	18,6%	15,5%	12,7%	10,3%	8,2%	6,4%
37.000	37,9%	31,6%	26,4%	22,0%	18,3%	15,1%	12,3%	9,9%	7,8%	5,9%
38.000	37,7%	31,4%	26,2%	21,7%	17,9%	14,7%	11,9%	9,5%	7,3%	5,5%
39.000	37,5%	31,1%	25,8%	21,4%	17,5%	14,3%	11,5%	9,0%	6,9%	5,0%
40.000	37,2%	30,8%	25,5%	21,0%	17,1%	13,8%	11,0%	8,6%	6,4%	4,5390%
41.000	36,9%	30,4%	25,1%	20,5%	16,7%	13,4%	10,5%	8,1%	5,9%	4,0%
42.000	36,5%	30,0%	24,6%	20,1%	16,2%	12,9%	10,1%	7,6%	5,4%	3,5%
43.000	36,1%	29,6%	24,2%	19,6%	15,7%	12,4%	9,6%	7,1%	4,9%	3,0%
44.000	35,6%	29,1%	23,7%	19,1%	15,2%	11,9%	9,0%	6,6%	4,4%	2,5%
45.000	35,1%	28,6%	23,2%	18,6%	14,7%	11,4%	8,5%	6,0%	3,9%	2,0%
46.000	34,6%	28,1%	22,6%	18,0%	14,2%	10,8%	8,0%	5,5%	3,4%	1,5%
47.000	34,1%	27,5%	22,1%	17,5%	13,6%	10,3%	7,4%	5,0%	2,8%	1,0%
48.000	33,5%	26,9%	21,5%	16,9%	13,0%	9,7%	6,9%	4,4%	2,3%	0,4%
49.000	32,8%	26,3%	20,9%	16,3%	12,4%	9,1%	6,3%	3,9%	1,7%	0,1%
50.000	32,2%	25,7%	20,2%	15,7%	11,8%	8,6%	5,7%	3,3%	1,2%	0,6%
51.000	31,5%	25,0%	19,6%	15,1%	11,2%	8,0%	5,2%	2,8%	0,7%	1,2%
52.000	30,8%	24,3%	19,0%	14,4%	10,6%	7,4%	4,6%	2,2%	0,1%	1,7%
53.000	30,1%	23,7%	18,3%	13,8%	10,0%	6,8%	4,0%	1,6%	0,4%	2,2%
54.000	29,4%	23,0%	17,6%	13,2%	9,4%	6,2%	3,4%	1,1%	1,0%	2,7%
55.000	28,6%	22,2%	16,9%	12,5%	8,8%	5,6%	2,9%	0,5%	1,5%	3,3%
56.000	27,9%	21,5%	16,2%	11,8%	8,1%	5,0%	2,3%	0,0%	2,0%	3,8%
57.000	27,1%	20,8%	15,5%	11,2%	7,5%	4,4%	1,7%	0,6%	2,6%	4,3%
58.000	26,3%	20,0%	14,8%	10,5%	6,9%	3,8%	1,1%	1,2%	3,1%	4,8%
59.000	25,5%	19,3%	14,1%	9,8%	6,2%	3,2%	0,5%	1,7%	3,6%	5,3%
60.000	24,7%	18,5%	13,4%	9,2%	5,6%	2,6%	0,0%	2,2%	4,1%	5,8%
61.000	23,9%	17,8%	12,7%	8,5%	5,0%	2,0%	0,6%	2,8%	4,7%	6,3%
62.000	23,1%	17,0%	12,0%	7,8%	4,3%	1,4%	1,2%	3,3%	5,2%	6,8%
63.000	22,3%	16,2%	11,3%	7,2%	3,7%	0,8%	1,7%	3,8%	5,7%	7,2%
64.000	21,4%	15,5%	10,6%	6,5%	3,1%	0,2%	2,3%	4,4%	6,2%	7,7%
65.000	20,6%	14,7%	9,9%	5,8%	2,5%	0,4%	2,8%	4,9%	6,7%	8,2%

H	CAS [kt]									
	210	220	230	240	250	260	270	280	290	300
0	9,9%	9,7%	9,6%	9,4%	9,3%	9,2%	9,1%	9,0%	9,0%	9,0%
1.000	10,3%	10,1%	9,8%	9,6%	9,4%	9,3%	9,2%	9,1%	9,0%	8,9%
2.000	10,7%	10,4%	10,1%	9,8%	9,6%	9,4%	9,2%	9,0%	8,9%	8,8%
3.000	11,0%	10,6%	10,3%	9,9%	9,7%	9,4%	9,2%	9,0%	8,8%	8,7%
4.000	11,3%	10,8%	10,4%	10,1%	9,7%	9,4%	9,2%	8,9%	8,7%	8,6%
5.000	11,6%	11,0%	10,6%	10,1%	9,8%	9,4%	9,1%	8,9%	8,6%	8,4%
6.000	11,8%	11,2%	10,7%	10,2%	9,8%	9,4%	9,1%	8,8%	8,5%	8,3%
7.000	12,0%	11,3%	10,7%	10,2%	9,7%	9,3%	9,0%	8,6%	8,3%	8,1%
8.000	12,1%	11,4%	10,8%	10,2%	9,7%	9,2%	8,8%	8,5%	8,1%	7,9%
9.000	12,2%	11,5%	10,8%	10,2%	9,6%	9,1%	8,7%	8,3%	7,9%	7,6%
10.000	12,3%	11,5%	10,8%	10,1%	9,5%	9,0%	8,5%	8,1%	7,7%	7,4%
11.000	12,4%	11,5%	10,7%	10,0%	9,4%	8,8%	8,3%	7,9%	7,5%	7,1%
12.000	12,4%	11,4%	10,6%	9,9%	9,2%	8,6%	8,1%	7,7%	7,2%	6,9%
13.000	12,4%	11,4%	10,5%	9,8%	9,1%	8,4%	7,9%	7,4%	7,0%	6,6%
14.000	12,3%	11,3%	10,4%	9,6%	8,9%	8,2%	7,6%	7,1%	6,7%	6,2%
15.000	12,2%	11,2%	10,2%	9,4%	8,6%	8,0%	7,4%	6,8%	6,4%	5,9%
16.000	12,1%	11,0%	10,0%	9,2%	8,4%	7,7%	7,1%	6,5%	6,0%	5,6%
17.000	12,0%	10,8%	9,8%	8,9%	8,1%	7,4%	6,8%	6,2%	5,7%	5,2%
18.000	11,8%	10,6%	9,6%	8,7%	7,9%	7,1%	6,5%	5,9%	5,3%	4,9%
19.000	11,6%	10,4%	9,3%	8,4%	7,6%	6,8%	6,1%	5,5%	5,0%	4,5%
20.000	11,4%	10,2%	9,1%	8,1%	7,2%	6,5%	5,8%	5,1%	4,6%	4,1%
21.000	11,1%	9,9%	8,8%	7,8%	6,9%	6,1%	5,4%	4,8%	4,2%	3,7%
22.000	10,8%	9,6%	8,4%	7,4%	6,5%	5,7%	5,0%	4,4%	3,8%	3,3%
23.000	10,5%	9,3%	8,1%	7,1%	6,2%	5,3%	4,6%	4,0%	3,4%	2,8%
24.000	10,2%	8,9%	7,7%	6,7%	5,8%	4,9%	4,2%	3,5%	2,9%	2,4%
25.000	9,9%	8,5%	7,4%	6,3%	5,4%	4,5%	3,8%	3,1%	2,5%	2,0%
26.000	9,5%	8,2%	7,0%	5,9%	4,9%	4,1%	3,3%	2,7%	2,0%	1,5%
27.000	9,1%	7,8%	6,5%	5,5%	4,5%	3,6%	2,9%	2,2%	1,6%	1,0%
28.000	8,7%	7,3%	6,1%	5,0%	4,1%	3,2%	2,4%	1,7%	1,1%	0,6%
29.000	8,3%	6,9%	5,7%	4,6%	3,6%	2,7%	1,9%	1,3%	0,6%	0,1%
30.000	7,8%	6,4%	5,2%	4,1%	3,1%	2,2%	1,5%	0,8%	0,2%	0,4%
31.000	7,4%	6,0%	4,7%	3,6%	2,6%	1,8%	1,0%	0,3%	0,3%	0,9%
32.000	6,9%	5,5%	4,2%	3,1%	2,1%	1,3%	0,5%	0,2%	0,8%	1,4%
33.000	6,4%	5,0%	3,7%	2,6%	1,6%	0,7%	0,0%	0,7%	1,3%	1,9%
34.000	5,9%	4,5%	3,2%	2,1%	1,1%	0,2%	0,5%	1,2%	1,8%	2,4%
35.000	5,3%	3,9%	2,7%	1,6%	0,6%	0,3%	1,1%	1,7%	2,3%	2,9%
36.000	4,8%	3,4%	2,1%	1,0%	0,0%	0,8%	1,6%	2,3%	2,9%	3,4%
37.000	4,3%	2,9%	1,6%	0,5%	0,5%	1,3%	2,1%	2,8%	3,4%	3,9%
38.000	3,9%	2,4%	1,2%	0,0%	0,9%	1,8%	2,6%	3,3%	3,9%	4,4%
39.000	3,4%	1,9%	0,7%	0,5%	1,4%	2,3%	3,1%	3,8%	4,4%	4,9%
40.000	2,9%	1,4%	0,2%	0,9%	1,9%	2,8%	3,6%	4,3%	4,9%	5,4%
41.000	2,4%	1,0%	0,3%	1,4%	2,4%	3,3%	4,1%	4,8%	5,4%	5,9%
42.000	1,9%	0,5%	0,8%	1,9%	2,9%	3,8%	4,6%	5,2%	5,8%	6,4%
43.000	1,4%	0,1%	1,3%	2,4%	3,4%	4,3%	5,1%	5,7%	6,3%	6,8%
44.000	0,9%	0,6%	1,8%	2,9%	3,9%	4,8%	5,5%	6,2%	6,8%	7,3%
45.000	0,4%	1,1%	2,3%	3,4%	4,4%	5,3%	6,0%	6,7%	7,3%	7,8%
46.000	0,2%	1,6%	2,8%	3,9%	4,9%	5,8%	6,5%	7,2%	7,7%	8,2%
47.000	0,7%	2,1%	3,3%	4,4%	5,4%	6,2%	7,0%	7,6%	8,2%	8,7%
48.000	1,2%	2,6%	3,8%	4,9%	5,9%	6,7%	7,4%	8,1%	8,6%	9,1%
49.000	1,7%	3,1%	4,3%	5,4%	6,4%	7,2%	7,9%	8,5%	9,1%	9,6%
50.000	2,2%	3,6%	4,8%	5,9%	6,8%	7,7%	8,4%	9,0%	9,5%	10,0%
51.000	2,7%	4,1%	5,3%	6,4%	7,3%	8,1%	8,8%	9,4%	10,0%	10,4%
52.000	3,3%	4,6%	5,8%	6,9%	7,8%	8,6%	9,3%	9,9%	10,4%	10,8%
53.000	3,8%	5,1%	6,3%	7,3%	8,2%	9,0%	9,7%	10,3%	10,8%	11,2%
54.000	4,3%	5,6%	6,8%	7,8%	8,7%	9,4%	10,1%	10,7%	11,2%	11,6%
55.000	4,8%	6,1%	7,2%	8,3%	9,1%	9,9%	10,5%	11,1%	11,6%	12,0%
56.000	5,3%	6,6%	7,7%	8,7%	9,6%	10,3%	11,0%	11,5%	12,0%	12,4%
57.000	5,8%	7,1%	8,2%	9,1%	10,0%	10,7%	11,4%	11,9%	12,4%	12,8%
58.000	6,3%	7,5%	8,6%	9,6%	10,4%	11,1%	11,8%	12,3%	12,7%	13,1%
59.000	6,7%	8,0%	9,1%	10,0%	10,8%	11,5%	12,1%	12,7%	13,1%	13,5%
60.000	7,2%	8,4%	9,5%	10,4%	11,2%	11,9%	12,5%	13,0%	13,5%	13,8%
61.000	7,7%	8,9%	9,9%	10,8%	11,6%	12,3%	12,9%	13,4%	13,8%	14,2%
62.000	8,1%	9,3%	10,4%	11,3%	12,0%	12,7%	13,3%	13,7%	14,2%	14,5%
63.000	8,6%	9,8%	10,8%	11,7%	12,4%	13,0%	13,6%	14,1%	14,5%	14,8%
64.000	9,0%	10,2%	11,2%	12,0%	12,8%	13,4%	14,0%	14,4%	14,8%	15,1%
65.000	9,5%	10,6%	11,6%	12,4%	13,1%	13,8%	14,3%	14,7%	15,1%	15,4%

H	CAS [kt]									
	310	320	330	340	350	360	370	380	390	400
0	8,9%	8,9%	8,9%	8,9%	8,9%	8,9%	8,9%	9,0%	9,0%	9,0%
1.000	8,8%	8,8%	8,7%	8,7%	8,7%	8,7%	8,7%	8,7%	8,7%	8,7%
2.000	8,7%	8,6%	8,6%	8,5%	8,5%	8,4%	8,4%	8,4%	8,4%	8,4%
3.000	8,6%	8,5%	8,4%	8,3%	8,2%	8,2%	8,2%	8,1%	8,1%	8,1%
4.000	8,4%	8,3%	8,2%	8,1%	8,0%	7,9%	7,9%	7,8%	7,8%	7,8%
5.000	8,2%	8,1%	7,9%	7,8%	7,7%	7,6%	7,6%	7,5%	7,5%	7,4%
6.000	8,0%	7,9%	7,7%	7,6%	7,4%	7,3%	7,3%	7,2%	7,1%	7,1%
7.000	7,8%	7,6%	7,5%	7,3%	7,2%	7,0%	6,9%	6,8%	6,8%	6,7%
8.000	7,6%	7,4%	7,2%	7,0%	6,8%	6,7%	6,6%	6,5%	6,4%	6,4%
9.000	7,4%	7,1%	6,9%	6,7%	6,5%	6,4%	6,3%	6,1%	6,1%	6,0%
10.000	7,1%	6,8%	6,6%	6,4%	6,2%	6,0%	5,9%	5,8%	5,7%	5,6%
11.000	6,8%	6,5%	6,3%	6,1%	5,9%	5,7%	5,5%	5,4%	5,3%	5,2%
12.000	6,5%	6,2%	5,9%	5,7%	5,5%	5,3%	5,2%	5,0%	4,9%	4,8%
13.000	6,2%	5,9%	5,6%	5,4%	5,1%	4,9%	4,8%	4,6%	4,5%	4,4%
14.000	5,9%	5,5%	5,3%	5,0%	4,8%	4,6%	4,4%	4,2%	4,1%	4,0%
15.000	5,5%	5,2%	4,9%	4,6%	4,4%	4,2%	4,0%	3,8%	3,7%	3,6%
16.000	5,2%	4,8%	4,5%	4,2%	4,0%	3,8%	3,6%	3,4%	3,3%	3,1%
17.000	4,8%	4,4%	4,1%	3,8%	3,6%	3,3%	3,1%	3,0%	2,8%	2,7%
18.000	4,4%	4,1%	3,7%	3,4%	3,2%	2,9%	2,7%	2,5%	2,4%	2,3%
19.000	4,0%	3,7%	3,3%	3,0%	2,7%	2,5%	2,3%	2,1%	2,0%	1,8%
20.000	3,6%	3,2%	2,9%	2,6%	2,3%	2,1%	1,9%	1,7%	1,5%	1,4%
21.000	3,2%	2,8%	2,5%	2,2%	1,9%	1,6%	1,4%	1,2%	1,1%	0,9%
22.000	2,8%	2,4%	2,0%	1,7%	1,4%	1,2%	1,0%	0,8%	0,6%	0,5%
23.000	2,4%	2,0%	1,6%	1,3%	1,0%	0,7%	0,5%	0,3%	0,2%	0,0%
24.000	1,9%	1,5%	1,1%	0,8%	0,5%	0,3%	0,1%	0,1%	0,3%	0,4%
25.000	1,5%	1,1%	0,7%	0,4%	0,1%	0,2%	0,4%	0,6%	0,8%	0,9%
26.000	1,0%	0,6%	0,2%	0,1%	0,4%	0,6%	0,9%	1,1%	1,2%	1,3%
27.000	0,6%	0,1%	0,2%	0,6%	0,9%	1,1%	1,3%	1,5%	1,7%	1,8%
28.000	0,1%	0,3%	0,7%	1,0%	1,3%	1,6%	1,8%	2,0%	2,1%	2,3%
29.000	0,4%	0,8%	1,2%	1,5%	1,8%	2,1%	2,3%	2,4%	2,6%	2,7%
30.000	0,9%	1,3%	1,7%	2,0%	2,3%	2,5%	2,7%	2,9%	3,1%	3,2%
31.000	1,4%	1,8%	2,1%	2,5%	2,8%	3,0%	3,2%	3,4%	3,5%	3,6%
32.000	1,8%	2,3%	2,6%	3,0%	3,2%	3,5%	3,7%	3,8%	4,0%	4,1%
33.000	2,3%	2,8%	3,1%	3,4%	3,7%	4,0%	4,2%	4,3%	4,5%	4,6%
34.000	2,8%	3,3%	3,6%	3,9%	4,2%	4,4%	4,6%	4,8%	4,9%	5,0%
35.000	3,3%	3,8%	4,1%	4,4%	4,7%	4,9%	5,1%	5,2%	5,4%	5,5%
36.000	3,8%	4,2%	4,6%	4,9%	5,2%	5,4%	5,6%	5,7%	5,8%	5,9%
37.000	4,4%	4,8%	5,1%	5,4%	5,7%	5,9%	6,1%	6,2%	6,3%	6,4%
38.000	4,9%	5,3%	5,6%	5,9%	6,2%	6,4%	6,6%	6,7%	6,8%	6,9%
39.000	5,3%	5,7%	6,1%	6,4%	6,6%	6,9%	7,0%	7,2%	7,3%	7,4%
40.000	5,8%	6,2%	6,6%	6,9%	7,1%	7,3%	7,5%	7,7%	7,8%	7,8%
41.000	6,3%	6,7%	7,1%	7,4%	7,6%	7,8%	8,0%	8,1%	8,2%	8,3%
42.000	6,8%	7,2%	7,5%	7,8%	8,1%	8,3%	8,4%	8,6%	8,7%	8,8%
43.000	7,3%	7,7%	8,0%	8,3%	8,5%	8,7%	8,9%	9,0%	9,1%	9,2%
44.000	7,7%	8,1%	8,5%	8,7%	9,0%	9,2%	9,3%	9,5%	9,6%	9,6%
45.000	8,2%	8,6%	8,9%	9,2%	9,4%	9,6%	9,8%	9,9%	10,0%	10,0%
46.000	8,7%	9,0%	9,3%	9,6%	9,8%	10,0%	10,2%	10,3%	10,4%	10,4%
47.000	9,1%	9,5%	9,8%	10,0%	10,3%	10,4%	10,6%	10,7%	10,8%	10,8%
48.000	9,5%	9,9%	10,2%	10,5%	10,7%	10,9%	11,0%	11,1%	11,2%	11,2%
49.000	10,0%	10,3%	10,6%	10,9%	11,1%	11,3%	11,4%	11,5%	11,6%	11,6%
50.000	10,4%	10,7%	11,0%	11,3%	11,5%	11,6%	11,8%	11,9%	11,9%	12,0%
51.000	10,8%	11,1%	11,4%	11,7%	11,9%	12,0%	12,1%	12,2%	12,3%	12,3%
52.000	11,2%	11,5%	11,8%	12,1%	12,2%	12,4%	12,5%	12,6%	12,6%	12,7%
53.000	11,6%	11,9%	12,2%	12,4%	12,6%	12,7%	12,9%	12,9%	13,0%	13,0%
54.000	12,0%	12,3%	12,6%	12,8%	13,0%	13,1%	13,2%	13,3%	13,3%	13,3%
55.000	12,4%	12,7%	12,9%	13,1%	13,3%	13,4%	13,5%	13,6%	13,6%	13,7%
56.000	12,8%	13,0%	13,3%	13,5%	13,7%	13,8%	13,9%	13,9%	14,0%	14,0%
57.000	13,1%	13,4%	13,6%	13,8%	14,0%	14,1%	14,2%	14,2%	14,3%	14,3%
58.000	13,5%	13,7%	14,0%	14,2%	14,3%	14,4%	14,5%	14,5%	14,6%	14,6%
59.000	13,8%	14,1%	14,3%	14,5%	14,6%	14,7%	14,8%	14,8%	14,9%	14,8%
60.000	14,1%	14,4%	14,6%	14,8%	14,9%	15,0%	15,1%	15,1%	15,1%	15,1%
61.000	14,5%	14,7%	14,9%	15,1%	15,2%	15,3%	15,4%	15,4%	15,4%	15,4%
62.000	14,8%	15,0%	15,2%	15,4%	15,5%	15,6%	15,6%	15,7%	15,7%	15,6%
63.000	15,1%	15,3%	15,5%	15,7%	15,8%	15,9%	15,9%	15,9%	15,9%	15,9%
64.000	15,4%	15,6%	15,8%	15,9%	16,0%	16,1%	16,2%	16,2%	16,2%	16,1%
65.000	15,7%	15,9%	16,1%	16,2%	16,3%	16,4%	16,4%	16,4%	16,4%	16,4%

Figure 5.4 Error of Rule of Thumb calculation of TAS for high altitudes and velocities (Lucht 2019)

6 Summary

The CAS to EAS Compressibility Correction Chart was developed using the Excel software, following a review of flight mechanics. Detailed derivations were provided to explain the process behind plotting the fundamental equations needed. The construction of the chart involved integrating plots that depict the compressibility correction relationship with Mach number and altitude, as well as its correlations with CAS and altitude.

A user guide was formed with all tabs within Excel spreadsheet, to aid the user in understanding the tool's structure and functionality. Additionally, a comparative analysis with the works of Dennis Lucht and Walter Bislin was conducted to offer insights into similarities and differences in results and methodologies.

7 Recommendations

There remains room for further optimization of this chart to enhance its utility for future applications. Several recommendations are proposed:

1. To streamline graph interpretations for users, an interactive model should be implemented, allowing users to hover over the chart and instantly view precise data including Mach number, pressure altitude, CAS, and ΔVc values at the point of interest.

2. Incorporating Dennis Lucht's supplementary rules of thumb enables the consideration of relative deviations in true airspeed and altitude ranges, along with accounting for temperature variations within the atmosphere. This ensures pilots are equipped with precise adjustments for utilizing the Compressibility Correction Chart during instrument flight under cruising conditions.

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Appendix A – VBA Macro for Column Selection

```
Sub SelectOddColumns()
 Dim selectedRange As Range
 Dim i As Integer
 Dim newRange As Range

 ' define selected range
 Set selectedRange = Selection

 ' for each column in the selected range
 For i = 1 To selectedRange.Columns.Count Step 2
  ' add every even column to a new range

  If newRange Is Nothing Then
   Set newRange = selectedRange.Columns(i)
  Else
   Set newRange = Union(newRange, selectedRange.Columns(i))
  End If

 Next i

 ' select new Range
 If Not newRange Is Nothing Then
  newRange.Select
 End If

End Sub
```