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Master Thesis

The Cubic Wing Loading Parameter in Passenger Aircraft Preliminary Sizing

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Abstract

Purpose – This thesis investigates the parameter Cubic Wing Loading (CWL). It is the mass of the aircraft divided by the wing surface area taken to the power of 3/2. As such the unit of the denominator is converted from m^2 to m^3 and CWL has the unit of kg/m^3. Classical Wing Loading (WL) is aircraft mass divided by wing area. It is investigated, if CWL (unlike WL) is independent of aircraft size, if it has advantages in preliminary aircraft design, and if it can be used as a basis for interesting correlations.
Methodology – Aircraft preliminary sizing equations for passenger jet aircraft are rewritten to replace WL with CWL. Aircraft statistical data are investigated with respect to CWL.
Findings – It is known that WL increases with aircraft size. Unfortunately, also CWL depends on aircraft size. However, CWL decreases with aircraft size. CWL introduced to preliminary sizing leads to additional (but manageable) iterations compared to preliminary sizing based on WL. Correlations with other aircraft design parameters are weak and no relation with accident rates for high CWL aircraft is found.
Research Limitations – 209 airplanes are studied for initial statistical correlations. Some correlations were limited to 72 airplanes due to lack of detailed data.
Practical Implications – There are no advantages to replace WL by CWL in passenger aircraft preliminary sizing.
Originality – This seems to be the first report to fully investigate CWL with respect to passenger aircraft and to offer a related user-friendly preliminary sizing spreadsheet.

HAW HAMBURG

DEPARTMENT OF AUTOMOTIVE AND AERONAUTICAL ENGINEERING

The Cubic Wing Loading Parameter in Passenger Aircraft Preliminary Sizing

Task for a *Master Thesis*

Background

Wing Loading, m/S is the mass of the aircraft divided by the wing surface area. Cubic Wing Loading is $m/S^{3/2}$. Here, the surface area is taken to the power of 3/2. As such the unit of the denominator is converted from m² to m³ and Cubic Wing Loading (CWL) has the unit of kg/m³, which is the unit of density. We know from statistics that Wing Loading (WL) unfortunately depends of aircraft size. Wing loading increases with aircraft size. We would like to investigate if CWL is rather a constant value for passenger aircraft of comparable design. CWL has been discussed in a study by S. Durmus in 2020 (https://perma.cc/923V-EU8T). This research can serve as a starting point for this thesis. CWL is also used for Radio Controlled (RC) aircraft. A related webpage is e.g. https://perma.cc/6UM8-L7GZ. RC plane designers claim that CWL is related to aircraft performance, handling, fuel efficiency, and structural characteristics. For this reason, it is widely and successfully used in RC aircraft design. It is however problematic that traditional WL-based equations for preliminary passenger aircraft sizing must be rewritten.

Task

Task of this project is to show potential advantages of using Cubic Wing Loading as a fundamental airplane parameter in aircraft performance, handling, and design of passenger jet aircraft. Following subtasks have to be considered:

- Literature review related to CWL.
- Fundamental description of CWL and related basic equations.
- Rewriting preliminary sizing equations after the introduction of the CWL parameter, including a "T/W versus CWL matching chart".
- Collecting aircraft data.
- Investigating statistical correlations based on the CWL parameter.

The report has to be written in English based on German or international standards on report writing.

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List of Symbols

a	Scale factor
AR_{wet}	Wetted area aspect ratio
b	Wingspan
B_s	Breguet factor
c	Sound´s speed
CD	Drag coefficient
CD_0	Parasitic drag coefficient
CD_f	Friction drag coefficient
CL	Lift Coefficient
$CL_{,max,L}$	Maximum lift coefficient with flaps in landing configuration
$CL_{,max,TO}$	Maximum lift coefficient with flaps in take-off configuration
D	Drag
E (L/D)	Aerodynamic efficiency
g	Gravity earth acceleration
h_{CR}	Cruise altitude
k_L	Landing factor
k_{TO}	Take-off factor
k_{vs}	Variable sweep constant
L	Lift
l	Length
L/D (E)	Aerodynamic efficiency
M	Mach number
m	Aircraft mass
m_F	Fuel mass
M_{ff}	Mission fuel fraction
$m_{F.Req}$	Required fuel mass
$m_{F.Rev}$	Reserve fuel mass
m_{ML}	Maximum landing mass
m_{MTO}	Maximum take-off mass
m_{OE}	Operating empty mass
m_{PL}	Payload mass
m_{ZF}	Zero fuel mass
n_e	Number of engines
p	Atmospheric pressure
R	Range
R^2	Square correlation coefficient
R_{gas}	Universal constant for gases
S	Wing surface
s	Span

S_{LFL}	Landing field length
S_{TOFL}	Take-off field length
S_{wet}	Wetted area
T	Thrust
T_{amb}	atmospheric temperature
T_{CR}	Cruise thrust
T_{TO}	Take-off thrust
V	Speed
V_{CR}	Cruise speed
V_{md}	Minimum drag speed
Vol	Volume
W	Weight

Greek Symbols

γ	Climb gradient
γ'	ratio of specific heats
ρ	Density
σ	Relative density
τ	Volume parameter
φ	efficiency factor

List of Abbreviations

AR	Aspect Ratio
BPR	By Pass Ratio
BWB	Blended Wing Body
CS	Certification Specification (EASA)
CWL	Cubic Wing Loading
DL	Displacement Loading
DOC	Direct Operating Costs
FAR	Federal Aviation Regulations (FAA)
FF	Form Factor
FI	Interference Factor
RC	Radio Controlled
RPF	Reynolds Performance Factor
SFC	Specific Fuel Consumption
WL	Wing Loading

List of Definitions

Blended Wing Body

"An aircraft that combines features of a conventional winged fuselage and a flying wing, in which the wings merge continuously with a flattened, aerodynamic body that provides much of the lift." (https://www.ahdictionary.com)

Displacement Loading

"The weight of an airplane divided by the displacement of the engine." (Reynolds 1989)

Power Loading

"The gross weight of an aircraft divided by the horsepower of the engine(s)." (AGARD 1980)

Waverider

"A lifting body, design for flight at supersonic or hypersonic speeds, which relies essentially on a shockwave, or system of shockwaves, beneath its lower surface for producing its lift force." (AGARD 1980)

1 Introduction

1.1 Motivation

Air traffic is expected to continue growing in the next decades. New planes, more efficient, faster, and safer will be required to be built by manufacturers. Therefore, aircraft design is and will continue to be an interest area of research and development. Nowadays, aircraft design theories are based on two basic parameters, the ratio between thrust of the engines and weight of the plane (Thrust to Weight Ratio; T/W), and the one between mass and wing area (Wing Loading; WL). Based on these two parameters the design of an aircraft which meets some specified requirements, can be determined.

Radio Controlled (RC) planes designer are used to work with another related parameter, the ratio between the mass of the aircraft and the wing surface to the power of 1.5, known as Cubic Wing Loading (CWL). This parameter is not dependent on size for RC planes and it allows to compare different scale models and it is direct related with their flight characteristics and their performance. Thus, it gives important information for designers and users.

Some authors have considered recently the possibility to introduce this parameter into real passenger aircraft design. However, how it relates to other design parameters or which advantages can this parameter bring to aircraft design has not been investigated comprehensively.

1.2 Title Terminology

Title: "The Cubic Wing Loading Parameter in Passenger Aircraft Preliminary Sizing".

Cubic Wing Loading Parameter
It is a new parameter based on traditional Wing Loading (m/S), where the surface in the denominator is raised to the power of 3/2. As a result, its units are kg/m^3.

This thesis aims to determine if the mentioned parameter presents some advantages when introducing it into passenger aircraft preliminary sizing.

Passenger Aircraft
The Cambridge Dictionary (https://dictionary.cambridge.org) defines the word *passenger* as:

"Someone who is travelling in a car, plane, etc., but not controlling it."

The Cambridge Dictionary defines the word *aircraft* as:

"A vehicle that flies."

Hence, a *passenger aircraft* is defined as an air vehicle where people travel. In this thesis, we will only consider large transport category airplanes used for passengers' transport.

Preliminary Sizing
The Cambridge Dictionary defines the word *preliminary* as:

"Coming before a more important action or event, especially introducing or preparing for it."

The word sizing refers to the action of giving something a concrete size.

Hence, for *preliminary sizing* refers to the conceptual design of an object, in regards into its dimensions and mass. In this thesis, we will focus on aircraft preliminary sizing.

Other definition of interest for this thesis:

Square-Cube Law
When a model plane is scaled, length dimensions are multiplied by a scale factor. Thus, its new surface is proportional to the square of the multiplier, and its new volume to the cube of the multiplier. Therefore, the increase in volume is faster than the increase in surface area. For easy understanding see Figure 1.1.

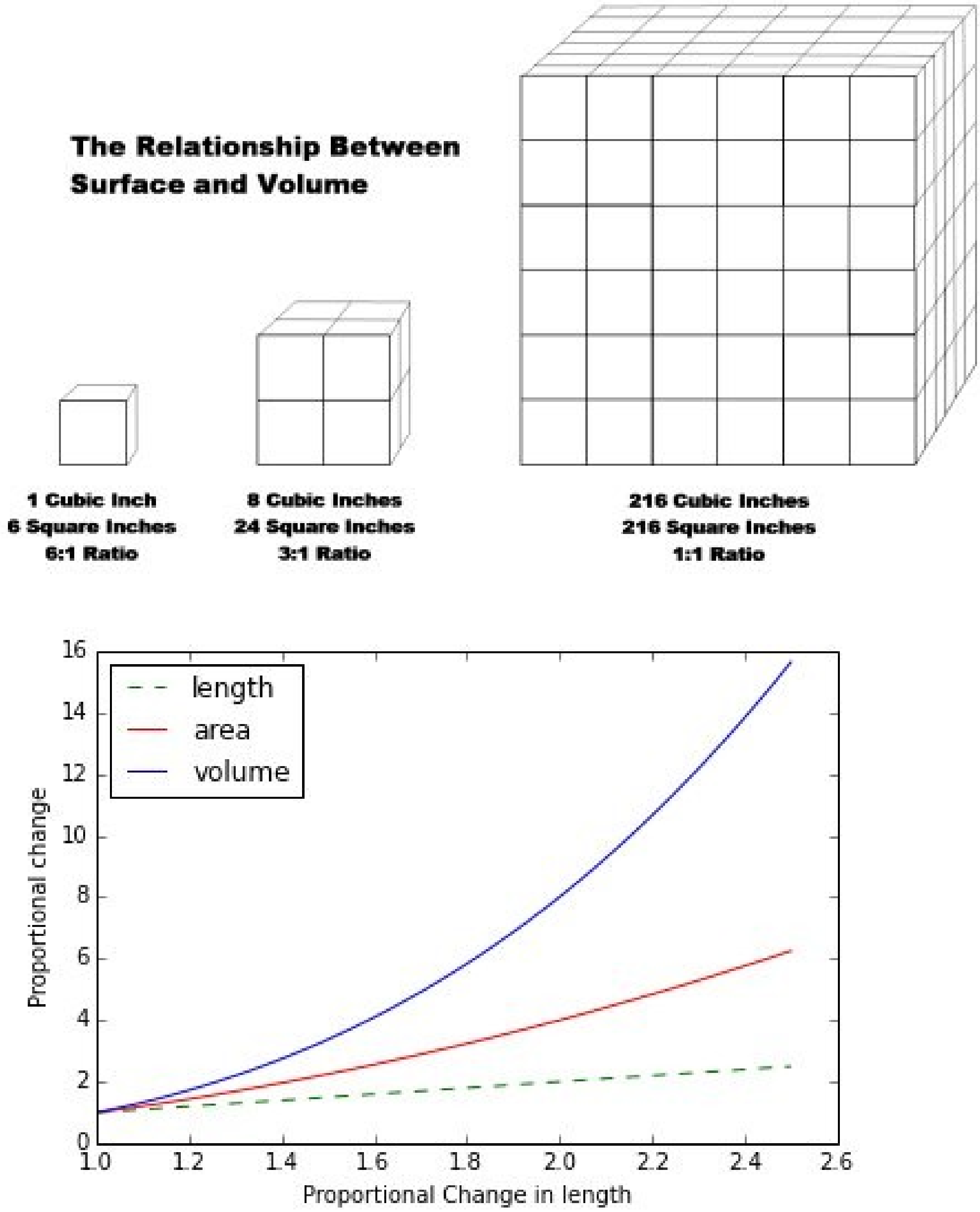

Figure 1.1 Square-Cube Law (Bartimaeus 2016).

Flyability

It is a term adopted by RC plane designers to refer the ease of flying of a RC model. It refers to the flight characteristics of the model and the skill level required to handle it. CWL parameter allows grouping of RC models in different flyability levels according to their performance and pilots' ability required to fly them. (Meyers 2018)

1.3 Objectives

The aim of this thesis is to introduce the cubic wing loading parameter into passenger aircraft design and decide if its use is reasonable, as well as which advantages can it provide. To achieve this objective the following question will be answered:

- Is cubic wing loading rather a constant value for passenger aircraft of comparable design?
- Is it possible to rewrite aircraft design theory based on this new parameter?
- What are the benefits or disadvantages of applying cubic wing loading into aircraft design?
- What is the current knowledge of this parameter and what can be investigated further?

For answering these main questions, a full analysis on statistics data is done over a wide range of planes, while the operations and necessary calculation to readapt aircraft preliminary sizing is addressed throughout this thesis.

1.4 Literature

The following documents contain the basic knowledge in cubic wing loading and it is introduced to aircraft designers. They all made the start point of this thesis:

- Durmus 2020, "Effects of Cubic Wing Loading Parameter on Airplane Sizing and Parasitic Drag", is a research article published in the Journal of Aeronautics and Aerospace Engineering, conducted by the Department of Aeronautics from Edremit School of Civil Aviation at Balikesir University, in Balikesir, Turkey. It concentrates on the physical meaning of cubic wing loading and relates it to relative wetted area and parasitic drag. Specifically of interest are the dimensional analysis carried out and the values of the parameter given for different types of aircraft. The document was the base of the present research. However, opposite to these document's statements, CWL was not found to be a constant parameter if considering passenger aircraft, neither was it a good parameter for preliminary sizing.

- Meyers 2018, "Wing Cube Loading", is an article published in the Silent Electric Flyers blog, which focus on the values of the parameters CWL and WL in RC planes. Highlighted is the classification of models in different flyability levels based on its CWL, making an interesting relation between these two characteristics.

- Reynolds 1989, "Model design & Technical Stuff', is an article published in Model Builder magazine which introduces the use of CWL in scale-model plane design. Of interest is the introduction of a new performance factor for models based on CWL. Some interesting figures are taken from the document.

- Küchemann 1978, "Aerodynamics Design of Aircraft", is a book which embraces the work of the German aerodynamicist Dietrich Küchemann, based upon material taught at Imperial College London. The influence of a volume parameter (and the square-cube law) on supersonic aircraft performance is deduced.

In the process of writing this thesis two other books compiling aircraft design knowledge were consulted:

- Kundu 2010, "Aircraft Design" is a didactic book published by Cambridge University Press. It compiles the required knowledge for both, passenger and military aircraft design. Of interest are sections referred to square-cube law and structural and fuel load of aircraft depending on their m_{MTO}.

- Raymer 1992, "Aircraft Design: A Conceptual Approach" is a scientific book published by the American Institute of Aeronautics and Astronautics where the established practices in aircraft conceptual design are exposed. Interesting for the research was the chapter dedicated to empty weight estimation for preliminary sizing.

Finally, of special relevance is the consultation of:

- Scholz 2015, "Aircraft Design Lecture Notes. Chapter 5: Preliminary sizing", written as lectures notes for Hamburg University of Applied Sciencies. Here, the whole preliminary sizing process of an aircraft is detailed and explained. The same structure for preliminary sizing was followed in this thesis but adpating it to the CWL parameter. These lecture notes will be referenced many times throughout the thesis.

1.5 Structure

This thesis is divided into five main chapters focusing on individual aspects of the present study.

- Chapter 2 gives the background information on the current circumstances and knowledge in CWL parameter for RC planes and former research that first introduced it into real aircraft design.

- Chapter 3 introduces fundamentals of CWL and takes care of mathematical basics through different equations.

- Chapter 4 deals with the transformation of preliminary sizing equations for conceptual aircraft design if they are based on CWL instead of traditional WL. Necessary changes are introduced and explained. As well as the correspondent consequences in the calculations.

- Chapter 5 presents statistical correlations of CWL with other relevant aircraft design parameters. Differences between narrow-body, wide-body and propeller aircraft are considered.

- Chapter 6 encompasses further information that CWL can provide to aircraft conceptual design. CWL is shown in relation with performance factor, accident rates, as well as fuel and structural efficiency.

All Excel sheets and MATLAB codes used for the research and referred in the thesis are collected in Harvard Dataverse: https://doi.org/10.7910/DVN/HELNOX. Details are also included in the appendices.

2 State of the Art

CWL has been historically used for RC planes designs. Recently it has been an attempt to introduce this parameter into real passenger aircraft design. The State of Art is presented below.

2.1 CWL in Radio Controlled Planes

In RC planes design, scale is an important factor, since it is common to compare different size planes. Therefore, a suitable parameter that takes size into account, is needed. Let us consider normal Wing Loading and Cubic Wing Loading parameters:

- WL is the ratio between the mass of the airplane and its wing surface (m/S). The maximum take-off mass and plain wing surface will be considered. Thus, its units are kg/m^2, as pressure has. Regarding into its dimensions, due to the square-cube law, and because mass scale as weight and it does it as volume, mass scales as the cubic of the length (a detailed study between mass and wing size relation in birds and bats is conducted in (Greenewalt 1975)), while surface does it as the square of the length, consequently:

$$\frac{m}{S} \sim \frac{l^3}{l^2} \sim l \tag{2.1}$$

The result is a parameter with dimensions of kg/m^2 which depends on the size of the plane.

- CWL is the ratio between the mass of the airplane and its wing surface to the power of 1.5. With this change, units change and now are kg/m^3, as density has; and both, numerator and denominator have a cubic form:

$$\frac{m}{S^{\frac{3}{2}}} \sim \frac{l^3}{l^{2\frac{3}{2}}} \sim \frac{l^3}{l^3} \sim 1 \tag{2.2}$$

This parameter does not depend on aircraft size but on the aircraft´s building materials and rigidity.

With inclusion of size-dependence in the analysis, the latter seems to be a good parameter for comparing different size planes, as opposed to the first parameter.

Additionally, CWL is an interesting comparative parameter as it relates the flight characteristics of the model. It should be pointed out that the wing loading determines how fast an aircraft must fly to remain in the air or to land, as it is deduced from the lift equation (when flying horizontally, the lift is equal to aircraft's weight)

$$W = L = \frac{1}{2}\rho S V^2 C_L \rightarrow V^2 = \frac{2}{\rho S C_L}\frac{W}{S} \quad . \tag{2.3}$$

The lower the WL is, the faster the aircraft must fly.

However, a scale effect should be considered, a high airspeed for a big model would have a similar effect as a low airspeed for a small model. Thus, the bigger plane will be harder to control, especially during landing, even though both have the same WL.

The importance of CWL is that it handles that difference in "flyability", as it handles the difference in size, and it indicates how skilled the pilot should be in order to fly the aircraft. To have some values in mind a beginner pilot should not start with a CWL higher than 8 kg/m^3. This is further explained in the next chapter.

2.1.1 Meyers' Contribution

Let us consider three different scaled-models of the same plane. A model of 0.3 m^2 and 1.7 kg will have a WL of 5.7, and a CWL of 10.35. Now, a reduced version of 0.15 m^2 and 0.6 kg will have a WL of 4 and a CWL of 10.33; and, an increased version of 0.65 m^2 and 5.4 kg will have a WL of 8.3 and a CWL of 10.30. The three of them will fly in a similar way. We must be careful as the last model has a WL much higher than the first one with a smaller aircraft, and it could lead to a misleading assumption: the aircraft was not built too heavy, but an incorrect comparative parameter is used. Values are collected in Table 2.1.

Table 2.1 Data for the examples models considered.

model	mass	wing surface	WL	CWL
small model	0.6	0.15	4.0	10.33
basic model	1.7	0.30	5.7	10.35
big model	5.4	0.65	8.3	10.30
	1.245	0.15	8.3	21.43

CWL provides a single-step comparative number since it is possible to know that the three models will have the same flight characteristics as their CWLs ~ 10.3. Then again, in order to understand how a WL = 5.7 plane will fly, the wing surface must be known too, and therefore it becomes a two-step comparative number. It is worth noting that WL values increase with models' size. A plane with WL = 8.3 and 0.65 m^2 wing surface will fly significantly different from one with 0.15 m^2 wing surface and the same WL value. The last example, with a mass of 1.245 kg and a CWL of 21.43 will be hard to fly, if it does.

Two charts are proposed for use of CWL for RC planes (Meyers 2018). They were simulated in MATLAB and are presented here in Figure 2.1.

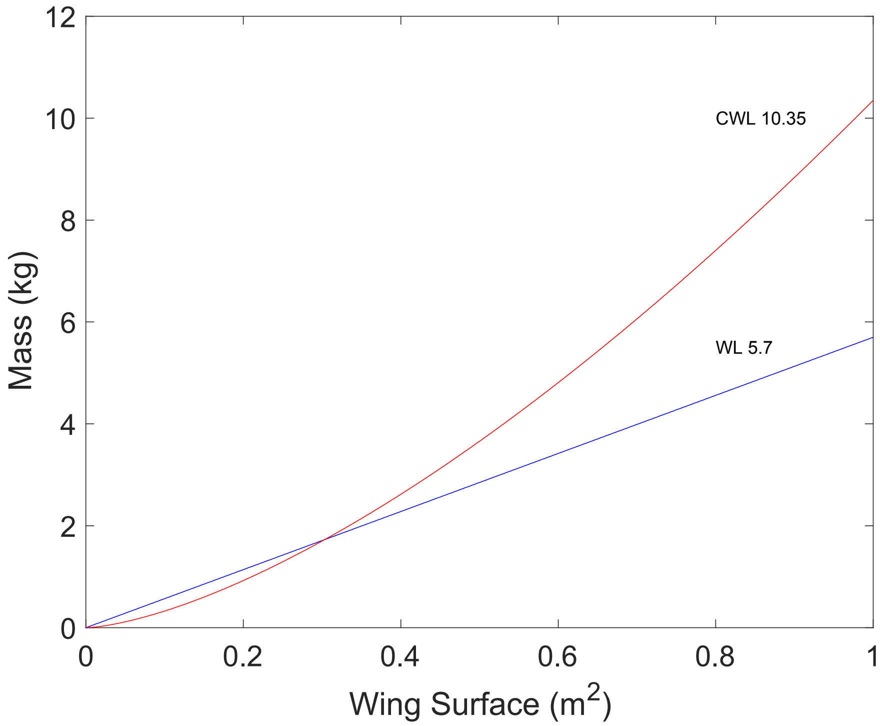

Figure 2.2 CWL and WL lines for basic model.

Figure 2.2 represents the lines for WL = 5.7 and CWL = 10.35 for the basic model. Note WL line is straight while CWL line is curved. This figure shows how, for a small range of wing surfaces, WL and CWL can be equally used to compare planes, but when the difference in wing surfaces gets higher and it approaches the larger end of the spectrum, WL cannot be used accurately as CWL can. Similar charts can be obtained for different models.

Figure 2.3 indicates how much a model must weigh if wing surface or mass are known and knowing it needs to fly similarly to another model with a CWL of 10.35. For example, for a 0.3 m^2 model, a mass of 1.7 kg will be needed, or a mass of 5.4 kg for a 0.65 m^2 model. Similar charts can be plotted for different CWL values.

Note both graphs represent the same CWL line. Meyers splits it in two different charts to emphasize the two concepts.

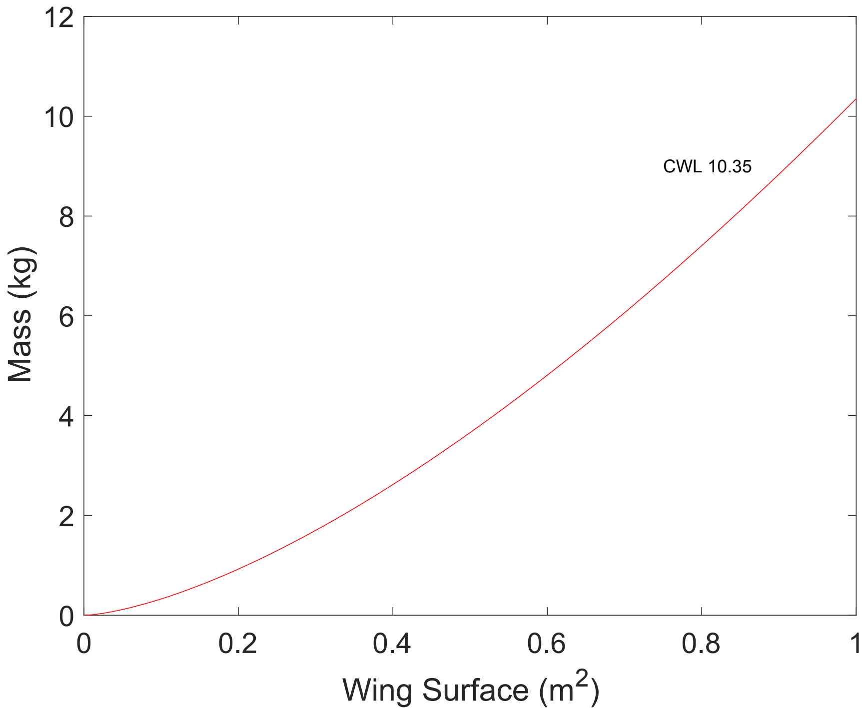

Figure 2.3 CWL line for basic model.

The author utilizes CWL values for grouping RC models in different "flyability" levels (Meyers 2018). A similar classification is presented in Table 2.2.

Table 2.2 "Flyability" levels based in RC planes' CWL.

level	CWL range	typical type(s)
1	0.00 … 2.99	indoor
2	3.00 … 4.99	backyard
3	5.00 … 6.99	park Flyers
4	7.00 … 9.99	sport planes & Trainers
5	10.00 … 13.99	advanced sport
6	14.00 … 16.99	expert types
7	17+	advanced expert Types

In the chart proposed by Meyers CWL range is evaluated in oz/ft^3. Values do not change substantially when converting them to kg/m^3.

2.1.2 Reynolds' Contribution

Reynolds (1989) introduced a new performance parameter related to CWL. This parameter is obtained by multiplying CWL with displacement loading (DL), i.e. the weight of the airplane divided by the displacement of the engine, preceded by a factor. CWL is an indicator of stall or landing speed, as WL, while DL is an indicator of maximum speed potential and rate of climb, as power loading. Therefore, together they are indicative of speed range. The lower the factor is, the higher the speed range and the better the aircraft's performance.

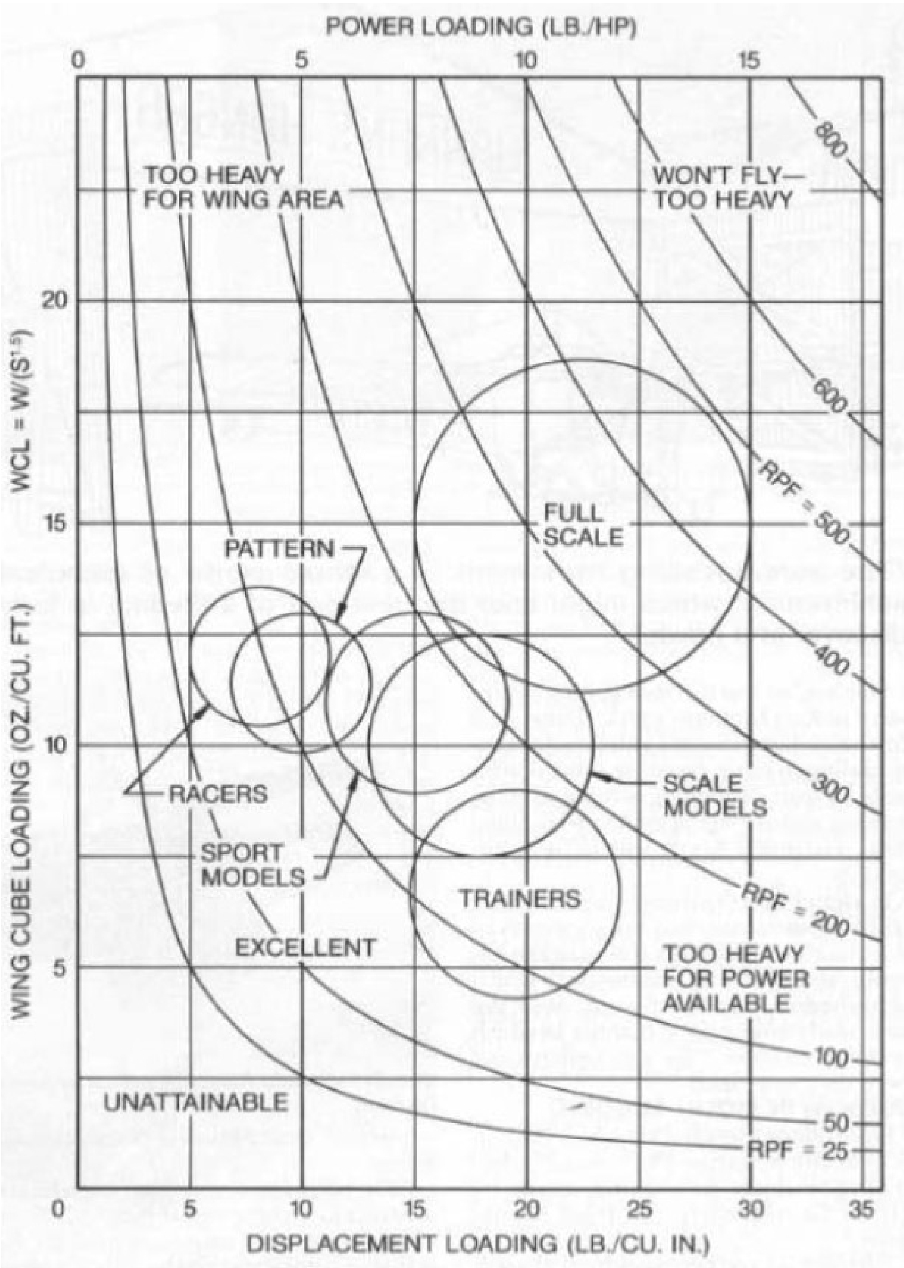

Figure 2.4 RPF parameter for RC planes (Reynolds 1989).

In Figure 2.4, from Reynolds' article, the lines represent constant values for the parameter explained, named as Reynolds Performance Factor (RPF), and the circles, where different types of models fit.

Despite all of this, it shouldn't be forgotten that basic design (power, airfoil) and other considerations of the design (center of gravity placement, angle of attack) have an enormous influence in the way planes fly the way they do. CWL offers an approximation for similar designs.

As described above, the advances that CWL provides to RC planes are notable. This does not apply to real passenger aircraft, which will be discussed in the following section.

2.2 CWL in Passenger Aircraft Preliminary Sizing

The most current knowledge in using CWL in passenger aircraft preliminary sizing is summarized in “Effects of Cubic Wing Loading Parameter on Airplane Sizing and Parasitic Drag” by Seyhun Durmus (2020).

However, Küchemann (1978) was the first author introducing a volume parameter (2.4) such as CWL is, into aircraft design. The volume parameter used affected waverider performance in supersonic flows and the wetted area over wing surface ratio.

$$\tau = \frac{Vol}{S^{\frac{3}{2}}} \tag{2.4}$$

This parameter increases when the waverider is thicker ($\tau \sim 0.1$) and decreases in thinner waverider ($\tau \sim 0.04$) for the same span over length ratio (s/l).

Pressure and skin friction drag in total drag computation for a lifting body is correlated to thickness: in aerodynamics, pressure drag importance increases when thickness does, whereas, skin friction matters more in aircraft with thin wings than in any other kind of aircraft. Therefore, drag is correlated with this volume parameter too.

The parameter was introduced into drag coefficient calculation and as a result, into aerodynamics efficiency estimation. L/D resulted to be a maximum when $\tau \to 0$, but, Küchemann also demonstrated that relatively high values of L/D could be obtained with quite high values of τ ($\tau \sim 0.08$), and that allowed designers space to accommodate liquid hydrogen tanks in the waverider.

Besides Küchemann, research about proper CWL introduction into real aircraft design was conducted by Seyhun Durmus, as mentioned before. CWL has shown to be an effective parameter for comparative study of different types of aircraft and he suggested it could be applied for wing sizing and parasitic drag calculation.

Wing size of an aircraft can be estimated from the average CWL value of aircraft built with similar materials from its definition:

$$\left(\frac{m_{MTO}}{CWL}\right)^{\frac{2}{3}} = S \tag{2.5}$$

Moreover, CWL can also be seen as a relation between wing and fuselage ratio. Wing and fuselage have different stiffness and density values, considering the aircraft mass as a whole, aircraft with larger wings have less density values and therefore, less CWL.

With respect to wetted area, Durmus stated high CWL aircraft are correlated to high relative wetted area and therefore, high parasitic drag, which supported Küchemann views.

Two more remarks are identified in Durmus´ study. Aspect Ratio (AR), another dimensionless parameter, offers along with CWL, an interesting comparative graph.

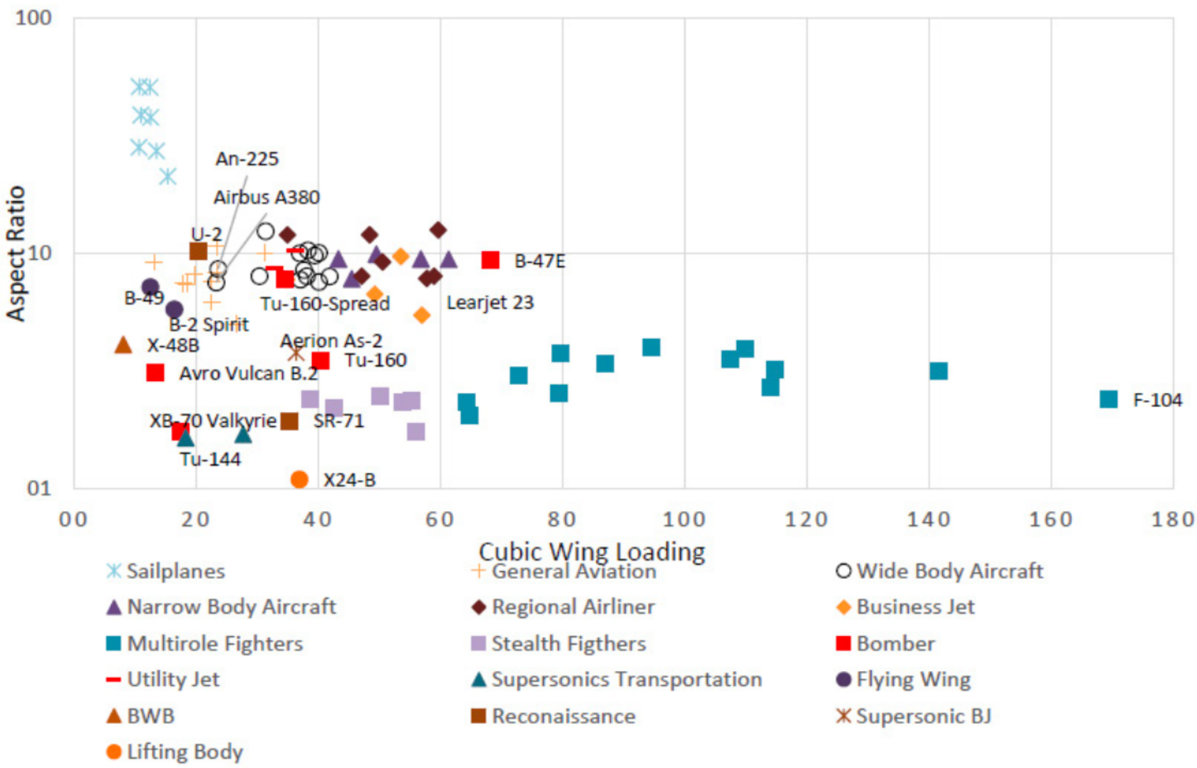

Figure 2.6 Aspect ratio and cubic wing loading chart of 81 aircraft. A base 10 log. Scale for AR was used for the readability of the scatter plot. (Durmus 2020)

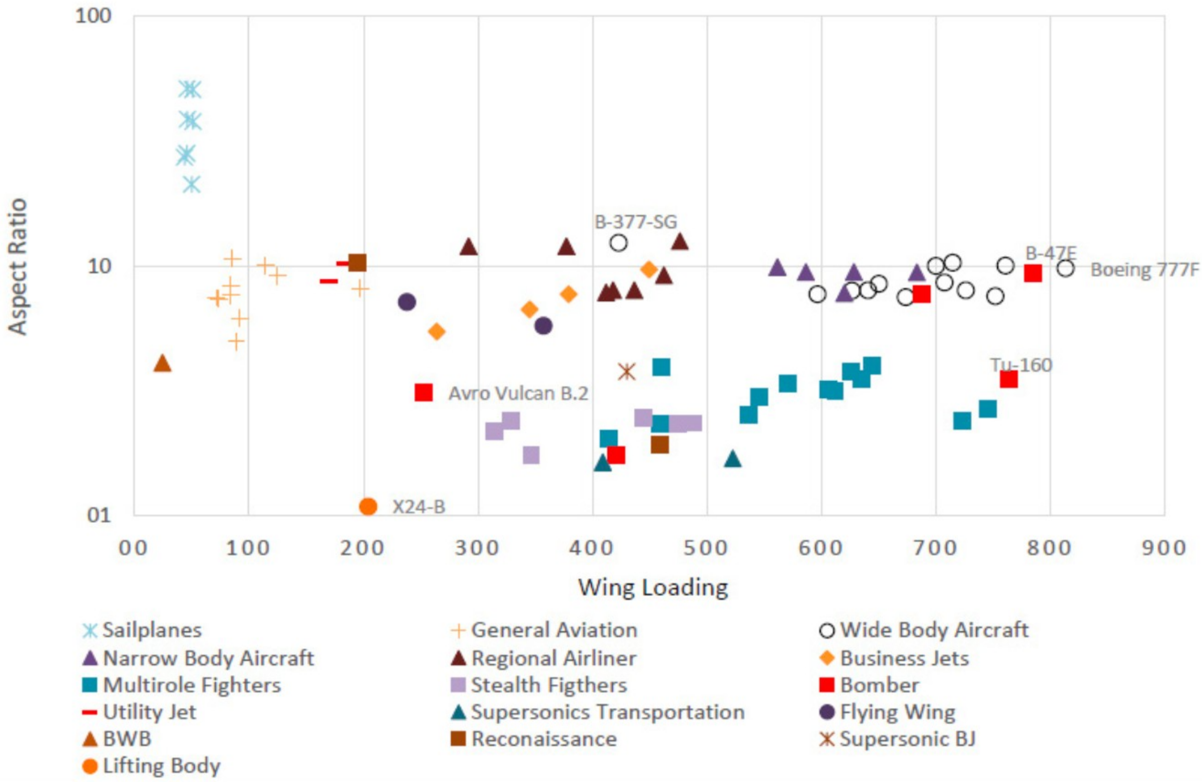

Figure 2.7 Aspect ratio and wing loading chart of 81 aircraft. A base 10 log. Scale for AR was used for the readability of the scatter plot. (Durmus 2020)

CWL is rather a constant value for similar aircraft design than normal WL. Figure 2.6 also shows fighters have much larger CWL than normal passenger aircraft, their wings are smaller and they are built with heavier materials (metallic alloys over composites). And sailplanes present the lowest CWL values, which is directly correlated with their high wing-fuselage ratio and so, the fact they land at lower speeds.

In Figure 2.7 it can be observed how types of aircraft are not separated regarding WL values. This gives CWL an initial advantage for comparing or identifying planes types.

To summarize, the research carried out in the past provides a detailed explanation on the physical meaning of CWL and introduces how it can help aircraft design. A detailed analysis related to passenger aircraft will be discussed in sections from 0 to 0.

3 Theoretical Basics

A mathematical analysis is conducted in this chapter to better understand CWL.

First, CWL expressed as a function of WL is addressed. Basic equations are

$$CWL = \frac{m_{MTO}}{S^{3/2}} \tag{3.1}$$

$$WL = \frac{m_{MTO}}{S} \quad . \tag{3.2}$$

From now on m stands for m_{MTO} through the whole chapter. Therefore

$$\frac{CWL}{WL} = \frac{m/_{S^{1.5}}}{m/_S} = \frac{S}{S^{1.5}} = \frac{1}{S^{0.5}} \tag{3.3}$$

$$\boxed{CWL = \frac{1}{S^{0.5}} WL \quad .} \tag{3.4}$$

Furthermore, if S is obtained from (3.2) then

$$\frac{1}{WL} = \frac{S}{m} \tag{3.5}$$

$$\frac{m}{WL} = S \tag{3.6}$$

$$CWL = \frac{1}{\left(\frac{m}{WL}\right)^{0.5}} WL \tag{3.7}$$

$$\boxed{CWL = \left(\frac{WL}{m}\right)^{0.5} \cdot WL = \frac{1}{m^{0.5}} WL^{1.5} \quad .} \tag{3.8}$$

Otherwise, if S is obtained from (3.1) the same expression is obtained with additional calculations

$$\frac{CWL}{m} = \frac{1}{S^{1.5}} \tag{3.9}$$

$$\frac{m}{CWL} = S^{1.5} \tag{3.10}$$

$$\left(\frac{m}{CWL}\right)^{\frac{1}{1.5}} = S \tag{3.11}$$

$$CWL = \frac{1}{\left(\left(\frac{m}{CWL}\right)^{\frac{1}{1.5}}\right)^{0.5}} WL \tag{3.12}$$

$$CWL = \frac{1}{\left(\frac{m}{CWL}\right)^{\frac{1}{3}}} WL \tag{3.13}$$

$$CWL = \left(\frac{CWL}{m}\right)^{\frac{1}{3}} WL \tag{3.14}$$

$$CWL^{\frac{2}{3}} = \frac{1}{m^{\frac{1}{3}}} WL \tag{3.15}$$

$$\boxed{CWL = \frac{1}{m^{0.5}} WL^{1.5} \quad .} \tag{3.16}$$

With the relation between CWL and WL, it is possible to study the relationship between other parameters and CWL that were originally expressed with WL, such as the proportion with the stall speed of an aircraft or, back to RC planes, the speed relation between two different scaled models.

Equation (2.3) allows to consider aircraft speed as proportional to the square root of WL. The following analysis establishes a relationship between this speed and CWL

$$V = \sqrt{\frac{2mg}{\rho S C_L}} \tag{3.17}$$

$$V \propto \sqrt{WL} \quad . \tag{3.18}$$

If we translate it to CWL then

$$\boxed{V \propto \sqrt{\frac{m}{S}} = \frac{m^{0.5}}{S^{0.5}} \frac{S}{S} = \frac{m^{0.5} S}{S^{1.5}} \frac{m^{0.5}}{m^{0.5}} = \frac{m}{S^{1.5}} \frac{S}{m^{0.5}} = CWL \frac{S}{m^{0.5}} \ or \ \sqrt{CWL\, S^{0.5}} \quad .} \tag{3.19}$$

The complex relationship becomes less intuitive than using traditional WL.

With respect to RC planes, as it can be deducted from (2.3), scaling a model implies changing its flying speed. A relationship between speeds for the original model and the scaled one could be estimated using WL first and CWL later

$$\frac{2\,mg}{\rho S} = V^2 \quad . \tag{3.20}$$

Sub-index 1 refers to original model

$$\frac{2\,m_1 g}{\rho S_1} = V_1^2 \quad . \tag{3.21}$$

The second model will be obtained scaling the first with a factor a. As stated earlier, in agreement with the square-cube law, mass scale as the cube of the length and surface as the square of it

$$\frac{2\,m_1 a^3 g}{\rho S_1 a^2} = V_2^2 \tag{3.22}$$

$$V_1^2 \frac{a^3}{a^2} = V_2^2 \tag{3.23}$$

$$V_2^2 = aV_1^2 \tag{3.24}$$

$$V_2 = V_1\sqrt{a} \quad . \tag{3.25}$$

Changing a for wing loading ratio in (3.25)

$$WL_1 = \frac{m}{S} \tag{3.26}$$

$$WL_2 = \frac{ma^3}{Sa^2} \tag{3.27}$$

$$\frac{WL_2}{WL_1} = a \tag{3.28}$$

$$\boxed{V_2^2 = \frac{WL_2}{WL_1} V_1^2 \quad .} \tag{3.29}$$

And now, WLs ratio for CWLs ratio

$$CWL = \frac{1}{S^{0.5}} WL \tag{3.30}$$

$$\frac{CWL_2}{CWL_1} = \frac{WL_2}{WL_1} \left(\frac{S_1}{S_2}\right)^{0.5} \tag{3.31}$$

$$\frac{WL_2}{WL_1} = \frac{CWL_2}{CWL_1} \left(\frac{S_1}{S_2}\right)^{0.5} \tag{3.32}$$

$$\frac{WL_2}{WL_1} = \frac{CWL_2}{CWL_1} \left(\frac{S_1 a^2}{S_1}\right)^{0.5} \tag{3.33}$$

$$\frac{WL_2}{WL_1} = \frac{CWL_2}{CWL_1} a \tag{3.34}$$

$$\boxed{V_2^2 = \frac{CWL_2}{CWL_1} a\, V_1^2 \quad .} \tag{3.35}$$

Again, in this example, relationship becomes simpler when using WL.

It is concluded that relationships become more complex when introducing CWL. This has been observed when showing the relationship with stall speed and minimum flight speeds for two differently-scaled RC models, and similarly it should become complex in other cases. The reason behind simpler equations are obtained with WL is because WL is directly obtained from lift Equation (2.3), while for obtaining CWL mathematical calculations need to be carried out over this equation (introducing exponents too). This is an important disadvantage for the use of CWL as a design parameter.

4 Preliminary Sizing Equations Introducing CWL Parameter

CWL is said to offer several advantages for RC plane design since, unlike traditional WL, it would be a non-size dependable parameter. The aim of this thesis is to check whether introducing this parameter into passenger jet aircraft design presents some advantage over WL-based preliminary sizing. As explained before, WL is mass over surface ratio, while CWL is mass over surface to the power of 3/2. For introducing CWL into preliminary sizing equations, they must be rewritten.

4.1 Preliminary Sizing

> *The preliminary sizing of an aircraft is carried out by taking into account requirements and constraints.* (Scholz 2015).

Those requirements are compiled in the airworthiness regulation codes EASA CS-25 / FAR Part 25.

Taking this in mind, a process for preliminary sizing must be followed. One was proposed by Loftin and it is summarized in Figure 4.1.

Figure 4.1 refers to jet aircraft. For propeller aircraft T must be exchanged by Power (P) in the equations and other changes are necessary.

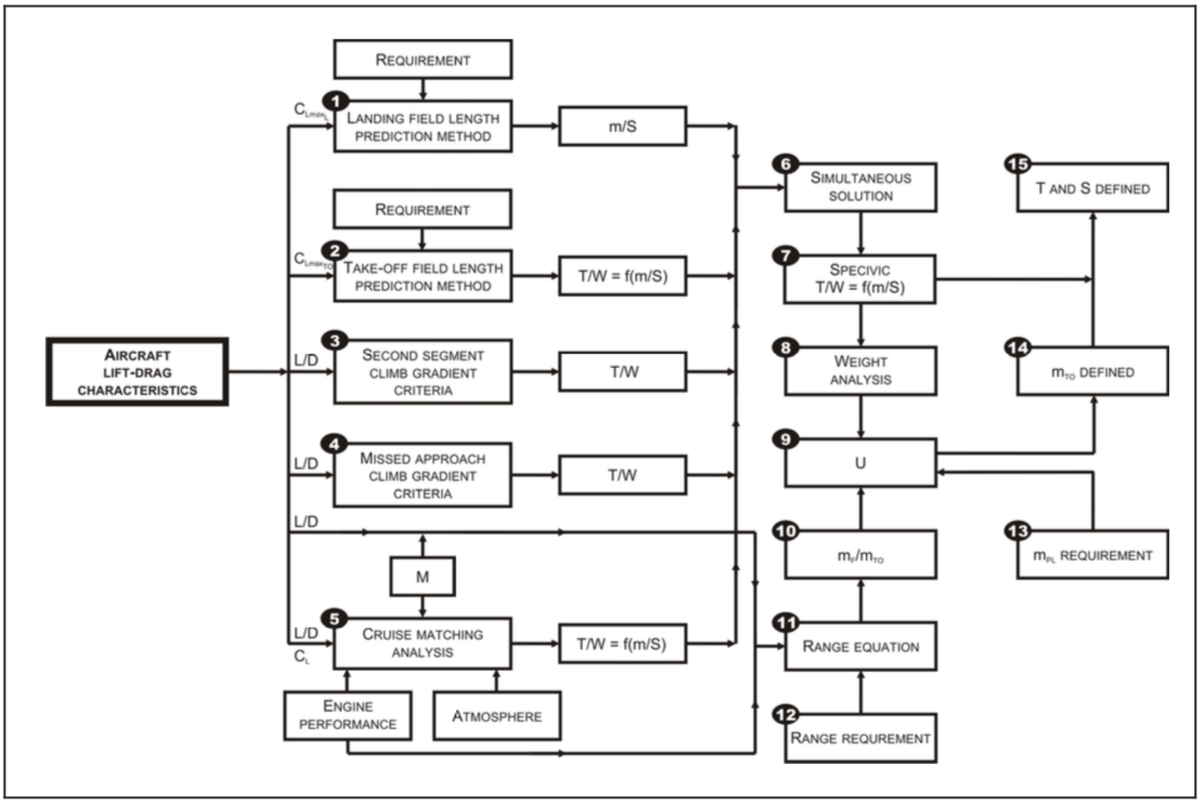

Figure 4.1 Flow chart of the aircraft preliminary sizing for jets (Scholz 2015). Based on (Loftin 1980).

Preliminary sizing expresses requirements and constraints in equations and considers all of them. They are transformed into a pair of appropriate values of T/W and WL for the plane to guarantee the compliance, regarding economical manners too. Now, it will be tried to rewrite equations with CWL and so, obtaining a pair of appropriate values of T/W and CWL.

What changes must be introduced into the equations is presented below. For a detailed understanding of preliminary sizing Scholz's lecture notes can be consulted (Scholz 2015). Here in the chapter, the same sequence is followed as in these lecture notes.

4.1.1 Landing Distance

Traditional landing distance equation provides a maximum value for the wing loading. This wing loading must not be exceeded if the aircraft is to meet requirements. Input values are maximum lift coefficient with flaps in landing configuration $C_{L,max,L}$, landing field length S_{LFL}, relative density σ, and factor k_L

$$\frac{m_{ML}}{S_w} = k_L S_{LFL} \sigma C_{L,max,L} \tag{4.1}$$

or referred to WL based on take-off mass

$$\frac{m_{MTO}}{S_w} = k_L S_{LFL} \sigma C_{L,max,L} \frac{m_{MTO}}{m_{ML}} \quad . \tag{4.2}$$

$C_{L,max,L}$ is selected by data in the literature, S_{LFL} is obtained according to CS/FAR from the landing distance and a safety factor (it is an input of the design problem) and k_L is giving by Loftin for jets airplanes and equal to 0.107 kg/m^3. σ depends on the airport's altitude and m_{ML}/m_{MTO} is calculated from statistics.

If we want to obtain CWL, all the terms in (4.2) must be raised to the power of 3/2 to get $S^{3/2}$

$$\frac{m_{ML}^{3/2}}{S_w^{3/2}} = (k_L S_{LFL} \sigma C_{L,max,L})^{3/2} \quad . \tag{4.3}$$

Now, it can be referred to CWL dividing both side by $m_{ML}^{3/2} / m_{MTO}$

$$\frac{m_{MTO}}{S_w^{3/2}} = \frac{m_{ML}^{3/2} / S_W^{3/2}}{m_{ML}^{3/2} / m_{MTO}} = \frac{(k_L S_{LFL} \sigma C_{L,max,L})^{3/2}}{m_{ML}^{\frac{3}{2}} / m_{MTO}} \quad . \tag{4.4}$$

As before, $m_{ML}{}^{3/2}/m_{MTO}$ has to be calculated from statistics. This was done with the aircraft data collected in Excel, detailed in Appendix A and shown in Table 4.1

Table 4.1 Statistical values of maximum landing mass to the power of 3/2 over maximum take-off mass $m_{ML}^{3/2}/m_{MTO}$ for jets of different design range.

design range classification	design range (NM)	design range (km)	$m_{ML}^{3/2}/m_{MTO}$
short range	up to 1000	up to 2000	119.6
medium range	1000-3000	2000-5500	188.3
long range	3000-8000	5500-15000	289.8
ultra-long range	more than 8000	more than 15000	323.7

4.1.2 Take-Off Distance

The take-off distance equation gives a value of WL and T/W which must not be undershot if the aircraft is to meet requirements. Input values are maximum lift coefficient with flaps in take-off configuration $C_{L,max,TO}$, take-off field length S_{TOFL}, relative density σ, and factor k_{TO}

$$\frac{T_{TO}}{m_{MTO}g} = \frac{k_{TO}}{s_{TOFL}\sigma\, C_{L,max,TO}} \frac{m_{MTO}}{S_w} \quad . \tag{4.5}$$

$C_{L,max,TO}$ is selected by data in the literature, S_{TOFL} is considered to be proportional to the take-off ground roll and is obtained according to CS/FAR (it is an input of the design problem) and k_{TO} is giving by Loftin for jets airplanes and is equal to 2.34 m^3/kg. σ depends on the airport's altitude.

To refer it to CWL, Equation (4.5) should be raised to the power of 3/2

$$\begin{aligned} \left(\frac{T_{TO}}{m_{MTO}g}\right)^{\frac{3}{2}} &= \left(\frac{k_{TO}}{s_{TOFL}\sigma\, C_{L,max,TO}}\right)^{3/2} \frac{{m_{MTO}}^{3/2}}{{S_w}^{3/2}} \\ &= \left(\frac{k_{TO}}{s_{TOFL}\sigma\, C_{L,max,TO}}\right)^{3/2} \sqrt{m_{MTO}} \frac{m_{MTO}}{{S_w}^{3/2}} \quad . \end{aligned} \tag{4.6}$$

A problem must be observed here, the second term depends now in m_{MTO} (further than the one included in CWL term), since it is an output of the preliminary sizing traditional process, it should be studied whether it is possible to calculate it up front or if an iteration is necessary for the CWL process. This appears to be an important limitation for preliminary sizing based on CWL. It will be discussed later.

4.1.3 Climb Rate during 2nd Segment

The equation gives a minimum value for T/W airplane should meet to guarantee the aircraft is able to climb with a specific gradient specified in the regulations even with one engine inoperative. Input values are aerodynamics efficiency *E* during take-off, climb gradient γ and the number of engines.

$$\frac{T_{TO}}{m_{MTO}g} = \frac{n_E}{n_E - 1}\left(\frac{1}{E} + sin\gamma\right) \tag{4.7}$$

E for take-off can be calculated as explained by Scholz notes (2015), γ is taken from CS/FAR, and number of engines is known from the design.

Since the equation does not depend on WL or CWL, no changes need to be applied.

4.1.4 Climb Rate during Missed Approach

The next equation is like the one obtained for the 2nd segment, only that mass corresponds now to maximum landing mass

$$\frac{T_{TO}}{m_{ML}g} = \frac{n_E}{n_E - 1}\left(\frac{1}{E} + sin\gamma\right) \quad . \tag{4.8}$$

To refer it to m_{MTO}, both sides must be divide by m_{MTO}.

$$\frac{T_{TO}}{m_{MTO}g} = \frac{n_E}{n_E - 1}\left(\frac{1}{E} + sin\gamma\right)\frac{m_{ML}}{m_{MTO}} \tag{4.9}$$

E for missed approach can be calculated as explained by Scholz (2015) as for take-off, γ is taken from CS/FAR, and the number of engines is known from the design. m_{ML}/m_{MTO} is calculated from statistics.

As before, the equation is not affected by WL or CWL and no changes are needed.

4.1.5 Cruise

In cruise, required values for T/W and WL are obtained separately and depend on altitude. Through this altitude dependence a relation can be established, later, between them.

- Thrust to Weight Ratio

The equation given for preliminary sizing is

$$\frac{T_{TO}}{m_{MTO}g} = \frac{1}{(T_{CR}/T_{TO})E} \quad . \tag{4.10}$$

Input values are aerodynamic efficiency during cruise E and the ratio between thrust for cruise and thrust for take-off T_{CR}/T_{TO}.

E for cruise can be calculated as explained by Scholz (2015), and T_{CR}/T_{TO} is a function of altitude and by-pass ratio (BPR). BPR is specified for the design. h_{CR} is the cruise altitude in meters.

$$\frac{T_{CR}}{T_{TO}} = (0{,}0013\ BPR - 0{,}0397)\frac{h_{CR}}{1000} - 0{,}0248\ BPR + 0{,}7125 \tag{4.11}$$

Equation (4.10) does neither depend on WL nor on CWL.

- Cubic Wing Loading

The equation given for preliminary sizing is

$$\frac{m_{MTO}}{S_w} = \frac{C_L M^2}{g} \frac{\gamma_{heat}}{2} p(h) \quad . \tag{4.12}$$

Input values are the lift coefficient during cruise C_L, Mach number M, the ratio of specific heats γ_{heat} (known as κ in relevant German literature), and the pressure.

C_L is calculated as explained by Scholz (2015), Mach number is specified for the design, γ is equal to 1.4 for air and pressure is a function of altitude.

If the equation wants to be referred to CWL, again, it must be raised to the power of 3/2

$$\left(\frac{m_{MTO}}{S_w}\right)^{3/2} = \left(\frac{C_L M^2}{g} \frac{\gamma}{2} p(h)\right)^{3/2} \tag{4.13}$$

and divided by $m_{MTO}^{1/2}$

$$\frac{m_{MTO}}{S_w^{3/2}} = \left(\frac{C_L \gamma}{2g} p(h)\right)^{3/2} \frac{M^3}{\sqrt{m_{MTO}}} \tag{4.14}$$

As in Equation (4.6) the second term depends on m_{MTO}. The problem will be addressed later.

Finally, T/W and CWL depend on altitude through T_{CR}/T_{TO} and pressure respectively. However, values can be given in a table like Table 4.2 and moved to a matching chart considering T/W as a function of CWL.

Table 4.1 Example table for the collection of cruise performance data.

altitude	CWL	T/W
…	…	…
…	…	…
5000 m	…	…
…	…	…
…	…	…

4.2 Matching Chart

Restrictions considered above for T/W and CWL can be captured in a matching chart since everything was referred to m_{MTO} although different flight phases were considered. An example is presented in Figure 4.2.

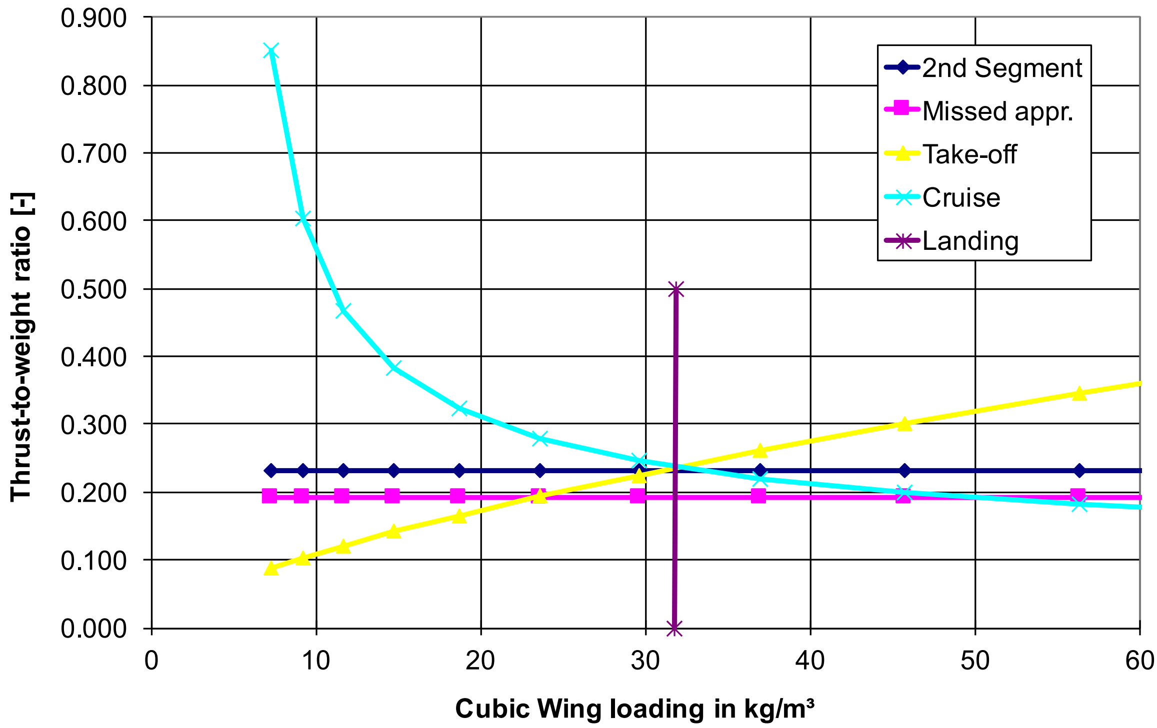

Figure 4.2 Matching chart. Lines mark restrictions as imposed by equations.

Considering minimum and maximum limit values for CWL and T/W imposed by equations, the area where the design must fit in the case studied to meet all requirements is between the take-off, cruise and landing lines. In Figure 4.3 those areas unsuitable are marked with hashed lines.

Now, since it was warned before, economical manners should be considered too. Then, the less thrust to weight ratio the better, as the power plant required for the plane will be cheaper. Consequently, the optimum design point is marked also in red in Figure 4.3. This leads to the values of CWL and T/W for the design that were being looked for.

Note V/V_{md} was given a value to bring the cruise line to the intersection of take-off and landing lines so that both, take-off, landing, and cruise, are sizing the aircraft at the same time.

V/V_{md} affects E and C_L calculation. Therefore, it has a direct effect in take-off and cruise curves through Equations (4.7), (4.9) and (4.14).

Figure 4.3 Matching chart: Design point regarding restrictions and economical manners.

4.3 Maximum Take-Off Mass

Now, it is time to address m_{MTO} estimation. In traditional preliminary sizing this can be done after all the calculation above, but for CWL preliminary sizing it is needed to get the value before, since, as it was warmed, m_{MTO} is an input value for Equations (4.6) and (4.14). The equation for m_{MTO} is

$$m_{MTO} = \frac{m_{PL}}{1 - \frac{m_F}{m_{MTO}} - \frac{m_{OE}}{m_{MTO}}} \tag{4.15}$$

It is required to calculate m_F/m_{MTO} and m_{OE}/m_{MTO} .

4.3.1 Relative Operating Empty Mass

Four approaches are given here to calculate the relative operating empty mass. All of them are based on statistics and head to different relative errors.

- Approach 1

Based on data collected, Raymer graphed m_{OE}/m_{MTO} versus m_{MTO} for different aircraft types, obtaining different curves for each and calculated an equation which fit these curves (Raymer 1992). The following equation was deducted

$$\frac{m_{OE}}{m_{MTO}} = A{m_{MTO}}^{c}k_{vs} \tag{4.16}$$

where parameters A and c, for jets transport, are equal to 1.02 and -0.06, while k_{vs} is 1 for fixed sweep wings, as normally considered.

Equation (4.16) depends on m_{MTO}, then, an iterative process for estimating m_{OE}/m_{MTO} is needed

- An approximate value for m_{MTO} is guessed
- The ratio m_{OE}/m_{MTO} is calculated through Equation (4.16)
- A new value for m_{MTO} is obtained from (4.15)

If the result does not match the guess value, a value between the two is used as the next guess. This will usually converge in just a few iterations.

The following approach considers equations calculated with an Excel sheet developed by student Jan Lehnert (Lehnert 2018) for the project "Methoden zur Ermittlung des Betriebsleermassenanteils im Flugzeugentwurf" at HAW Hamburg university. It was published with associated data from the project in Harvard Dataverse. To adapt equations to only already known input values in the preliminary sizing process, the original document and its equations have been modified. Different equations are considered. Only the one with less error rate are explained.

Equation (4.16) was also included in the mentioned Excel sheet and an error rate of 6.3% was obtained for m_{OE}/m_{MTO} estimation.

- Approach 2

A simple equation relates m_{OE}/m_{MTO} with range R through two parameters, k and u

$$\frac{m_{OE}}{m_{MTO}} = k + uR \tag{4.17}$$

R is known from design condition. When k and u takes 0.5967 and $-1.657 \cdot 10^{-5}$ 1/m as values respectively, an error rate of 4.48% is obtained (see Table 4.3).

Table 4.3 Parameters' values and error rate for equation (4.17).

k	0.5967
u	$-1.657\cdot10^{-5}$ 1/m
error	4.48%

- Approach 3

Another option works with a more complex equation. Input values are range R, maximum pay load m_{PL}, number of engines n_e and cruise speed V_{CR}. Four parameters are needed in this case, w, m, n and z

$$\frac{m_{OE}}{m_{MTO}} = R^w m_{PL}^m n_e^n V_{CR}^z \tag{4.18}$$

All input values are known from design specifications and are given in SI units based on m, kg, and s. V_{CR} is known since the Mach number is specified. It also depends on flight altitude, which needs to be estimated. The values parameters ought to take are expressed in Table 4.4.

Table 4.4 Parameters' values and error rate for equation (4.18).

w	-0.1054
m	$-1.76\cdot10^{-2}$
n	$-1.30\cdot10^{-2}$
z	0.0664
error	4.27%

Note, the error is roughly the same as in Equation (4.17) although the equation takes into account a higher number of parameters and input values, which makes it more complex. Therefore, in later calculation (4.17) is preferred over (4.18).

- Approach 4

It can also be noticed that, in Equation (4.17) u takes a value almost zero. So, it is guessed that it might work even simpler with

$$\frac{m_{OE}}{m_{MTO}} = k \quad , \tag{4.19}$$

where $k = 0.5289$. However, the error goes up until 8% (see Table 4.5). Thus, Equation (4.17) is still preferred.

Table 4.5 Parameters' values and error rate for equation (4.19).

k	0.5289
error	8.05%

Equations (4.16) and (4.17) are considered possible options for preliminary sizing. With (4.17) the iteration process can be omitted.

4.3.2 Relative Fuel Mass

Relative fuel mass is calculated from the equation

$$\frac{m_F}{m_{MTO}} = 1 - M_{ff} \tag{4.20}$$

where M_{ff} stands for mission fuel fraction, which is obtained from

$$M_{ff} = \frac{m_{SO}}{m_T}\frac{m_T}{m_L}\frac{m_L}{m_{DES}}\frac{m_{DES}}{m_{CR,alt}}\frac{m_{CR,alt}}{m_{CLB}}\frac{m_{CLB}}{m_{MA}}\frac{m_{MA}}{m_{DES}}\frac{m_{DES}}{m_{LOI}}\frac{m_{LOI}}{m_{CR}}\frac{m_{CR}}{m_{CLB}}\frac{m_{CLB}}{m_{TO}} = \frac{m_{SO}}{m_{TO}} \tag{4.21}$$

Each fraction represents the fuel mass consumed in a different flight phase represented in Figure 4.7 from take-off (TO) to switch-off (SO) of the engines at the destination airport. Cruise, loiter and cruise to alternative airport fractions are calculated according to the Breguet Equation (4.22), while others are estimated statistically.

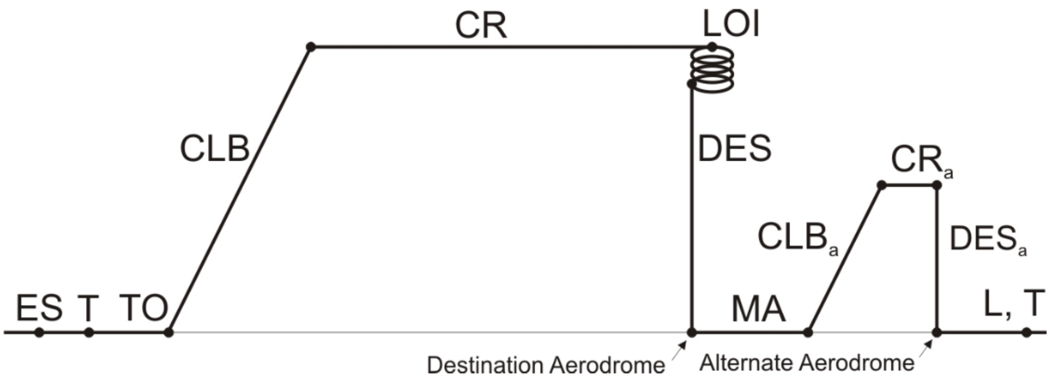

Figure 4.4 Typical flight phase of a civil aircraft flight mission (Scholz 2015).

The Breguet equation is

$$\frac{m_{i+1}}{m_i} = e^{-\frac{S_i}{B_s}} \tag{4.22}$$

where m_{i+1}/m_i take the form of m_{LOI}/m_{CR}, m_{DES}/m_{LOI} or m_{DESa}/m_{CRa} for each one of the phases mentioned respectively, s_i is the distance covered in that phase and B_s is Breguet factor.

s_i is specified within design requirements and B_s is calculated for jets planes as

$$B_s = \frac{{}^{L}/_{D}\, V}{SFC_T g} \tag{4.23}$$

This equation depends on thrust specific fuel consumption SFC_T, the earth's acceleration g due to gravity, speed V, and aerodynamic efficiency L/D. SFC_T is known from other planes or statistics and L/D for cruise can be calculated ias explained in Scholz (2015). At last, speed depends on altitude and therefore, the Breguet factor does too.

The altitude that the airplane flies at is unknown. Recalling (4.11) and assigning a value for T_{CR}/T_{TO}, the altitude can be identified. Therefore, now, an iterative process is compulsory in order to solve preliminary sizing based on CWL.

Estimating an appropriate value for T_{CR}/T_{TO} led to an altitude value and finally to a value for m_{MTO} which can be used to complete preliminary sizing. With the results and (4.10), a new value for T_{CR}/T_{TO} can be obtained and it is possible to check the accuracy of the original assumption. From there on, an iteration process can be conducted starting with the newly obtained T_{CR}/T_{TO} value obtained and identified. At the end, a precise value for altitude and m_{MTO} to complete preliminary sizing. Values as obtained from the iteration process are given in Appendix B. It is important to remember that with relative fuel mass and operating empty mass, m_{MTO} can be determined with (4.15).

4.4 Final Calculations

Finally, take-off mass and wing area can easily be calculated from T/W and CWL values obtained in the matching chart by their own definition

$$T = \frac{T}{W}\, m_{MTO} g \tag{4.24}$$

$$S_w = \left(\frac{m_{MTO}}{CWL}\right)^{\frac{2}{3}} \tag{4.25}$$

After this calculation, preliminary sizing is completed.

It is worth noting that other parameters such as maximum landing mass m_{ML}, operating empty mass m_{OE}, fuel mass m_F, required fuel mass $m_{F.Req}$, reserve fuel mass $m_{F.Rev}$ and zero fuel mass m_{ZF} can easily be calculated in the same way as in traditional preliminary sizing.

Finally, make sure m_{ML} is larger than $m_{ZF} + m_{F,Rev}$ after completing preliminary sizing. If not, increase the ratio $m_{ML\,/\,mMTO}$.

5 Statistical Correlations

In the second part of this study is to check relationships between CWL and other aircraft parameters. In order to accomplish this task, a wide range of data from different airplane models built through history was collected. Data can be extracted from Excel on Harvard Dataverse and graphs obtained are presented below.

5.1 General Observations

A comparative study between parameters' behavior with respect to CWL and traditional WL is shown in two charts, one for each parameter. Aircraft were classified in three different groups:

- Narrow-body aircraft
- Wide-body aircraft
- Propeller aircraft

The first type is represented by blue dots, second type to orange dots and third type to grey dots. In general, wide-body aircraft present a lower range of values for CWL and higher for WL than narrow-body and propeller planes. This statement will be explained in Chapter 5.2.

A difference can be perceived comparing CWL with WL. While CWL range is roughly equal for propeller, narrow-body and wide-body aircraft, WL tends to be lower for propeller aircraft than for jet planes. This is because CWL is a non-size dependable parameter, as opposed to WL. The reason why it is not completely constant in passenger aircraft is because they are not scaled proportionally (whereas RC planes are).

As a rule, aircraft that have upper, lower, right and left limits in each chart are identified, as well as other whose data may be out of the general trend. This enables a better and more exhaustive understanding of the charts.

Aircraft with the lower and higher values of CWL are Ilyushin 14M and Boeing 737-400. IL-14M is an ancient plane which has an oversized wing surface for its weight and Boeing 737-400 is a stretched and increased m_{MTO} version of B737 models 100, 200, 300, which led to an under-sized wing.

Planes with the lower and higher values of WL are Piper PA-31 Navajo and Airbus models 340-500 and 340-600 respectively. Piper PA-31 Navajo, (or Embraer 820) is a light aircraft for 7 passengers closer to leisure flying planes (generally with lower WL which requires less

T/W and lower landing speeds), while the high m_{MTO} and four engines of both Airbus 340-500 and 340-600 and their under-sized wing results in a high WL value.

Supersonic planes, Tupolev 144D and British Aerospace/Aérospatiale Concorde, pop also out of regular planes in charts. Their completely different design for supersonic flight gives parameters unusual values compared to traditional planes.

A table next to each chart provides information about linear trend lines and R^2 values (R^2 is an indicator of the correlation between two parameters. The closer to 1, the highest the correlation is). Generally, correlation is low and it remains under 0.5. For a detailed analysis see Chapter 7.2.

5.2 Maximum Take-Off Mass and CWL

Maximum take-off mass is a variable directly related to CWL, the higher it is, the higher CWL is for the same wing surface. Nevertheless, when increasing m_{MTO}, wing surface tends to increase too and it has the opposite effect on CWL:

$$CWL = \frac{m_{MTO}}{S^{3/2}} \tag{5.1}$$

The main difference between CWL and WL graphs is the trend-lines' gradient, negative for CWL and positive for WL.

If increase in size of planes results in a proportional weight and surface increase, according to the square-cube law, CWL would be a constant parameter. However, they do not. Larger planes have an increase in surface greater than the weight increase. Therefore, as warned earlier, CWL range is lower for wide-body planes. The reasoning behind that type of built style, will be described in Chapter 0. On the other hand, WL is a size dependable parameter, for larger planes and because of the square-cube law, WL increases.

$$WL = \frac{m_{MTO}}{S} \tag{5.2}$$

Airbus 380 is the upper limit of the chart. The two-story plane is considerable the biggest than other models. Propeller planes have the lowest values, which are also related to their smaller sizes. Only propeller aircraft Tu-114 reaches values similar to narrow or even wide-body values as this plane was a long-range four-engines turboprop with a striking different design and range compared to ordinary propeller planes, generally used for regional routes.

The information is collected in Figure 5.1, Figure 5.2, and Table 5.1, Table 5.2.

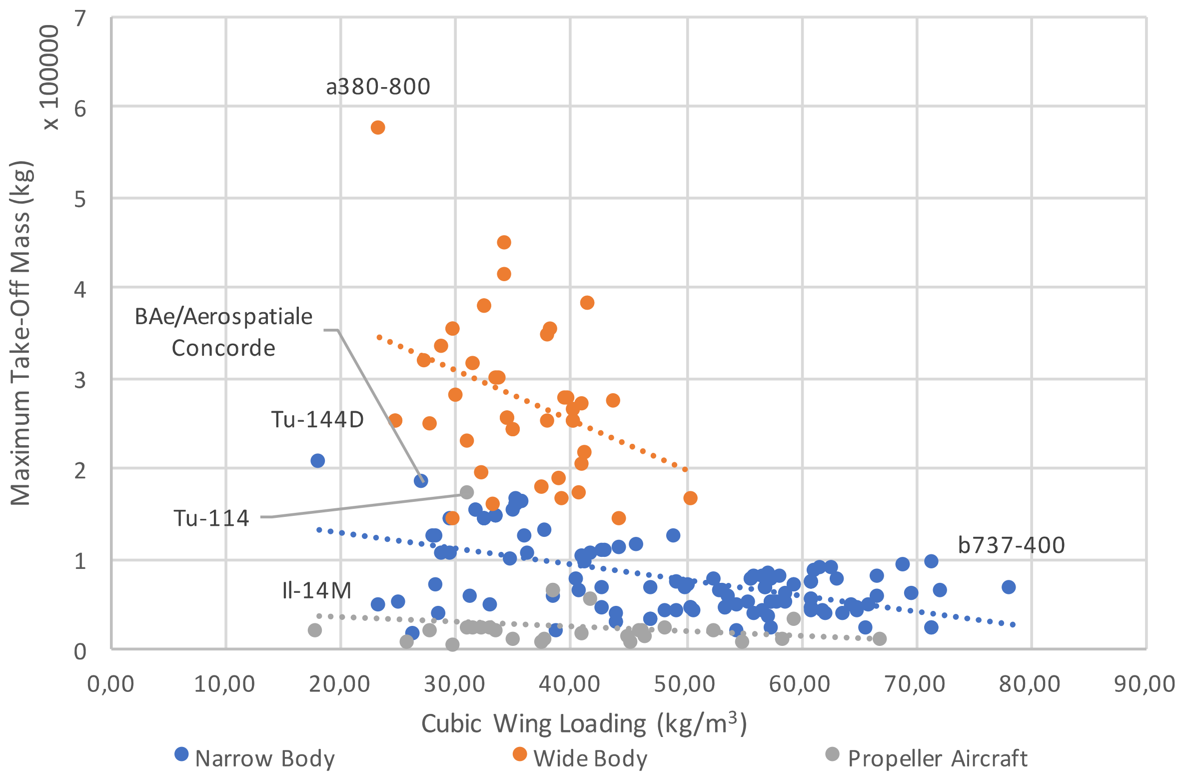

Figure 5.1 Maximum Take-Off Mass and Cubic Wing Loading.

Table 5.1 Trend-line´s data for Figure 5.1.

aircraft type	equation	R^2
narrow body	m_{MTO} = -1761.7·CWL + 164697	0.31702
wide body	m_{MTO} = -5469.3·CWL + 472135	0.13043
propeller aircraft	m_{MTO} = -484.49·CWL + 45340	0.02778

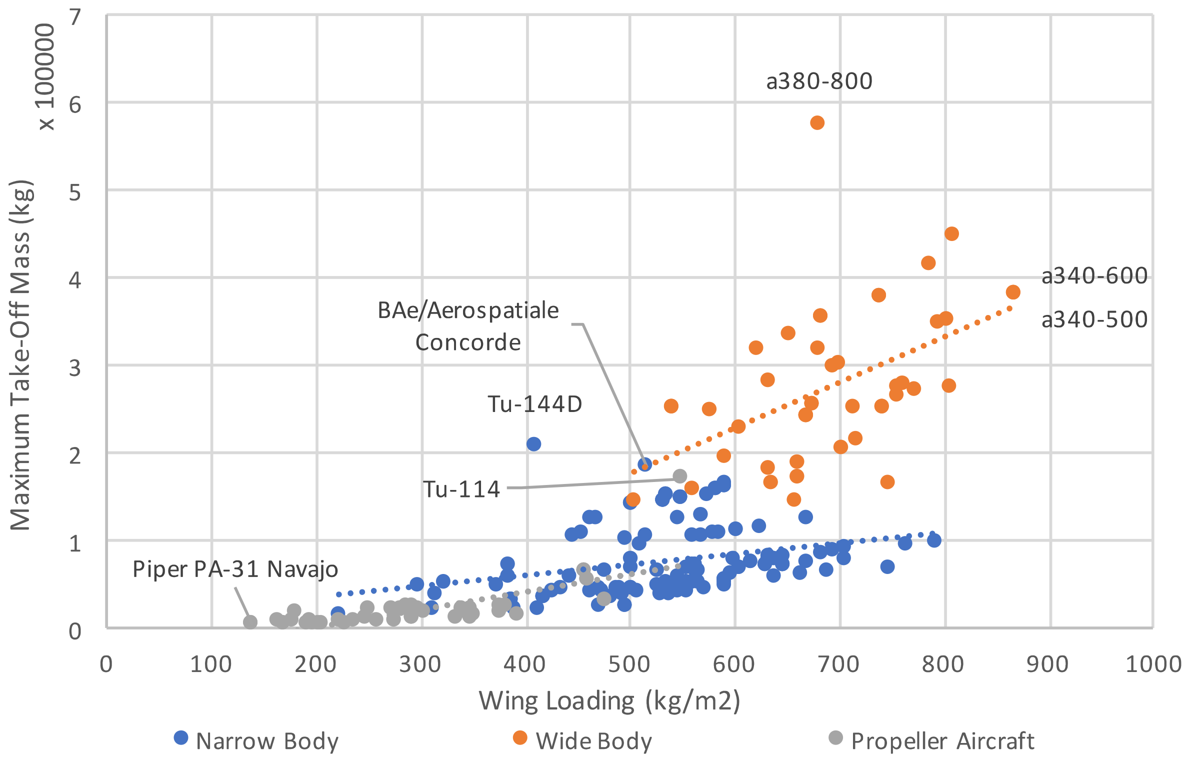

Figure 5.2 Maximum Take-Off Mass and Wing Loading.

Table 5.2 Trend-line´s data for Figure 5.2.

aircraft type	equation	R^2
narrow body	m_{MTO} = 118.34·WL + 14717	0.07563
wide body	m_{MTO} = 521.38·WL - 84075	0.24774
propeller aircraft	m_{MTO} = 200.71·WL - 37729	0.5211

5.3 Wing Surface and CWL

The other parameter which directly affects CWL is wing surface.

When increasing CWL, wing surface decreases (Figure 5.1). Logically, wide-body and larger airplanes have larger wing surfaces, and propeller have lower ones. Airbus 380 is again in the upper limit of the chart and Tu-114 stands out among traditional propeller planes.

However, the trend changes for WL: in propeller aircraft, there is a positive correlation and in narrow and wide bodies, WL remains almost constant. It is worth recalling that the increase in weight is higher when increasing the aircraft size due to the square-cube law and this difference in proportion is not corrected with traditional WL (Figure 5.2).

Out of the general trend is the Yakolev 40, a soviet regional jet that is closer to a propeller aircraft utility and has a reduced maximum take-off weight compared to other jet planes. It should be noted the difference between the biggest and smallest wing surface among wide-body planes, with an extremely different wing area (219 m^2 versus 845 m^2) but with a similar proportional increase in weight and wing surface (against square-cube law) and hence, a close WL value: Airbus' models 310 and 380.

The information is collected in Figure 5.3, Figure 5.4, and Table 5.3, Table 5.4.

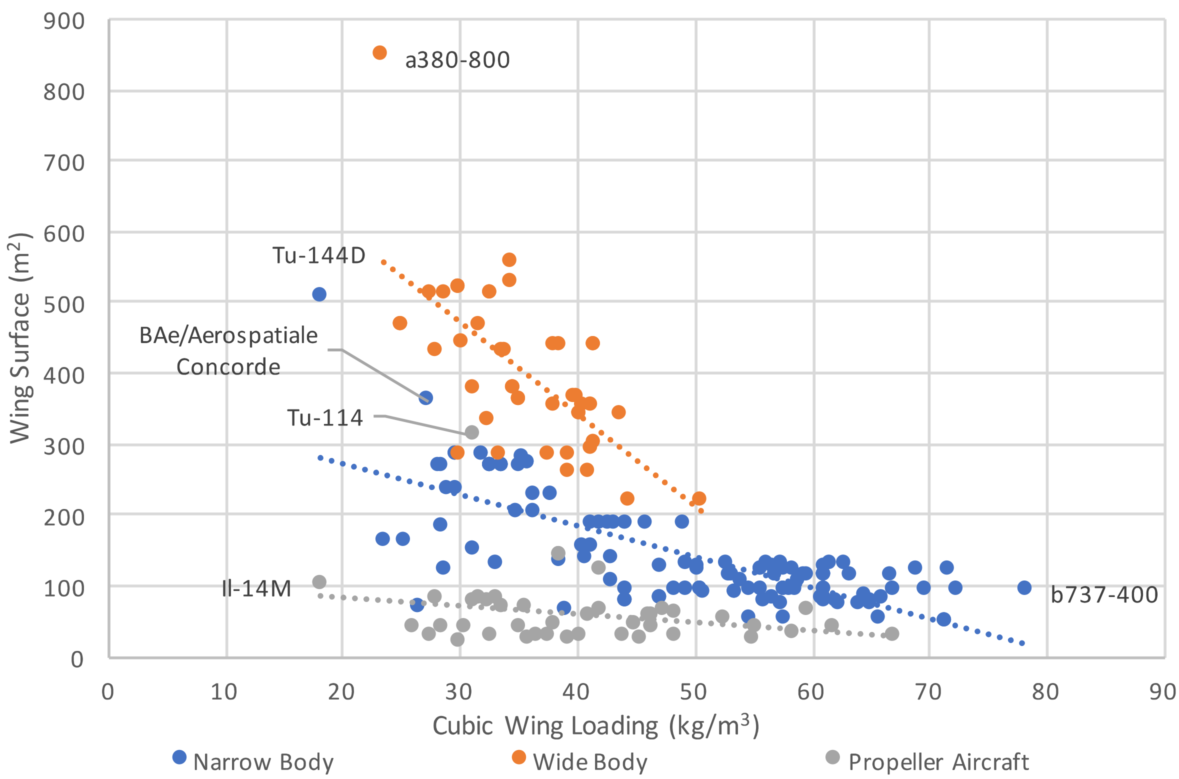

Figure 5.3 Wing Surface and Cubic Wing Loading.

Table 5.3 Trend-line´s data for Figure 5.3.

aircraft type	equation	R^2
narrow body	S_w = -4.4022·*CWL* + 360.58	0.55782
wide body	S_w = -12.984·*CWL* + 860.53	0.45212
propeller aircraft	S_w = -1.1277·*CWL* + 104.97	0.05893

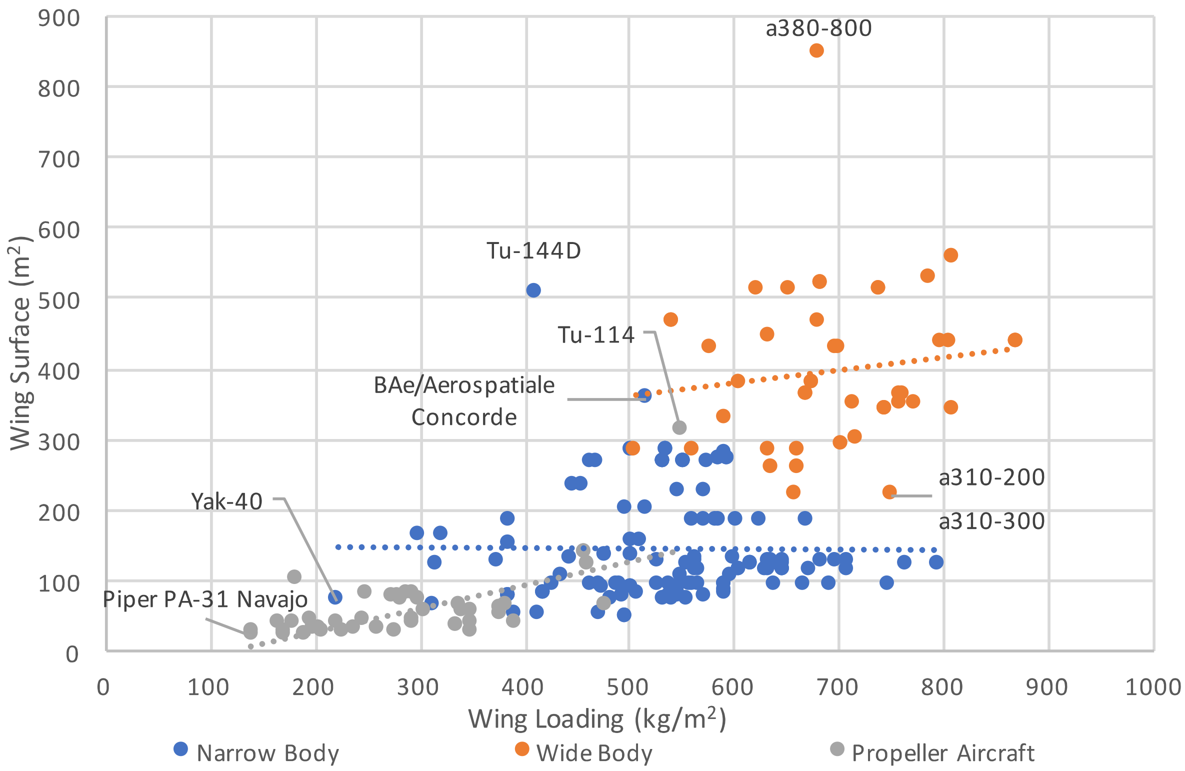

Figure 5.4 Wing Surface and Wing Loading.

Table 5.4 Trend-line´s data for Figure 5.4.

aircraft type	equation	R^2
narrow body	S_w = -0.008·WL + 149.79	9.7E-05
wide body	S_w = 0.1751·WL + 276.95	0.01719
propeller aircraft	S_w = 0.3303·WL – 36.227	0.43843

5.4 Aspect Ratio and CWL

An increasing tendency for Aspect Ratio AR – generally known as A in the literature – with CWL can be noticed. A direct relation can be established between these two parameters when an increase of CWL is due to a decrease in wing surface. It is worth recalling that the AR is the span of the aircraft divided by the square of the wing surface ($AR = b^2/S_w$).

The lower values in AR are correlated to supersonic aircraft with delta-wings (Tu-144 and Concorde). On the contrary, the highest value of AR corresponds to the turboprop DHC-8 Q300. In general, propeller and smaller planes have greater AR, which maximizes aerodynamics' efficiency but generates bigger stress in aircraft's structure and therefore, it would have a negative impact in heavier planes.

AR also increases with WL. Again, the Yak-40 has an AR value higher than generally narrow-body jet aircraft for its WL and closer to propeller aircraft's value, as noted earlier. Similar for the propellers Ilyushin 18 and 18D, a lower AR is needed for their WL due to their four-engines long-range design, which makes them closer to narrow-body aircraft.

The information is collected in Figure 5.5, Figure 5.6 and in Table 5.5, Table 5.6.

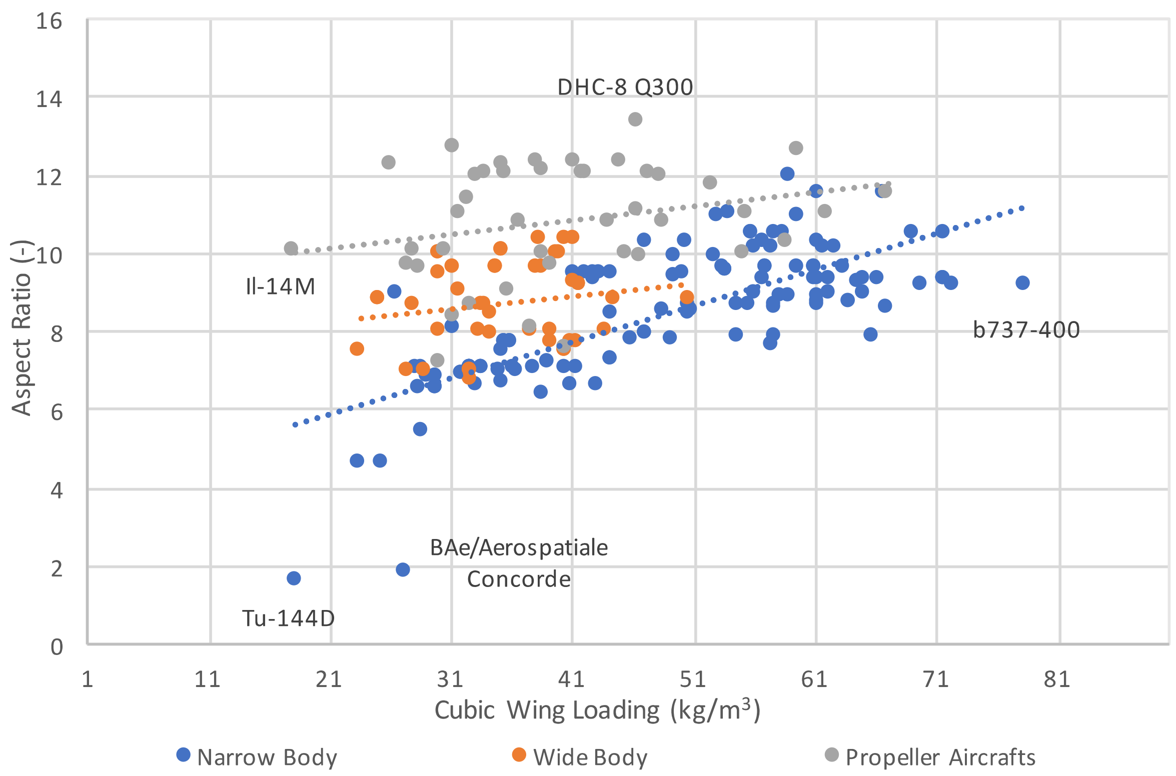

Figure 5.5 Aspect Ratio and Cubic Wing Loading.

Table 5.5 Trend-line´s data for Figure 5.5.

aircraft type	Equation	R^2
narrow body	$AR = 0.0929 \cdot CWL + 3.9134$	0.54659
wide body	$AR = 0.0324 \cdot CWL + 7.5508$	0.03081
propeller aircraft	$AR = 0.0347 \cdot CWL + 9.4294$	0.06251

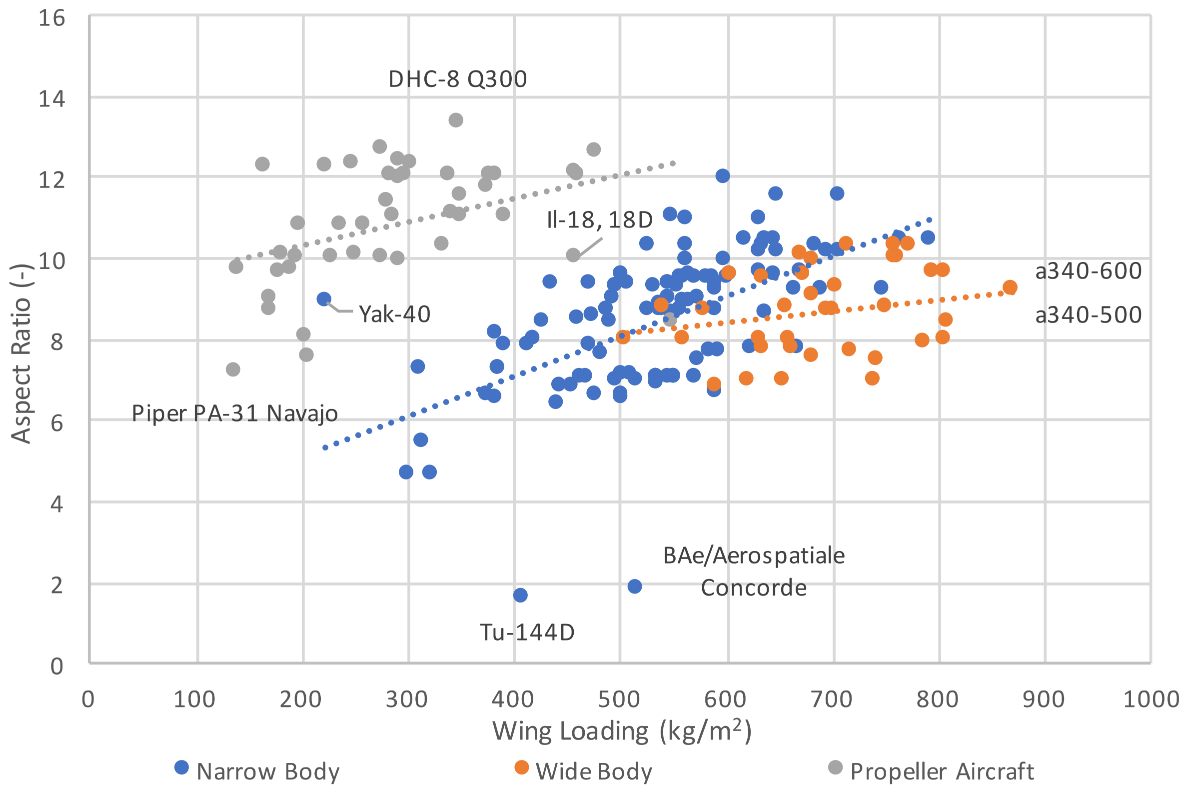

Figure 5.6 Aspect Ratio and Wing Loading.

Table 5.6 Trend-line´s data for Figure 5.6.

aircraft type	equation	R^2
narrow body	$AR = 0.0099 \cdot WL + 3.1209$	0.32653
wide body	$AR = 0.0027 \cdot WL + 6.8302$	0.04478
propeller aircraft	$AR = 0.0058 \cdot WL + 9.1542$	0.1521

5.5 Aircraft´s Speed and CWL

In the following section, 4 charts are presented. Maximum cruise speed is usually referred to maximum Mach number for jet planes which fly faster, while, for propeller aircraft, maximum speed is normally given in km/h. Similarly, in soviet planes, it is common to find these data in km/h instead of Mach number even if the plane is propelled by jet engines. For that reason, analyses are carried out in two different charts for CWL and WL depending on type of aircraft.

Maximum Mach number is rather a constant value for any CWL and close to $M = 1$. This is because airplanes tend to fly as fast as possible without exceeding sound´s speed which will result in a considerably increase in drag and hence in fuel consumption. For wide-body aircraft, maximum Mach number used to be slightly higher than for narrow-body planes, since their range is higher, a slightly higher speed against worst fuel consumption is appealing for airlines because of time saved. The supersonic aircraft developed by Tupolev and British Aeroespace/Aérospatiale can be clearly differentiated in the chart. Yak-40 flies slower since it was similar to propeller aircraft, which tend to fly slower (Fokker-50).

The information is collected in Figure 5.7, Figure 5.8 and in Table 5.7, Table 5.8.

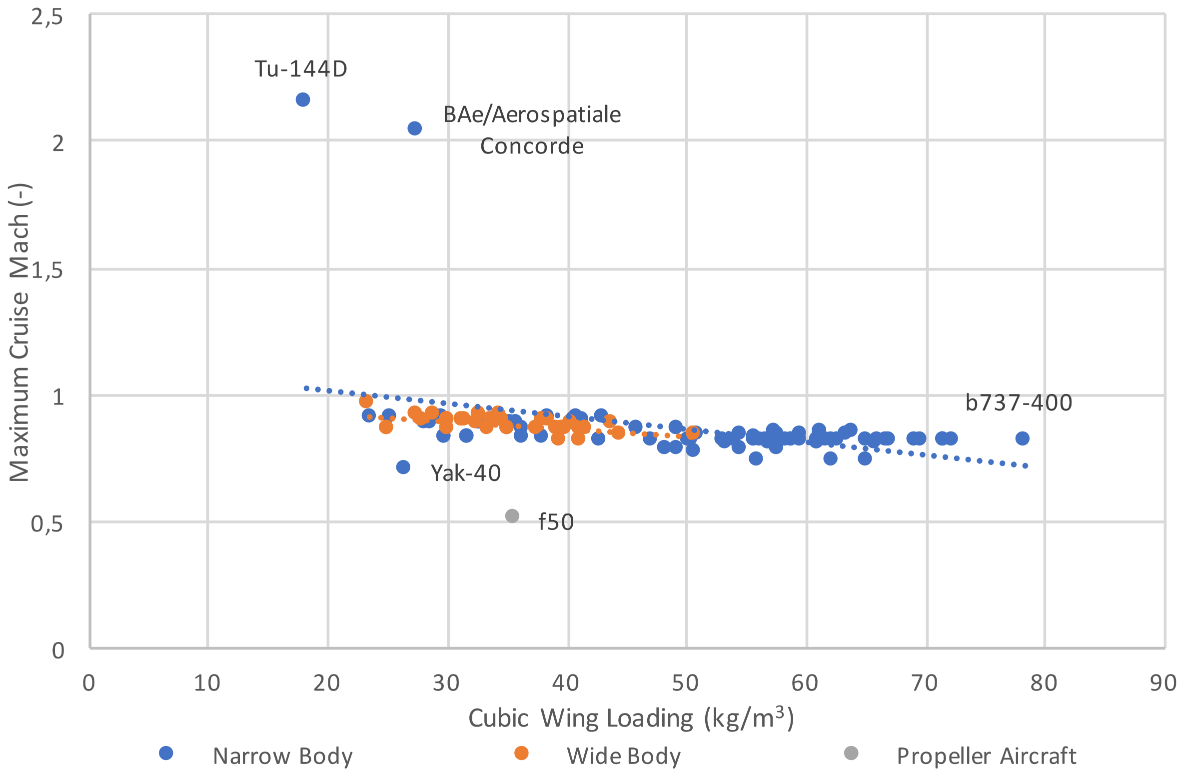

Figure 5.7 Maximum Cruise Mach and Cubic Wing Loading.

Table 5.7 Trend-line´s data for Figure 5.7.

aircraft type	equation	R^2
narrow body	$M_{max,cr}$ = -0.0051·CWL + 1.1179	0.15418
wide body	$M_{max,cr}$ = -0.0028·CWL + 0.9777	0.32699

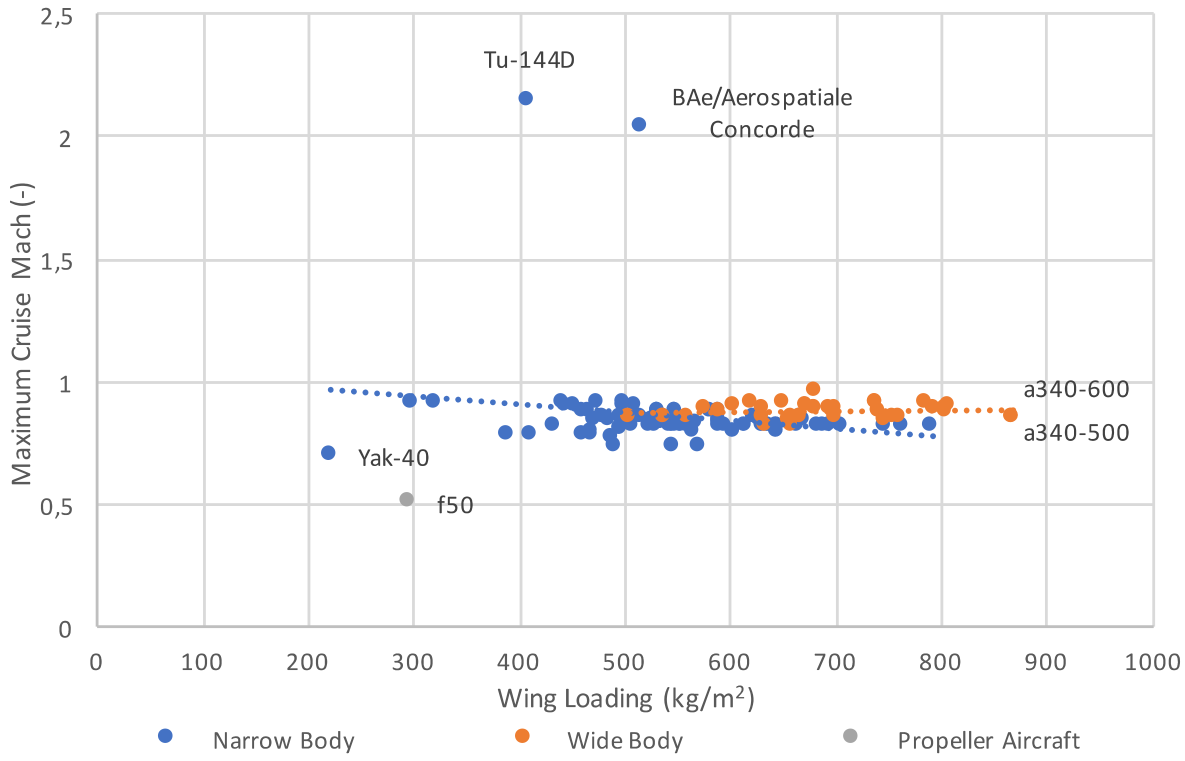

Figure 5.8 Maximum Cruise Mach and Wing Loading.

Table 5.8 Trend-line´s data for Figure 5.8.

aircraft type	equation	R^2
narrow body	$M_{max,cr} = -0.0003 \cdot WL + 1.0376$	0.02654
wide body	$M_{max,cr} = 4 \cdot 10^{-5} \cdot WL + 0.8519$	0.0149

Regarding plot for maximum speed in km/h versus CWL, propeller aircraft linear proportion increases with CWL as with WL. (3.19) and (3.20) showed a direct correlation between these two parameters. Nonetheless, note R^2 value is higher for CWL. Narrow-body and wide-body aircraft are again brought to their maximum possible speed and close to $M = 1$.

Tu-114 and VFW-Fokker 614 take an unusual place. Tu-114 was a four-engine long-range turboprop closer to narrow-body designs, while the low speed of VFW 614 is related to its special upper-wing engines design. As a result, both aircraft have a non-traditional design.

The information is collected in Figure 5.9, Figure 5.10 and in Table 5.9, Table 5.10.

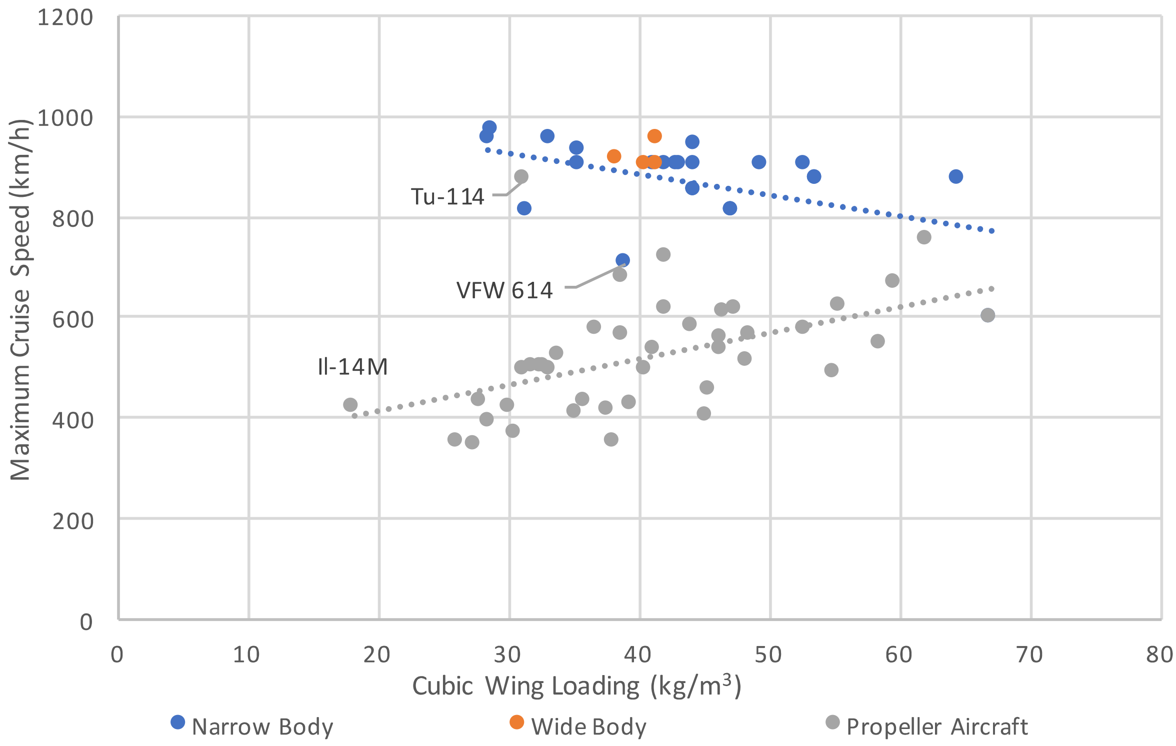

Figure 5.9 Maximum Cruise Speed and Cubic Wing Loading.

Table 5.9 Trend-line´s data for Figure 5.9

aircraft type	equation	R^2
narrow body	$V_{max,cr}$ = -4.2416·CWL + 1055	0.25573
propeller aircraft	$V_{max,cr}$ = 5.2159·CWL + 307.96	0.23946

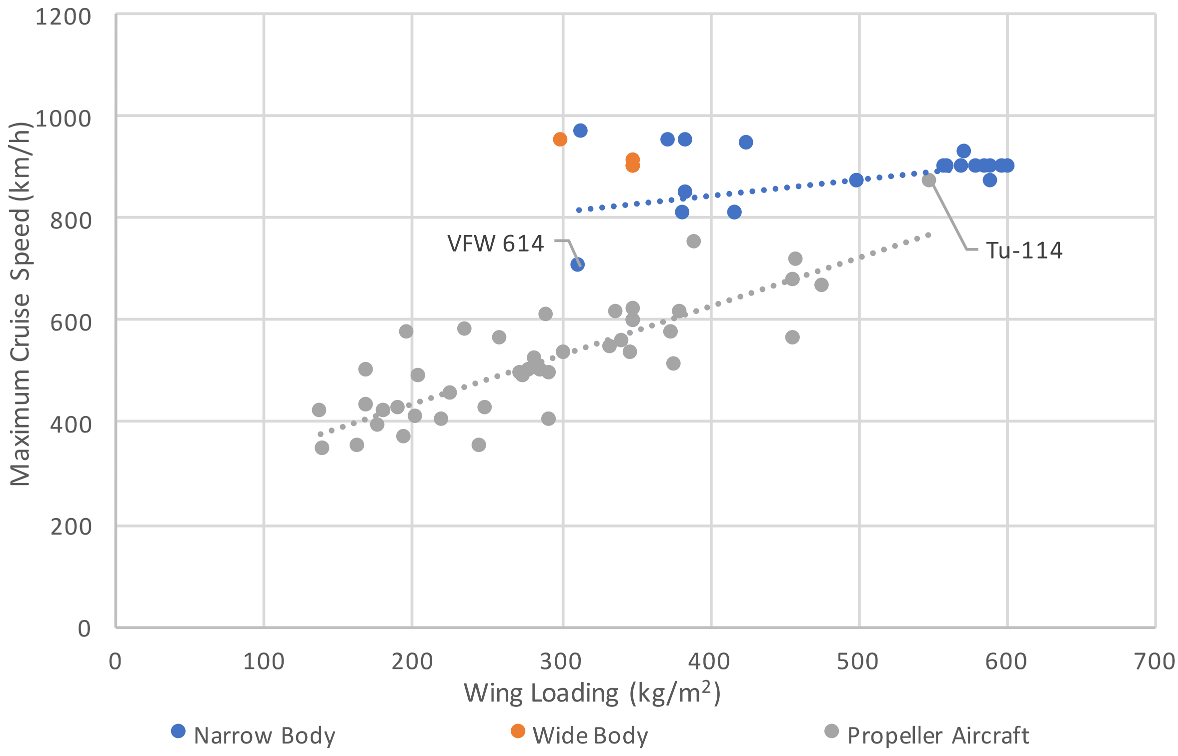

Figure 5.10 Maximum Cruise Speed and Wing Loading.

Table 5.10 Trend-line´s data for Figure 5.10.

aircraft type	equation	R^2
narrow body	$V_{max,cr} = 0.3166 \cdot WL + 717.46$	0.15243
propeller aircraft	$V_{max,cr} = 0.9514 \cdot WL + 244.98$	0.69432

5.6 Range and CWL

In this chapter the relationship between range and CWL is studied. However, R^2 values are especially low for this parameter, since many factors affect range (speed, specific fuel consumption, aerodynamic efficiency…), but it is not directly correlated with CWL or WL. Regardless, the gradient obtained with statistical data is negative for CWL and positive for WL.

Boeing 777-200 LR, a long-range version of Boeing 777, has the longest range for planes. Note higher ranges for narrow-bodies corresponds to the DC-8 Series, and to Tu-114 within the propeller aircraft. DC-8, was a four-engines narrow-body long-range plane with closer characteristics to a wide-body aircraft. Generally, range is higher in wide-body planes and lower in propellers. The lowest value for wide-body planes corresponds to the short to medium-range wide-body model developed in ancient URSS, Il-86.

The information is collected in Figure 5.11 Figure 5.12 and in Table 5.11, Table 5.12.

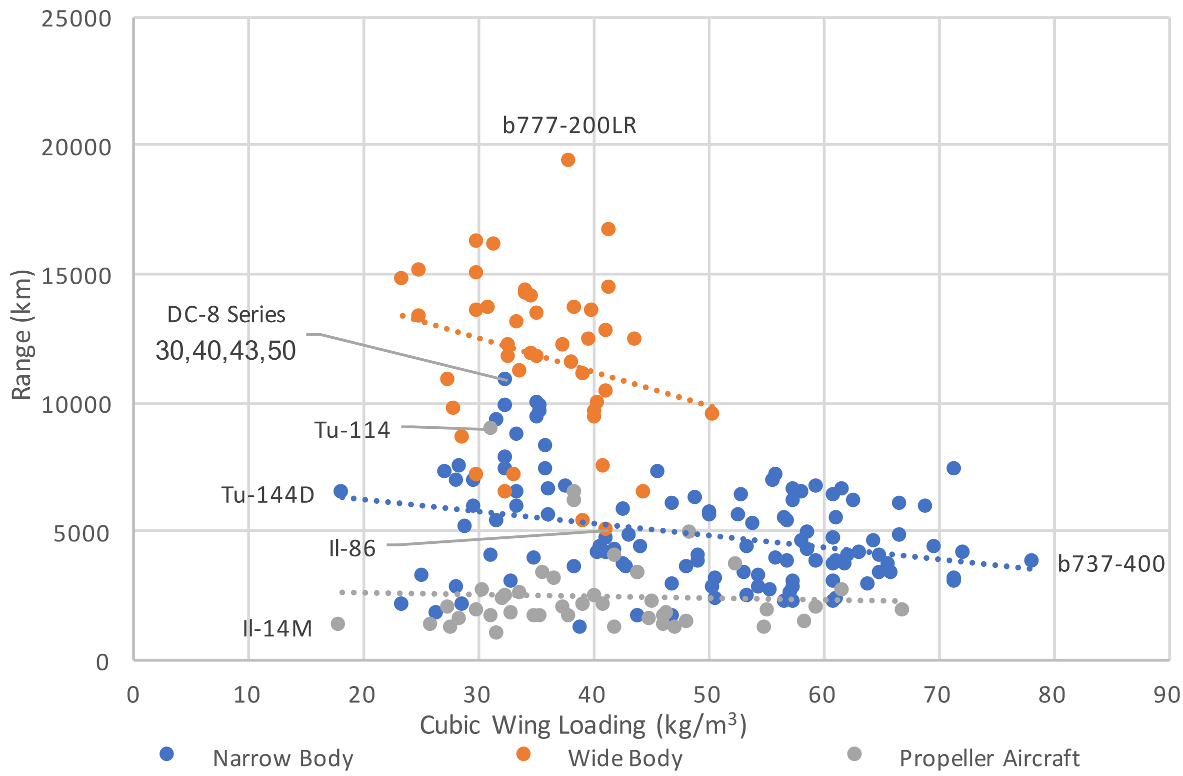

Figure 5.11 Range and Cubic Wing Loading.

Table 5.11 Trend-line´s data for Figure 5.11.

aircraft type	equation	R^2
narrow body	R = -46.842·CWL + 7198.7	0.09174
wide body	R = -133.08·CWL + 16529	0.05961
propeller aircraft	R = -6.2926·CWL + 2713	0.00177

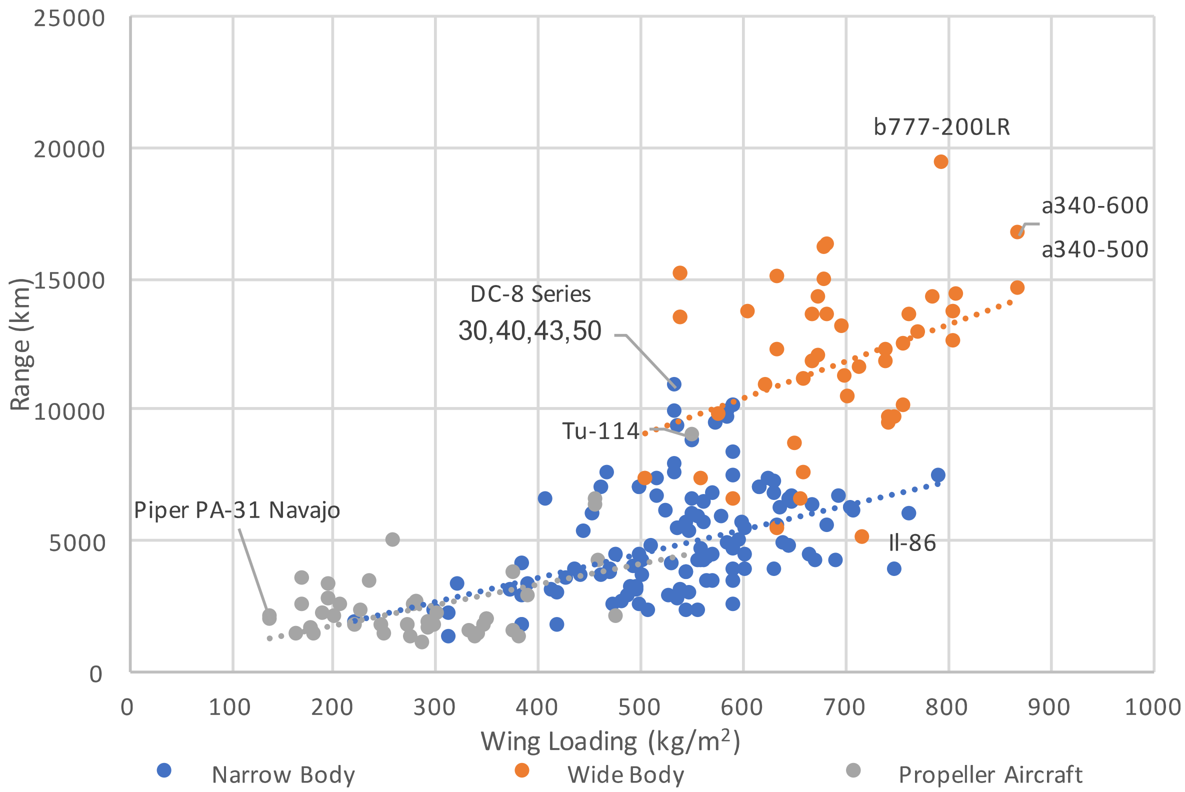

Figure 5.12 Range and Wing Loading.

Table 5.12 Trend-line´s data for Figure 5.12.

aircraft type	equation	R^2
narrow body	R = 9.2301·CWL – 73.809	0.18834
wide body	R = 14.053·CWL + 2047.3	0.13894
propeller aircraft	R = 7.9883·CWL + 148.37	0.24667

5.7 Wetted Area and CWL

Durmus (2020) claimed CWL would be highly proportional to relative wetted area. However, wetted area is not a usually given parameter for planes like wing area, hence it was not possible to calculate the ratio between both. An estimated value of 6~6.2 is considered for relative wetted area in conceptual aircraft design (Scholz 2015). With this, it is not possible to study the proportion. Considering a ratio of 6.1, S_{wet} over CWL or WL could be plotted, but it would result in the same graphs obtained in Chapter 0 with surface values multiplied by 6.1.

5.8 Cubic Wing Loading Evolution in Time

Studying how CWL and WL values have evolved in time may also be of interest. A limited research study is present in the following chapters, regarding only Airbus and Boeing models (72 models). Nevertheless, in commercial aircraft market, globally 72.6% of planes are produced by these two manufacturers (Cubas 2021).

The overall trend is to have increased CWL values over time for narrow-body aircraft and decreased values for wide-body aircraft. A reduction on CWL is expected because of the introduction of composites materials in aeronautical industry, lighter than original metal alloys, which makes density values decrease. Manufacturer's have an increased use of composites in their new wide-body models Boeing 787 and Airbus 350 (Bowler 2014). The use of these materials can reduce the weight of an aircraft by up to 10% (Apropedia 2009).

Airbus 380 has a low CWL value, according to Durmus (2020) it was especially designed for improving fuel over structural efficiency (this will be discussed in Chapter 0). Airbus 310 is a smaller version of the wide-body plane Airbus 300, and its design characteristics make it behave closer to narrow-body aircraft. First and second generation of Boeing 737s have a high CWL value.

The oldest model is the first Boeing 707 and the newest are Boeing 777-8 and 777-9.

For WL, similarly, the parameter used to be higher in time for narrow-body while for wide-body the proportion is roughly constant. This change in wide-body planes trend makes sense since the introduction of composites let building aircraft with a higher increase in wing surface over weight (in proportion and compared to what the square-cube law says) and hence, CWL values, where surface has a bigger influence (with a power of 3/2), decrease but WL do not.

A stretched version, and heavier Airbus 321neo from Airbus 320 led to a high wing loading. Boeing 767-200, as the Airbus 310 competitor, has a behavior closer to narrow-body models. On the top, Airbus 340-500 and 340-600 have the higher WL values. The lowest value is found in Boeing 720 with an over-sized wing for its m_{MTO}.

The information is collected in Figure 5.13 Figure 5.14 and in Table 5.13, Table 5.14.

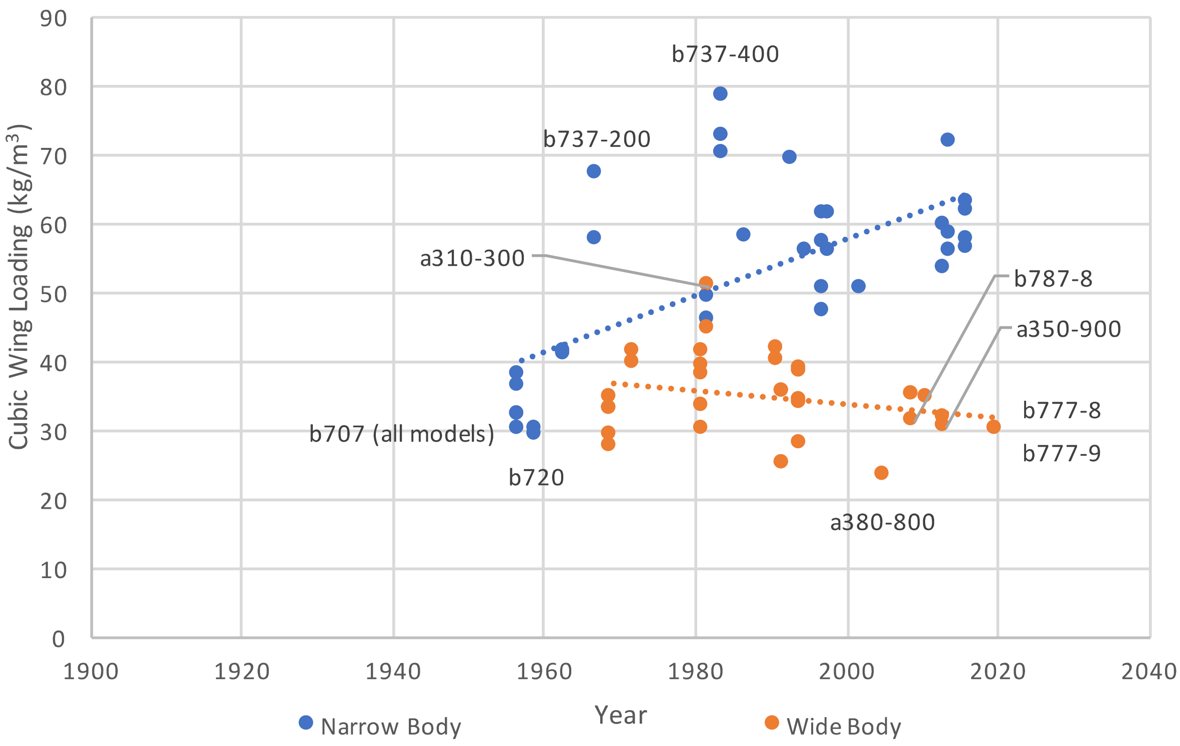

Figure 5.13 Cubic Wing Loading through Years.

Table 5.13 Trend-line´s data for Figure 5.13.

aircraft type	equation	R^2
narrow body	$CWL = 0.4091 \cdot Year - 760.71$	0.43019
wide body	$CWL = -0.1024 \cdot Year + 238.47$	0.0693

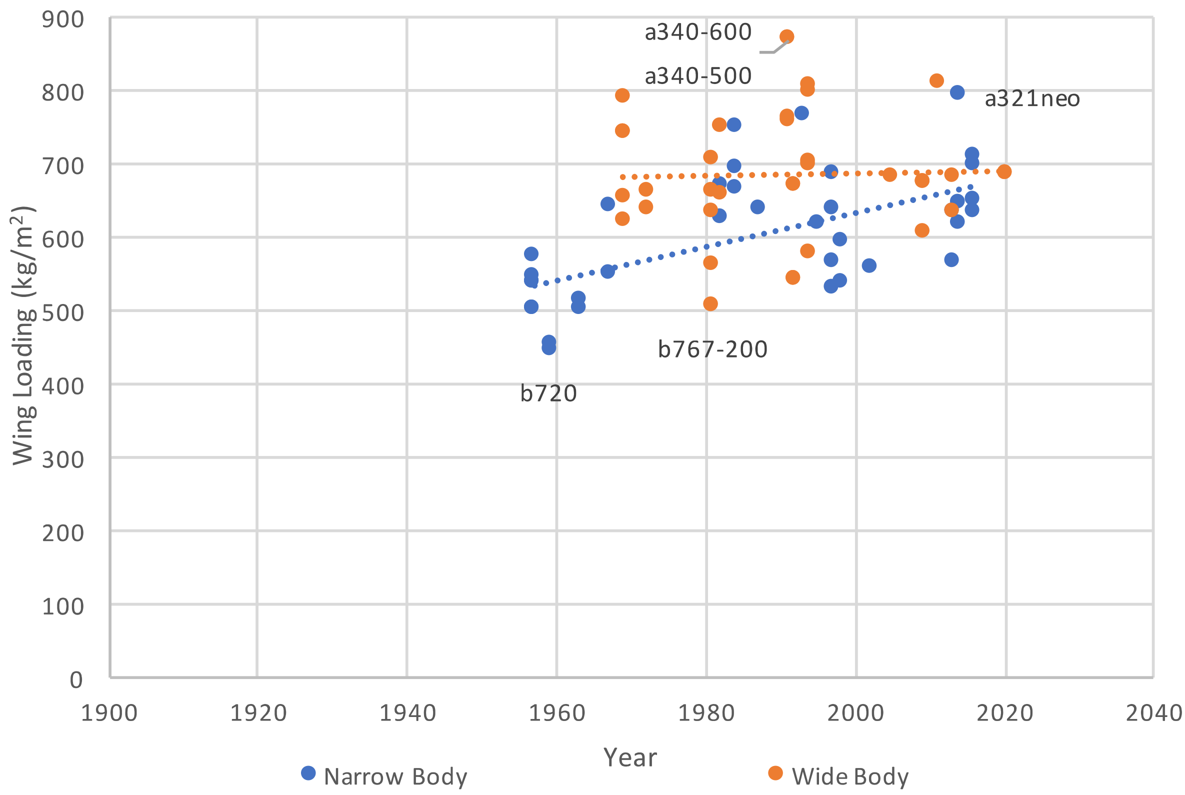

Figure 5.14 Wing Loading through Years.

Table 5.14 Trend-line´s data for Figure 5.14.

aircraft type	equation	R^2
narrow body	$WL = 2.2998 \cdot Year - 3966.3$	0.35197
wide body	$WL = 0.1885 \cdot Year + 309.94$	0.00108

5.9 Aspect Ratio Evolution in Time

Clearly AR has increased from the first designed aircraft to the new ones. This positive gradient represents the effort to improve aerodynamic efficiency in new models since a higher AR reduces induced drag (sailplanes, with AR higher than 20, have the higher aerodynamics efficiency within different types of aircraft). New generation aircraft, such as Boeing 777 and Airbus 220 reach for Airbus, and almost for Boeing, the highest AR value of their respective categories. In 60 years, AR has increased from 6.56 for Boeing 707-320 to 10.97 for Airbus 220. Again, Airbus 380, with its over-sized wing has a lower value according to Figure 5.15, while Airbus 330 has the highest among wide-body planes.

$$AR = \frac{b^2}{S_w} \tag{5.3}$$

The information is collected in Figure 5.15 Figure 5.16 and in Table 5.15, Table 5.16.

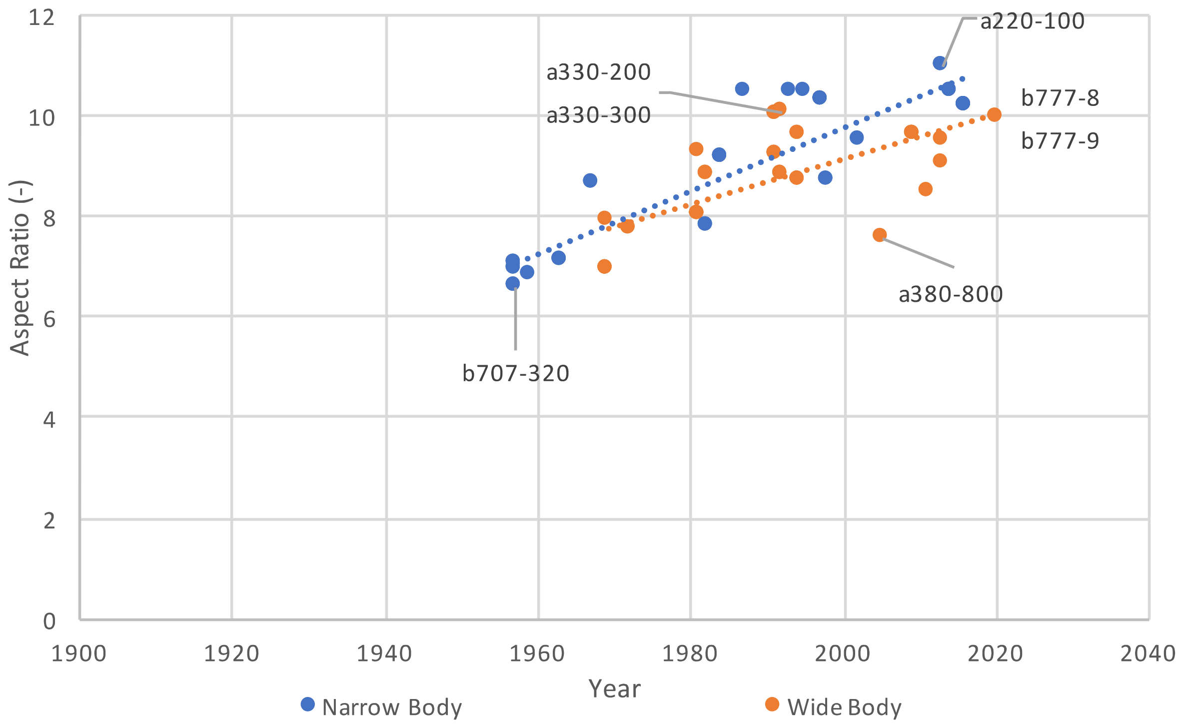

Figure 5.15 Aspect Ratio through Years.

Table 5.15 Trend-line´s data for Figure 5.15.

aircraft type	equation	R^2
narrow body	$AR = 0.0628 \cdot Year - 115.89$	0.81332
wide body	$AR = 0.0448 \cdot Year - 80.535$	0.4756

6 Other Relations

6.1 Performance Factor

It was explained in Chapter 1 that Francis Reynolds obtained a performance factor in RC planes by multiplying CWL with displacement loading, and that it was related to speed range – the difference between maximum and minimum possible speeds for an aircraft according to Reynolds (1989). The same study was conducted in this thesis using real aircraft with the following findings:

Displacement loading is not a useful parameter for passenger aircraft, mostly used with jet engines or propellers. The first idea was to try to change displacement loading by weight to thrust ratio, but no correlation with speed range was found (Figure 6.1, Table 6.1). However, when considering thrust to weight ratio directly, a commonly used parameter for identifying aircraft and preliminary sizing, the proportion worked (Figure 6.2, Table 6.2).

The new factor is hence the result of multiplying CWL with T/W. The higher it is, the higher the speed range for the aircraft. A high-speed range means the difference between aircraft's highest and lowest speed is greater.

To analyze the results, the maximum and minimum speed for the 72 Airbus and Boeings models were calculated and plotted over the product of CWL and T/W. For minimum or stall speed, from lift Equation (2.3):

$$V = \sqrt{\frac{2\,mg}{\rho S C_L}} \tag{6.1}$$

To obtain the lower speed possible, C_L was given its maximum value, which corresponds to the landing configuration, and was estimated to be 2.6 (Scholz 2015). Nevertheless, it was just an approximation since this value is different for each airplane.

For maximum speed, maximum cruise Mach number was converted to IS' speed units (m/s) with Equation (6.2) and (6.3)

$$V = Mc \tag{6.2}$$

$$c = \sqrt{\gamma' R_{gas} T_{amb}} \tag{6.3}$$

M stands for Mach number and c for the speed of sound. γ is the ratio of specific heats and equal to 1.4 for air, R_{gas} is the universal constant for gases (287 kg mol^{-1} K$^{-1)}$, and T_{amb} the atmospheric temperature for flight altitude (216.15 K), since every aircraft was considered to have its ceiling over 11000 m.

With maximum and minimum values, speed range can be obtained as a simply difference. Data for calculation is shown in Appendix C.

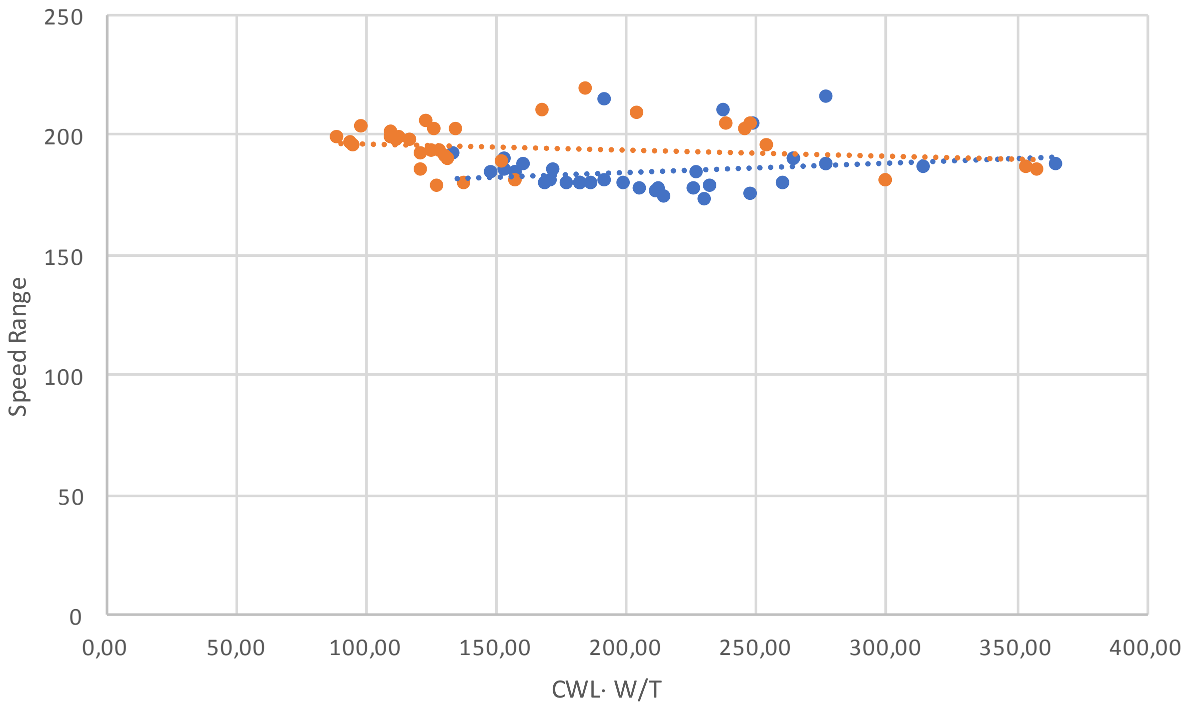

Figure 6.1 Speed Range and Performance Factor for 72 Airbus and Boeing's models (CWL·W/T).

Table 6.1 Trend-line´s data for Figure 6.1.

aircraft type	equation	R^2
narrow body	*Sp. range* = -0.0237·CWL·W/T + 198.57	0.03599
wide body	*Sp. range* = 0.0357·CWL·W/T + 177.34	0.03074

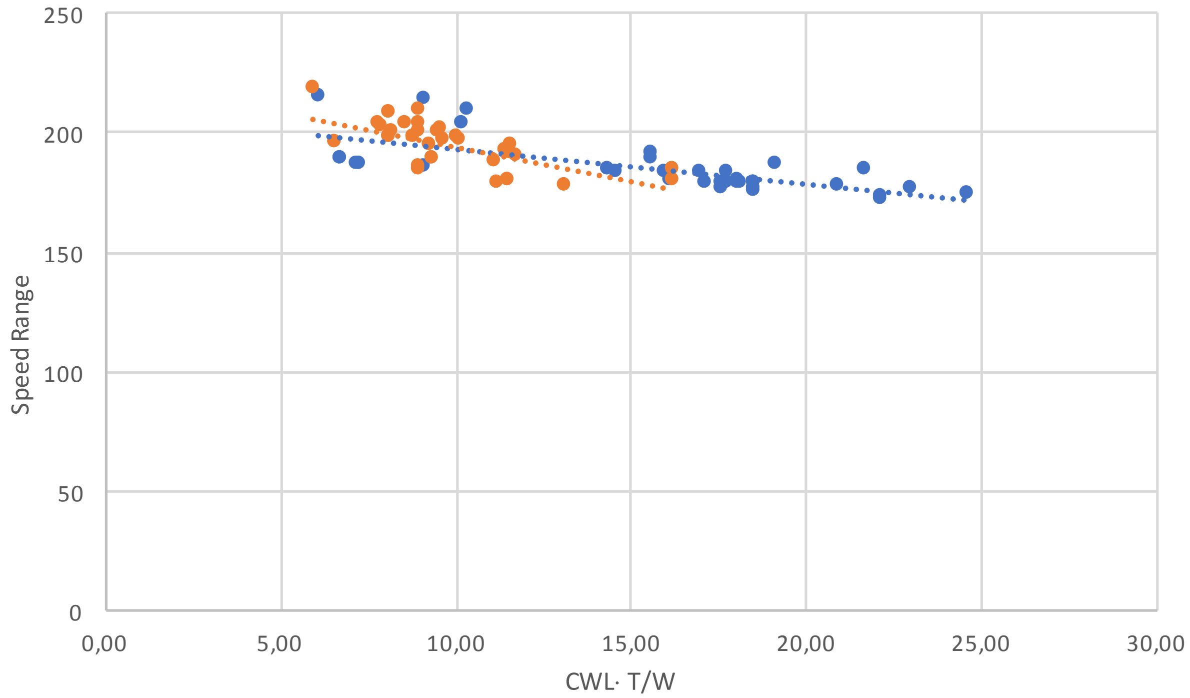

Figure 6.2 Speed Range and Performance Factor for 72 Airbus and Boeing's models (CWL·T/W).

Table 6.2 Trend-line´s data for Figure 6.2.

aircraft type	equation	R^2
narrow body	*Sp. range* = -1.4146·CWL·T/W + 206.96	0.49984
wide body	*Sp. range* = -2.889·CWL·T/W + 223.15	0.48615

The first performance factor is $R^2 \sim 0.03$, which indicates there is no linear proportion between the two parameters considered. The second one is $R^2 \sim 0.5$, which indicates there is not a high correlation but it works.

A similar chart to Reynolds´ was plotted in MATLAB. Again, blue points correspond to narrow-body and oranges ones to wide-body aircraft. Constant CWL·T/W values are represented in black lines.

In Figure 6.3, within aircraft with higher speed range ancient models such as Boeings 707-120 or 720 can be identified in the same region as new ones like Airbus 380-800. On the other end, with lower speed range, the Boeings 737 second generation models 400 and 300 and new Airbus 321neo are identified. Middle values are obtained in Boeing models 737-100 or 767-300.

A lower speed range can indicate a lower cruise speed or a faster landing speed.

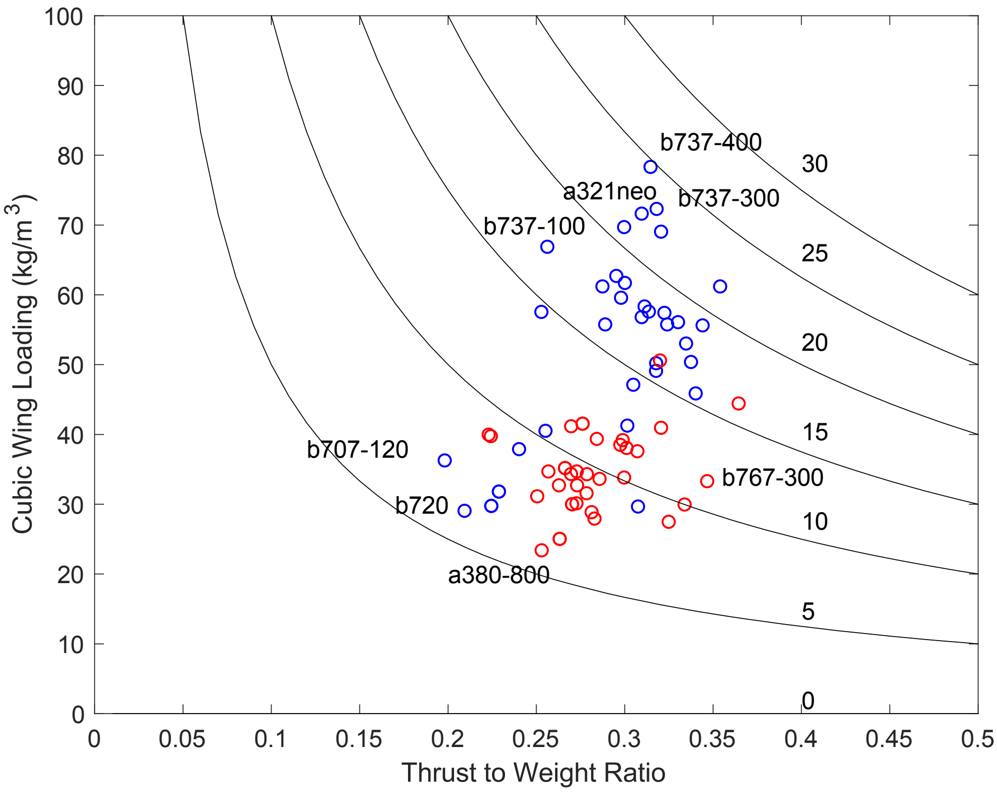

Figure 6.3 Performance Factor for Airbus and Boeing´s models.

6.2 Accident Rate

Durmus (2020) warned in his work that aircraft accident rates could be related to high CWL. Using the previously collected data, the research was addressed in this thesis and the results are shown in this chapter.

Figure 6.4 was captured from Boeing "Statistical Summary of Commercial Jet Airplane Accidents" (Boeing 2020). It shows hull-loss accident rate per million departures for different commercial aircraft models.

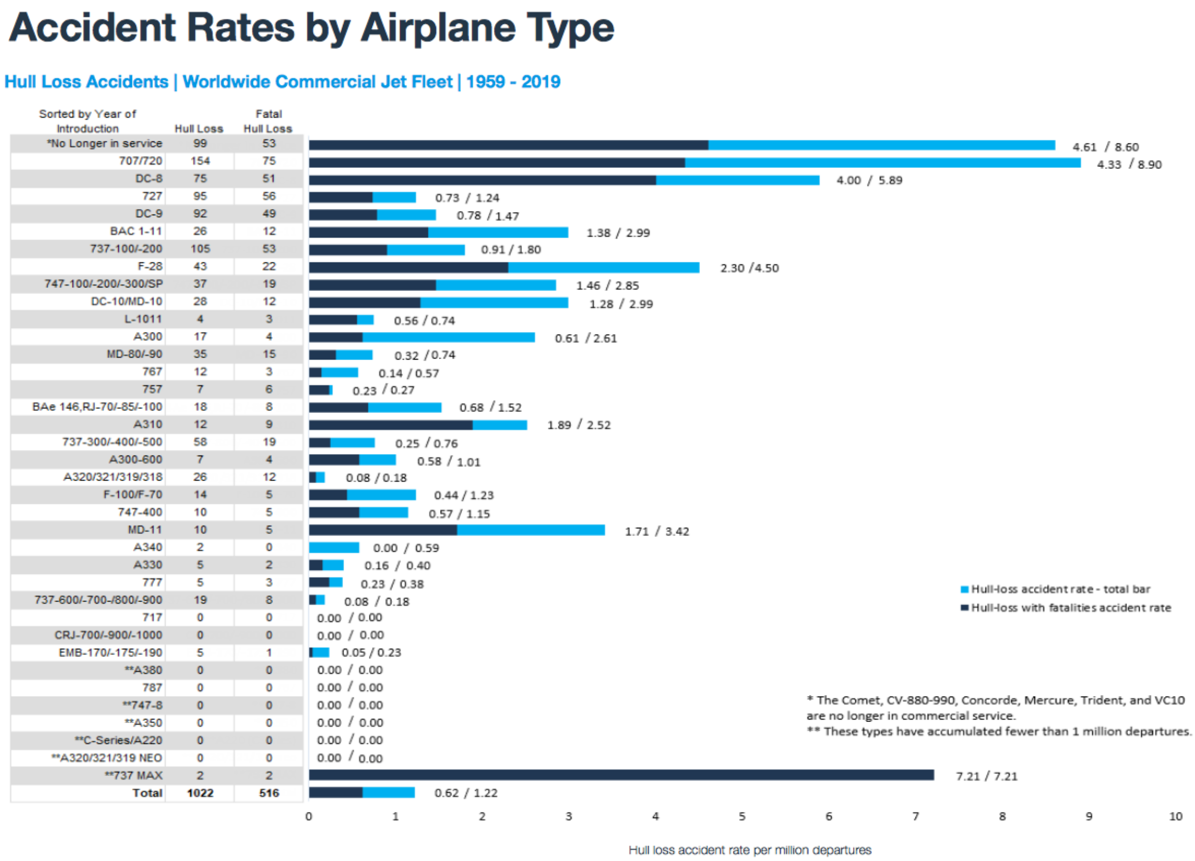

Figure 6.4 Hull-loss accident rates by airplane type (Boeing 2020).

These data were plotted over CWL values (Figure 6.5; Table 6.3). As some rates refer to several models inside a family, the CWL value of the most manufactured plane inside the family was considered in these cases as the most relevant plane within that group.

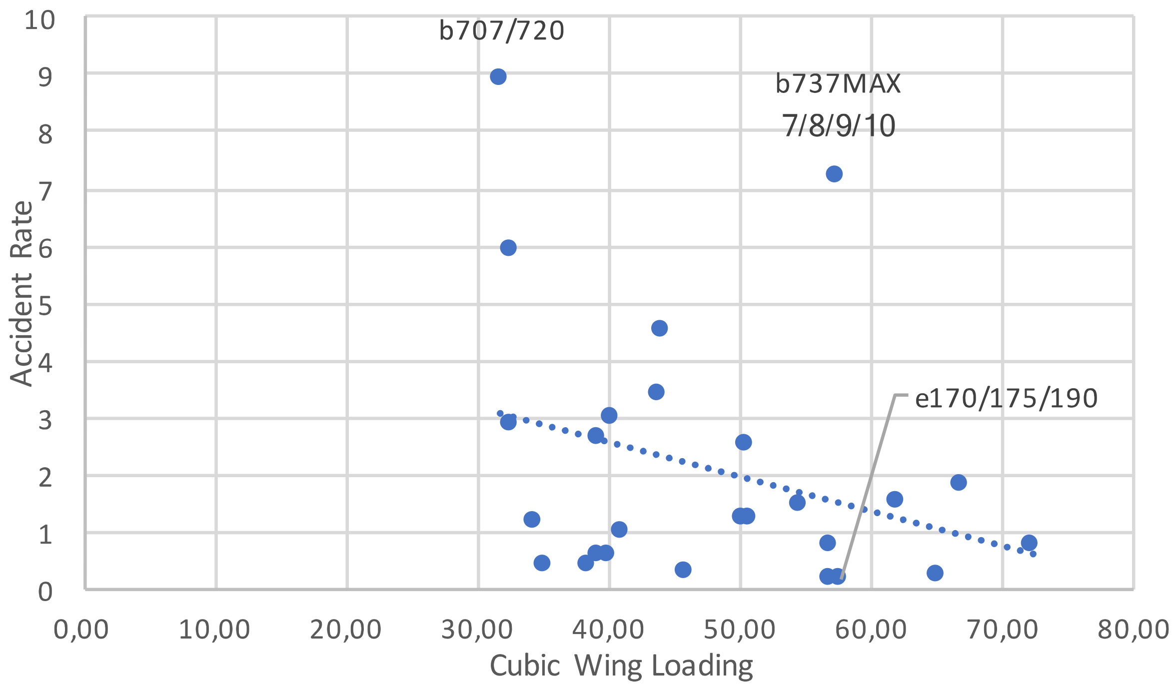

Figure 6.5 Accident Rate over Cubic Wing Loading.

Table 6.3 Trend-line´s data for Figure 6.5.

equation	R^2
Acc.Rate = -0.0601·*CWL* + 4.9658	0.09457

No proportion can be observed, not even if narrow and wide body are plotted separately (Figure 6.6; Table 6.4).

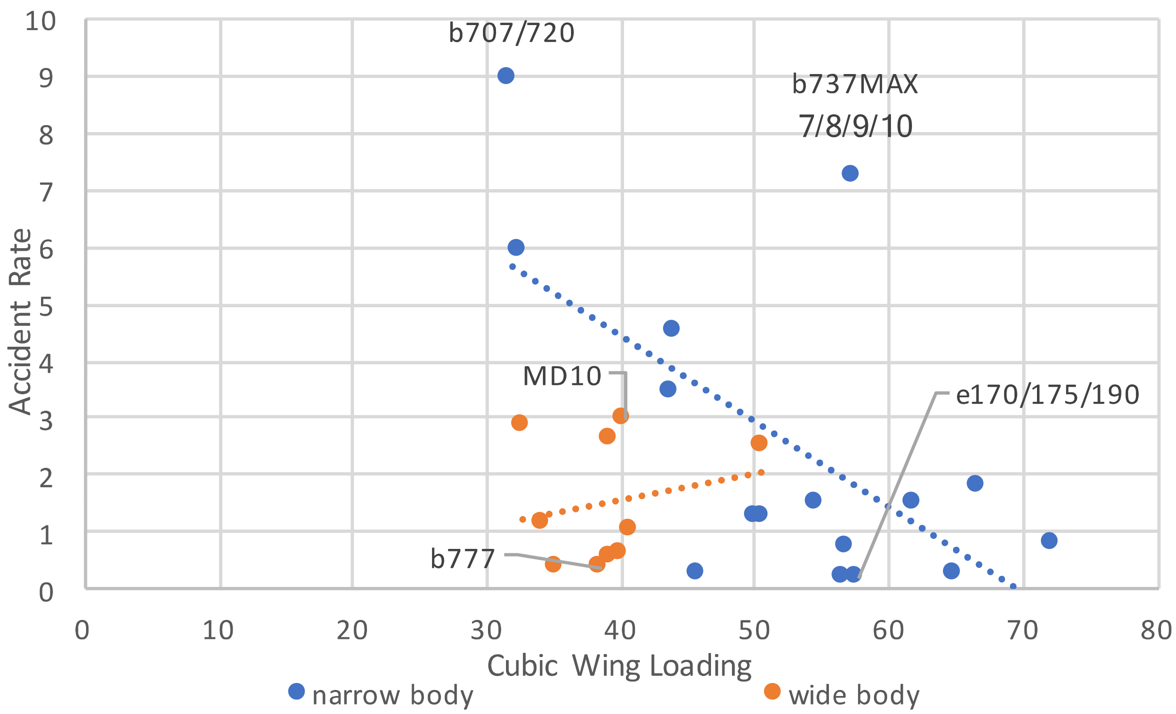

Figure 6.6 Accident Rate over Cubic Wing Loading for narrow-body and wide-body aircraft.

Table 6.4 Trend-line´s data for Figure 6.6.

aircraft type	equation	R^2
narrow body	*Acc.Rate* = -0.1508·*CWL* + 10.472	0.39723
wide body	*Acc.Rate* = 0.0465·*CWL* – 0.3139	0.04351

To explain this phenomenon, accidents are caused by numerous and a wide variety of circumstances which are not directly related with the original aircraft design as long as the design is a rational and checked design.

6.3 Fuel and Structural Efficiency

The square-cube law was considered in aircraft design in Kundu's book (Kundu 2010). As it was explained before, increasing linear dimension (e.g. span) leads to a higher increase in volume than in area. The increase in volume is associated with an increase in weight and therefore a stiffening of the structured is needed, which leads to a further increase in weight in a cyclical manner. As a result, beyond a size limit, aircraft design may be not be feasible due to an important growth of weight, heading to incredibly high WL values.

However, larger aircraft have a better structural efficiency while fuel economy is worse.

The better structural efficiency achieved for larger aircraft is shown in Figure 6.7. Figure 6.7(a) shows two lines, the line with an almost linear trend represents m_{OE} versus m_{MTO}, while the line with a decreasing trend represents the ratio m_{OE}/m_{MTO} versus m_{MTO}. Figure 6.7(b) is the same graph focused only on only midrange-aircraft.

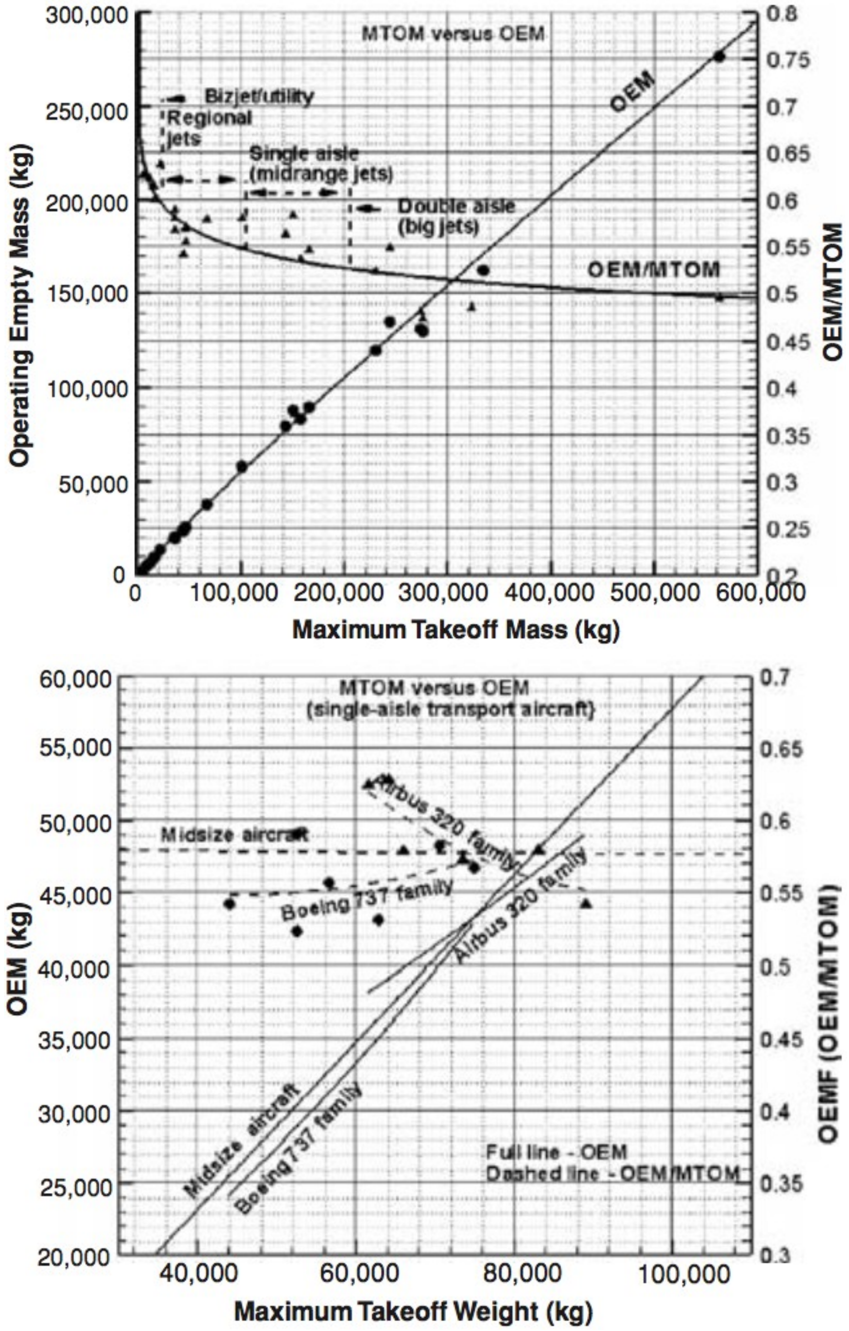

Figure 6.7 m_{OE} (OEM) versus m_{MTO} (MTOM) and m_{OE} / m_{MTO} lines (Kundu 2010).

It can be easily observed how m_{OE} increases with increasing m_{MTO}, but how m_{OE}/m_{MTO} takes the highest values for lighter planes, i.e., short-range low-passenger aircraft is ~0.6, sharply decreasing for narrow-body to ~0.56, and finally leveling out for long-haul wide-body airplanes to ~0.483. The introduction of lighter materials in aircraft manufacturing is reducing this fraction in recent designs.

Figure (b) corresponds to a detailed graph for midrange aircraft. It is interesting to observe how derivative aircraft from Boeing 737 and Airbus 320 are placed in the graph. As Boeing was pioneer, it had to learn while designing: Its baseline model was the smaller 737-100 whose growth led to a similar required growth in the different aero-structures maintaining as much as possible components commonality. Airbus learned from Boeing and its baseline model was the 320, in the middle of the family. The bigger or smaller versions were made adding/detaching fuselage sections in the after or forward fuselage, maintaining common structures. Although the variant aircrafts were not optimized in size, it turned to a decrease in manufacturing costs and a reduction of Direct Operating Costs (DOC).

In regards to fuel economy, larger aircraft have a bigger fuel load as shown in Figure 6.8. It can be noticed fuel load gets sharply higher for bigger aircraft (worst fuel economy).

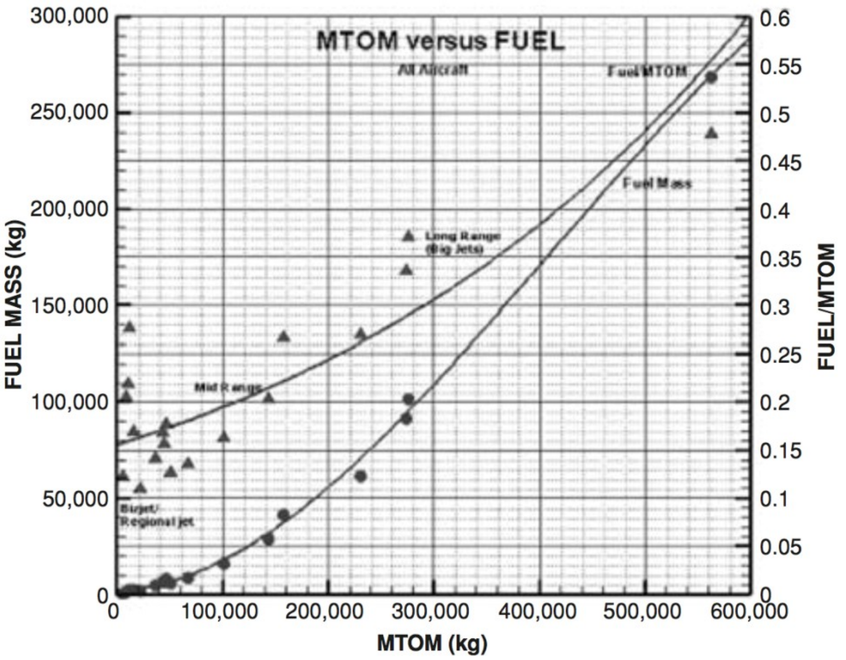

Figure 6.8 Fuel load versus m_{MTO} (MTOM) and m_F/m_{MTO} (M_f/MTOM) lines (Kundu 2010).

Equivalent to Figure 6.8, Figure 6.9 displays thrust over m_{MTO} and the ratio between them, getting this ratio a worst (lower) value for bigger aircraft. Table 6.5 shows trend-line information for this chart.

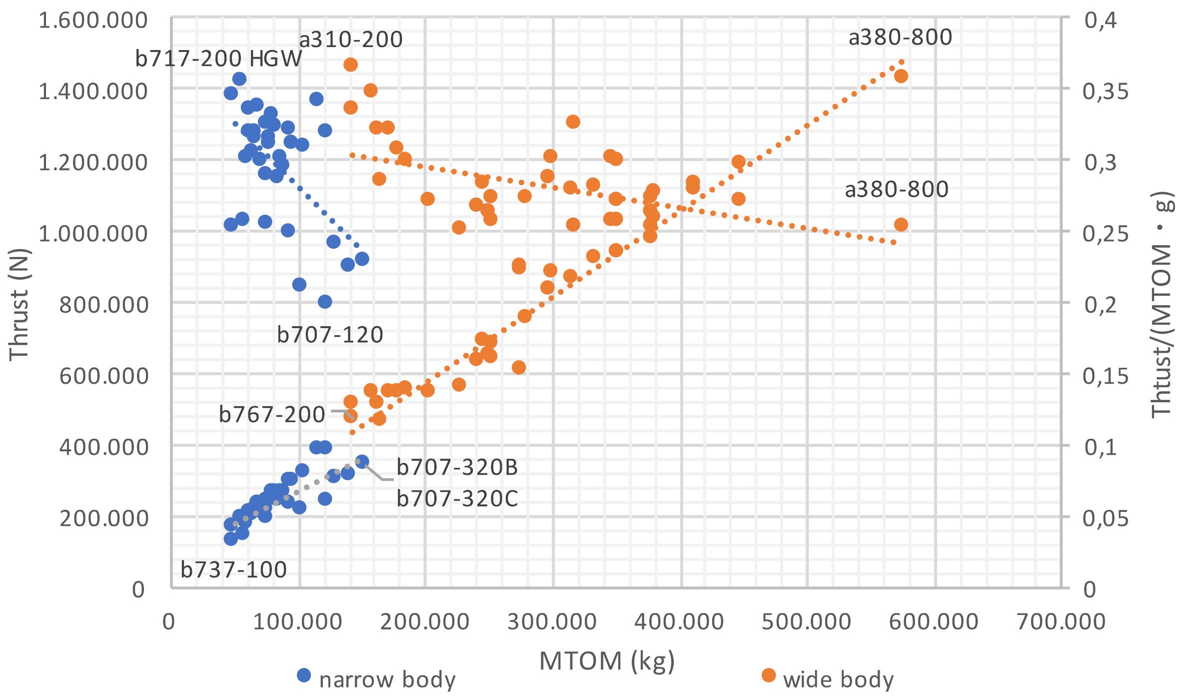

Figure 6.9 T versus m_{MTO} (MTOM) and $T/(m_{MTO} \cdot g)$ =T/W (T/(MTOM·g)) lines.

Table 6.5 Trend-line´s data for Figure 6.9.

aircraft type	equation	R^2
narrow body T line	$T = 1.8018 \cdot m_{MTO} + 86455$	0.70318
wide body T line	$T = 2.4264 \cdot m_{MTO} + 86868$	0.93359
narrow body T/W line	$T/W = -9 \cdot 10^{-7} \cdot m_{MTO} + 0.3702$	0.36738
wide body T/W line	$T/W = -1 \cdot 10^{-7} \cdot m_{MTO} + 0.3231$	0.20046

This graph was not included in Kundu’s book (Kundu 2010), but was done with data collected in the present study. It can be easily observed how thrust increases with m_{MTO} almost lineally, but also, as mentioned earlier, how $T/(m_{MTO}g)$ decreases for larger aircraft.

Advances in aircraft manufacture technology are responsible for bigger aircraft with bigger span and wing surface. However, not such an increase in weight can results in an aircraft built defying the square-cube law. Airbus 380 is a good example.

This lower increase in weight gives these planes a lower CWL value, (23,41 for Airbus 380), and if plotted in Figures 6.8 and 6.9, the aircraft would be placed not as much as it should to the right of the diagrams.

This, led Durmus (2020) to claim that aircraft built against the square-cube law (with low CWL) provided worse structural efficiency and reduced the maximum take-off mass and the operating empty mass ratio, but, on the contrary, they provided a better power to weight ratio (higher) and fuel economy (lower fuel load).

7 Discussion

7.1 Preliminary Sizing

CWL does not present advantages at the origin of aircraft design as opposed to traditional WL.

CWL can be easily obtained in traditional equations as it involves two basic parameters: mass and wing surface. However, the power of 3/2 gives the equations a complex form and makes them unpractical.

Preliminary sizing can be carried out but an iterative process is needed. This makes computational cost and time for designing the aircraft higher.

If applied to an example airplane, with the original spreadsheet for preliminary sizing based on WL and designed by Scholz (Scholz 2015) a m_{MTO} of 254,272 kg is the result. With the new spreadsheet, using the new equations, the results obtained are shown in Table :

Table 7.1 m_{MTO} and relative errors.

	m_{MTO} (kg)	relative error (%)	V/V_{md}
WL based preliminary sizing	254,272	-	1.36
m_{OE}/m_{MTO} acc. to Raymer	259,053	1.88	1.31
m_{OE}/m_{MTO} acc. to eq. 4.17	250,371	1.53	1.43
m_{OE}/m_{MTO} acc. to sts. (original)	255,102	0.33	1.36

Remember V/V_{md} value must be changed so that take-off and landing are sizing the aircraft at the same time.

As it can be seen, a different result is obtained for each m_{OE}/m_{MTO} value considered, calculated according to Raymer (4.16), equation from approach 1 (4.17) and a value given according to statistics.

In the third case, the same m_{OE}/m_{MTO} obtained in the original spreadsheet according to Loftin (Loftin 1980) is used to minimize result deviation and compare both spreadsheets calculation.

Relative error increases as error in m_{OE}/m_{MTO} calculation increases (6.3% for (4.16), 4.48% for (4.17)). Just about the same result is obtained when m_{OE}/m_{MTO} is given the same value as the one used in original spreadsheet (0.33%).

7.2 Statistical Correlations

Regarding statistical correlations, all correlations established are weak if considering R^2 values.

Higher correlations are obtained for wing surface over CWL for narrow-body aircraft ($R^2 = 0.55782$ against $9.7 \cdot 10^{-5}$ over WL) and aspect ratio over CWL for narrow-body aircraft too ($R^2 = 0.54659$ against 0.32653 over WL).

An inverse relationship is found between CWL and wing surface directly from its definition (3.1). Thus, wing size for an aircraft can be estimated with CWL value from aircraft built with similar materials and its m_{MTO} using the same Equation (3.1).

Durmus (2020) claimed a high proportion could be established also between CWL and relative wetted area. This study couldn't be conducted since in preliminary sizing an average relative wetted area of 6.1 is considered for every aircraft and no difference is established between different aircraft models.

As parasitic drag is proportional to relative wetted area (7.1), similarly, CWL was said to be proportional to parasitic drag, high CWL aircraft are related with high relative wetted area and high parasitic drag according to Durmus (2020). This supports the idea presented in Chapter 6.3, lower CWL aircraft require lower T/W.

$$C_{D0} = C_{Df}\left(\frac{S_{wet}}{S}\right) FF\ FI \tag{7.1}$$

A similar proportion with $R^2 \sim 0.5$ can be observed for CWL and aspect ratio for narrow-body planes. In this case, a direct correlation can be established between these two parameters when a reduction in AR is due to an increase in wing surface (7.2). Note the effect in CWL (3.1) is higher and it is not a linear relation (because of the power of 3/2).

$$AR = \frac{b^2}{S_w} \tag{7.2}$$

However, proportion is not observed for the majority of aircraft (wide and propeller mainly), this is directly related with their different and less proportional weight variation with CWL for wide and propeller planes ($R^2 = 0.13043$ and 0.02778 respectively for m_{MTO} versus CWL).

Therefore, a low AR is related to a low CWL but also a high AR if a big span and/or low weight is considered (e.g sailplane, AR ~ 21 … 51 CWL ~ 10…15). Many parameters play an important role.

It was also showed how AR has tended to increase in time (from 6.56 to 10.97) ($R^2 =$ 0.81332 for narrow-body and 0.4756 for wide-body planes). It was said to be consistent with aerodynamic efficiency.

However, in the future, this trend may be broken with the introduction of Blended Wing Body designs (BWB) with low AR ~ 4 and CWL ~ 8.

Why would a blended wing body might be a possible design for future aircraft? This is because AR is associated with aerodynamic efficiency (L/D) and designers look for it to be as high as possible. Generally, higher AR planes (e.g sailplanes) have higher L/D, but high AR also enters in conflict with stress-engineers' interests and structural stiffness. An all-wing design can lead to similar L/D values taking care also of stress engineers' interests with a low AR. This was shown by Torenbeek and Roskam and recompiled by Kundu (Kundu 2010) and it is since drag is the addition of a parasitic and an induced term (7.3): the second gets higher with lower AR, but the first is related to overall wetted area. Since the all-wing aircraft precludes the need for a separate fuselage, a reduced wetted area and skin friction is obtained, balancing the addition.

$$C_D = C_{D0} + \frac{{C_L}^2}{\pi AR\varphi} \tag{7.3}$$

A wetted-area aspect ratio was defined as follow (Kundu 2010):

$$AR_{wet} = \frac{b^2}{S_{wet}} = \frac{AR}{S_{wet}/_S} \tag{7.4}$$

and indicates how close a configuration is to the all-wing configuration.

7.3 Other Relations

The new performance factor allows to place aircraft according to their speed range. For the aircraft studied it goes from 172 m/s to 215 m/s for narrow-body airplanes and from 178 m/s to 218.5 m/s for wide-body ones.

Note the speed range runs between similar values for both types of aircraft but average value for the first ones is 185 m/s and for the second ones is 194.7 m/s, and therefore 69.4% of narrow-body planes have a performance factor between 5 and 15 (lower speed range) and 94.4% of wide-body planes have it between 15 and 30 (higher speed range). Precision is affected by how the minimum and maximum speed are calculated, as it was warned.

Calculation are compiled in Appendix C. This parameter helps to understand how an airplane flies and recall the idea for why CWL was used in RC planes.

Regarding to accident rates no correlation can be established ($R^2 = 0.09457$). Accidents in air traffic are associated to a wide range of factors and high CWL aircraft can't be said to be more prone to suffer an accident with a preliminary study as the one carried out in this thesis.

It can be noticed some of the variants within a family with the highest CWL have the highest number of accidents within that family (Skybrary 2021). This is due to that model is also the one most manufactured and with a major number of flights, therefore, it is not relevant when considering rates.

According to last section, larger aircraft are showed to have better structural efficiency with a decrease of m_{OE}/m_{MTO} with m_{MTO}, amassing a total reduction of 0.117 (Figure 6.7); and worst fuel economy with an increase of fuel load over m_{MTO}, gathering a total increase of 0.37 (Figure 6.8). The thrust to weight ratio is also reduced in 0.1 with m_{MTO} (Figure 6.9). These numbers were directly obtained from graphs.

However, aircraft may be built nowadays against the square-cube law, as Airbus 380, and get lower values of CWL thanks to not such a relative increase in weight with wing surface. This is possible thanks to new technology incorporated to aircraft manufacturing and design as better strength--to--weight ratio materials with introduction of composites, lighter and miniaturized equipment, better high-lift devices, which let accommodate higher wing loadings, and fuel economy.

The reduction of m_{MTO} places these planes more to the right in previous figures, giving them higher power to weight ratio and lower fuel loads, but worse structural efficiency as they should. Then, this is a characteristic for low CWL designs.

How much is the advantage obtained? Nowadays, the 50% of the structure of an aircraft can be built using composites. Composites structure can reduce the weight a maximum of 20%, hence a global reduction of 10% in airplane´s weight can be diminished as maximum (Apropedia 2009). According to Figure 6.7, Figure 6.8 and Figure 6.9, taking as reference the Boeing 787-10, which is currently the aircraft with more percentage of composite materials in its structure (Apropedia 2009), a maximum increase of 0.005 of m_{OE}/m_{MTO}, reduction of 0.01 of m_F/m_{MTO} and increase of 0.0025 of T/W can be forwarded (only reduction of weight due to the use of composite materials in aircraft´s structure is considered here). These numbers were directly obtained from graphs, comparing the values obtained for a plane with the mass of the Boeing 787 and the mass it would have according to its wing surface if it had been scaled following the square-cube law (taking into account its CWL value and (3.1)). The analysis was done for the three different models 800, 900, 1000 and the maximum values were considered.

Even though Airbus 380 stopped its production recently, since bigger planes are unprofitable for airlines when too many seats go unfilled, the same idea is behind smaller, modern and more successful Airbus 350 and the already mentioned Boeing 787 design. Hence, CWL is expected to decrease with years.

This time, the possible introduction of BWB designs would lead to a low CWL value too.

As exposed, CWL does not present as many advantages for real passenger aircraft as for radio control planes designers. It main use, related with “flyability” makes no sense when speaking of passenger aircraft. They are much less sensitive to atmospheric conditions than RC planes and the performance of the aircraft is quite similar among commercial designs. Just comparing with military planes some differences may be observed. These planes have a much larger CWL value (see Figure 2.4) as their wing surface is made smaller to be adapted to higher performance and more stiffness in structures is required. Their handling is also more complicated.

To summarize, as CWL cannot be obtained directly from the lift equation, calculations are intricate and it does not correlate with other aircraft parameters. Not enough advantages were found in the present study to justify the use of CWL in passenger aircraft design.

8 Summary and Conclusions

Cubic wing loading presents several advantages for radio controlled (RC) planes designers. However, this well-known parameter for RC planes had never been implemented in passenger aircraft design.

After presenting the current knowledge of this parameter and its use among RC planes designers as an indicator of flyability, its fundamentals were explained and mathematical basics relationship with respect to the more traditional WL parameter was described in a series of equations. As a result, expressions became more complex when introducing CWL.

The first analysis was focused on whether preliminary sizing as it is currently done using WL, could be conducted based on CWL. The process can be completed creating similar matching chart for T/W over CWL, but equations have to be changed. Introducing the exponent of 3/2 led to much more complex expressions. Moreover, in order to solve them, m_{MTO} needs to be calculated prior to obtaining the T/W and CWL for the aircraft.

In order to accomplish this, m_{OE} and m_F need to be known before calculations. m_{OE} can be calculated using several equations defined statistically, considering different input values. Some values include m_{MTO} and hence an iterative process is required, other values do not require it and therefore, it is not mandatory. Nevertheless, for calculating $m_{F,}$ the Breguet factor is needed, which is dependent on altitude. Flight altitude is unknown, and the equation to obtain it depends on thrusts ratio T_{CR}/T_{TO}, which at the same time is a function of T/W. Since when using CWL-based preliminary sizing T/W is still unknown, an estimation for T_{CR}/T_{TO} must be done and an iterative process is compulsory at this stage of the calculations.

Both arguments make using preliminary sizing with CWL useless over WL-based design.

Furthermore, a detailed analysis of CWL statistical correlations with other useful aircraft parameters was conducted. However, R^2s were low and no clear correlations were established. Moreover, it was demonstrated CWL is not a constant value for different passenger aircraft.

The aspect ratio and CWL were plotted versus time and a good increasing correlation was found with aspect ratio for narrow-body planes ($R^2 = 0.81332$).

Then, how can CWL be useful in passenger aircraft design? Chapter 0 presented some advantages.

A research study was carried out and concluded the performance factor presented by Reynolds for RC planes could be adapted to obtain a new performance factor which provided similar information for real passenger aircraft. The new factor changed displacement loading considered in Reynolds´s factor by T/W, a much more relevant parameter for real aircraft design. Besides being more practical, proportion worked better. This parameter gives an idea about speed range of an aircraft and as a result, about its performance.

Next, a relation between CWL and accident rate was evaluated, but the high influence of other factors did not allow isolating the relation of these two parameters, even though Durmus (2020) had deemed it worth studying.

Both Durmus (2020) and Kundu (2010) indicated that designs defying square-cube law with newer and lighter materials led to better fuel economy and thrust to weight ratio in larger planes, but reduced relative operating empty mass fraction and structural efficiency.

Finally, all results were analyzed and discussed.

9 Recommendations

The present report introduces CWL into real passenger aircraft preliminary sizing. Statistical correlations include 209 different types of aircraft. They may be extended in order to improve statistics by the addition of other planes and models, ancient designs or other kinds of planes, such as business jet. Only airliners were considered in this thesis.

CWL can also be introduced into other type of real aircraft designs, such as sailplanes or military planes. Even though it does not provide much information within commercial planes a wider vision could be conducted, since difference between military, commercials and sailplanes are higher.

Some parts of the research were based only in Airbus and Boeing's airplanes data (72 models) lacking detailed data for other planes. The study of CWL on design of other planes, mainly propeller aircraft, may be a possible subject of study.

Finally, as mentioned earlier, relative wetted area over wing surface is considered roughly constant for models in preliminary sizing and conceptual design. Therefore, Durmus (2020) claims CWL to be proportional to parasitic drag. This could not be studied in this thesis and can be investigated in future research.

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Appendix A – $m_{ML}{}^{3/2}/m_{MTO}$

Statistical values of maximum landing mass to the power of 3/2 over maximum take-off mass $m_{ML}{}^{3/2}/m_{MTO}$ for jets of different design range need to be calculated for completing preliminary sizing process based on CWL and were estimated statistically in Excel (see Table A.1, Table A.2, Table A.3, Table A.4). m_{ML}/m_{MTO} was also calculated taking advantage of data collected and since it is also needed for preliminary sizing. Nevertheless, this was also calculated in (Scholz 2015) since it is needed for traditional preliminary sizing too. Wide-body planes are marked in orange.

Table A.1 $m_{ML}{}^{3/2}/m_{MTO}$ for aircraft with a range under 2000 km.

<2000 km

Model	m_{MTO}	m_{ML}	m_{ML}/m_{MTO}	$m_{ML}{}^{3/2}/m_{MTO}$
DHC-7	19958	19051	0.955	131.75
F28-1000	29480	26770	0.908	148.57
F28-2000	29480	26770	0.908	148.57
F28-4000	33110	31525	0.952	169.05
YAK-40	15500	15500	1.000	124.50
		Average:	0.945	119.59

Table A.2 $m_{ML}{}^{3/2}/m_{MTO}$ for aircraft with a range between 2000-5500 km.

2000-5500 km

Model	m_{MTO}	m_{ML}	m_{ML}/m_{MTO}	$m_{ML}{}^{3/2}/m_{MTO}$
CRJ100	24041	21319	0.887	129.48
CRJ200	24041	21319	0.887	129.48
CRJ700	34019	30390	0.893	155.73
CRJ900	38330	34020	0.888	163.71
CRJ1000	41640	36968	0.888	170.70
A300B4-200	165000	134000	0.812	297.29
B707-320C	151500	112000	0.739	247.41
B717-200 HGW	54884	49900	0.909	203.10
B720	104000	79380	0.763	215.05
B727-200	95100	79150	0.832	234.15
B737-900ER	85139	66361	0.779	200.79
ERJ 135LR	20000	18500	0.925	125.81
ERJ 140LR	21100	18700	0.886	121.19
ERJ 145XR	24100	20000	0.830	117.36
E170	38600	33300	0.863	157.43
E175	40370	34100	0.845	155.98
E190	51800	44000	0.849	178.18
E195	52290	45800	0.876	187.45

E175-E2	44800	40000	0.893	178.57
E190-E2	56400	49050	0.870	192.61
E195-E2	61500	54000	0.878	204.04
		Average:	0.900	188.27

Table A.3 $m_{ML}^{3/2}/m_{MTO}$ for aircraft with a range between 5500-15000 km.

5500-15000 km

Model	m_{MTO}	m_{ML}	m_{ML}/m_{MTO}	$m_{ML}^{3/2}/m_{MTO}$
A220-300	70900	60600	0.855	210.41
A300-600R	171700	140000	0.815	305.09
A310-300	164000	140000	0.854	319.41
A318	68000	57500	0.846	202.76
A319NEO	75500	63900	0.846	213.95
A320NEO	79000	67400	0.853	221.49
A321NEO	97000	79200	0.816	229.78
A330-900NEO	251000	191000	0.761	332.56
A340-600	380000	265000	0.697	358.99
A380-800	575000	394000	0.685	430.11
B707-320B	151500	112000	0.739	247.41
B720B	106200	79379	0.747	210.59
B737 MAX 10	89765	74344	0.828	225.82
B747-8	447700	312000	0.697	389.26
B757-300	123830	101600	0.820	261.53
B767-400ER	204100	158760	0.778	309.93
B777-9	352400	266000	0.755	389.30
B787-10	254011	202000	0.795	357.42
		Average:	0.788	289.77

Table A.4 $m_{ML}^{3/2}/m_{MTO}$ for aircraft with a range upper to 15000 km.

>15000 km

Model	m_{MTO}	m_{ML}	m_{ML}/m_{MTO}	$m_{ML}^{3/2}/m_{MTO}$
A330-800NEO	251000	186000	0.741	319.59
A340-500	380000	246000	0.647	321.08
A350-900	280000	207000	0.739	336.35
A350-1000	316000	236000	0.747	362.81
B777-200LR	347452	223168	0.642	303.43
B777-8	352400	223168	0.633	299.17
		Average:	0.692	323.74

Appendix B – Iteration Processes

m_{TO} iteration process is presented in Table B.1 and Table B.2. The first step is to estimate a value for T_{CR}/T_{TO} (~ 0.1 … 0.3).

Table B.1 m_{MTO} iteration IT1.

Parameter	Symbol	Value	Units
Thrust ratio	$(T_{CR}/T_{TO})_{CR}$	**0.200**	
Conversion factor	m -> ft	0.305	m/ft
Cruise altitude	h_{CR}	**10124**	**m**
Cruise altitude	h_{CR}	**33216**	**ft**
Temperature, troposphere	$T_{Troposphäre}$	222.34	K
Temperature, h_{CR}	$T(h_{CR})$	222.34	K
Speed of sound, h_{CR}	a	299	m/s
Cruise speed	V_{CR}	**254**	**m/s**
Conversion factor	NM -> m	1852	m/NM
Design range	R	**8500**	**NM**
Design range	R	15742000	m
Distance to alternate	$s_{to\ alternate}$	200	NM
Distance to alternate	$s_{to\ alternate}$	370400	m
Chose: FAR Part121-Reserves?	domestic	**yes**	
	international	**no**	
Extra-fuel for long range		5%	
Extra flight distance	s_{res}	370400	m
Spec.fuel consumption, cruise	SFC_{CR}	**1.40E-05**	**kg/N/s**
Breguet-Factor, cruise	B_s	3.14E+07	m
Fuel-Fraction, cruise	$M_{ff,CR}$	0.605	
Fuel-Fraction, extra fliht distance	$M_{ff,RES}$	0.988	
Loiter time	t_{loiter}	2700	s
Spec.fuel consumption, loiter	SFC_{loiter}	**1.40E-05**	**kg/N/s**
Breguet-Factor, flight time	B_t	123371	s
Fuel-Fraction, loiter	$M_{ff,loiter}$	0.978	
Fuel-Fraction, engine start	$M_{ff,engine}$	**0.990**	
Fuel-Fraction, taxi	$M_{ff,taxi}$	**0.990**	
Fuel-Fraction, take-off	$M_{ff,TO}$	**0.995**	
Fuel-Fraction, climb	$M_{ff,CLB}$	**0.980**	
Fuel-Fraction, descent	$M_{ff,DES}$	**0.990**	
Fuel-Fraction, landing	$M_{ff,L}$	**0.992**	
Fuel-Fraction, standard flight	$M_{ff,std}$	0.580	

Fuel-Fraction, all reserves	$M_{ff,res}$	0.938		
Fuel-Fraction, total	M_{ff}	0.544		
Mission fuel fraction	m_F/m_{MTO}	0.456		
Realtive operating empty mass	m_{OE}/m_{MTO}	0.478		acc. to Raymer
Realtive operating empty mass	m_{OE}/m_{MTO}	0.456		acc. (4.17)
Realtive operating empty mass	m_{OE}/m_{MTO}	**0.472**		acc. to statistics
Realtive operating empty mass	m_{OE}/m_{MTO}	**0.472**		**<<< Choose**
Choose: type of a/c	short / medium or	**yes**		
	long range	**no**		
Mass: Passengers, including baggage	m_{PAX}	93.0	kg	
Number of passengers	n_{PAX}	**217**		
Cargo mass	m_{cargo}	**0**	**kg**	
Payload	m_{PL}	**20181**	**kg**	
Max. Take-off mass	m_{MTO}	**306923**	**kg**	acc. to Raymer
Max. Take-off mass	m_{MTO}	**229743**	**kg**	acc. to (4.17)
Max. Take-off mass	m_{MTO}	**281477**	**kg**	acc. to statistics
Max. Take-off mass	m_{MTO}	**281477**	**kg**	
Thrust-to-weight ratio @ landing m_{MTO}/S_W	$T_{TO}/(m_{MTO}*g)$	**0.243**		

Table B.2 m_{MTO} iteration.

Parameter	Value IT1		Value IT5	Units	
$(T_{CR}/T_{TO})_{CR}$	**0.200**	…	0.251		
		…			
h_{CR}	**10124**	…	**8248**	**m**	
h_{CR}	**33216**	…	**27060**	**ft**	
$T_{Troposphäre}$	222.34	…	234.539	K	
$T(h_{CR})$	222.34	…	234.539	K	
a	299	…	307	m/s	
V_{CR}	**254**	…	**261**	**m/s**	
		…			
B_s	3.14E+07	…	3.22E+07	m	
$M_{ff,CR}$	0.605	…	0.613		
$M_{ff,RES}$	0.988	…	0.989		
		…			
B_t	123371	…	123371	s	
$M_{ff,loiter}$	0.978	…	0.978		
		…			
$M_{ff,std}$	0.580	…	0.587		
$M_{ff,res}$	0.938	…	0.938		
M_{ff}	0.544	…	0.551		
m_F/m_{MTO}	0.456	…	0.449		
		…			
m_{OE}/m_{MTO}	0.478	…	0.480		acc. to Raymer

m_{OE}/m_{MTO}	0.456	…	0.456		acc. to (4.17)
m_{OE}/m_{MTO}	**0.472**	…	**0.472**		acc. to statistics
m_{OE}/m_{MTO}	**0.472**	…	**0.472**		
		…			
m_{MTO}	**306923**	…	**284361**	**kg**	acc. to Raymer
m_{MTO}	**229743**	…	**211864**	**kg**	acc. to (4.17)
m_{MTO}	**281477**	…	**255102**	**kg**	acc. to statistics
m_{MTO}	**281477**	…	**255102**	**kg**	**<<< Result**
		…			
$T_{TO}/(m_{MTO}*g)$	**0.243**	…	**0.235**		

If m_{OE}/m_{MTO} is calculated according to Raymer, another iterative process is needed as explained for (4.16). (See Table B.3).

Table B.2 m_{OE}/m_{MTO} iteration.

IT1	iteration 1		iteration 15		
m_{MTO}	**100000**	…	**306923**	**kg**	
m_{OE}/m_{MTO}	0.51121	…	0.47795		
IT2	iteration 1		iteration 15		
m_{MTO}	**306923**	…	**287630**	**kg**	
m_{OE}/m_{MTO}	0.47795	…	0.47981		
IT3	iteration 1		iteration 15		
m_{MTO}	**287630**	…	**284835**	**kg**	
m_{OE}/m_{MTO}	0.47981	…	0.48009		
IT4	iteration 1		iteration 15		
m_{MTO}	**284835**	…	284422,439	**kg**	
m_{OE}/m_{MTO}	0.48009	…	0.48013		
IT5	iteration 1		iteration 15		
m_{MTO}	**284422**	…	**284361**	**kg**	**<<< Result**
m_{OE}/m_{MTO}	0.48013	…	0.48014		

Appendix C –
Speed Range and Performance Factor Data

Speed range and performance factor values for Airbus and Boeing models are presented in Table C.1 and Table C.2. Speed range was calculated as the difference between maximum and minimum speed for the aircraft, obtained, respectively, with (6.1) and (6.2). (6.1) was particularized for $C_{L,max} = 2.6$, as general maximum value (Scholz 2015) and (6.2) for $T_{amb} = 216.15\ K$, considering maximum speed is achieved at the ceiling of the aircraft over 11000 m. Wide-body planes are marked in orange.

Table C.1 Speed range and performance factor values for Airbus and Boeing models (narrow-body).

Model	Minimum Speed (m/s)	Maximum Speed (m/s)	Difference	CWL*T/W
A220-100	58.83	241.93	183.10	17.75
A220-300	62.36	241.93	179.57	17.75
A318	58.50	241.93	183.43	15.96
A319-100	61.64	241.93	180.29	18.07
A319NEO	61.64	241.93	180.29	16.11
A320-200	62.65	241.93	179.28	18.07
A320NEO	63.05	241.93	178.88	18.16
A321-200	68.60	241.93	173.34	22.13
A321NEO	69.87	241.93	172.06	22.18
B707-120	57.98	244.89	186.90	7.19
B707-120B	59.26	244.89	185.63	9.10
B707-320	55.54	244.89	189.35	6.68
B707-320B	57.43	244.89	187.46	7.28
B707-320C	57.43	244.89	187.46	7.28
B707-420	55.54	244.89	189.35	6.68
B717-200 BASIC	57.49	244.89	187.40	19.14
B717-200 HGW	60.29	244.89	184.59	21.66
B720	52.32	267.31	214.98	6.08
B720B	52.87	267.31	214.43	9.12
B727-100	55.57	265.54	209.97	10.34
B727-200	61.88	265.54	203.66	10.22
B737-100	58.17	241.93	183.77	14.55
B737-200	62.70	241.93	179.23	17.13
B737-300	65.20	241.93	176.74	23.00
B737-400	67.85	241.93	174.08	24.64
B737-500	64.01	241.93	177.92	20.89
B737-600	56.92	241.93	185.01	14.37

B737-700	58.86	241.93	183.07	17.00
B737-800	62.50	241.93	179.43	17.59
B737-900ER	64.88	241.93	177.06	17.59
B737 MAX 7	62.40	241.93	179.53	18.52
B737 MAX 8	63.14	241.93	178.79	18.52
B737 MAX 9	65.45	241.93	176.48	18.52
B737 MAX 10	65.99	241.93	175.95	18.52
B757-200	62.02	253.74	191.72	15.61
B757-300	64.17	253.74	189.57	15.61

Table C.2 Speed range and performance factor values for Airbus and Boeing models (wide-body).

Model	Minimum Speed (m/s)	Maximum Speed (m/s)	Difference	CWL*T/W
A300B4-200	62.52	241.92	179.39	11.18
A300-600R	63.78	241.92	178.14	13.13
A310-200	63.64	247.82	184.17	16.19
A310-300	67.92	247.82	179.90	16.19
A330-200	64.21	253.72	189.51	9.37
A330-300	64.21	253.72	189.51	9.37
A330-800NEO	57.66	253.72	196.05	6.59
A330-900NEO	57.66	253.72	196.05	6.59
A340-200	68.30	253.72	185.41	8.91
A340-300	68.49	253.72	185.23	8.91
A340-500	73.16	253.72	180.55	11.47
A340-600	73.16	253.72	180.55	11.47
A350-900	62.47	262.57	200.10	8.22
A350-1000	64.75	262.57	197.82	8.79
A380-800	64.74	283.22	218.48	5.92
B747-SP	61.87	271.42	209.55	8.93
B747-100	63.40	271.42	208.02	8.12
B747-200B	67.49	271.42	203.93	8.60
B747-300	67.49	271.42	203.93	8.93
B747-400	69.60	271.42	201.82	9.56
B747-8	70.56	265.52	194.96	9.26
B767-200	55.74	253.72	197.98	10.00
B767-200ER	62.42	253.72	191.30	11.54
B767-300	58.76	253.72	194.96	11.54
B767-300ER	63.75	253.72	189.97	11.71
B767-400ER	65.76	253.72	187.95	11.11
B777-200	59.66	262.57	202.91	7.90
B777-200ER	65.46	262.57	197.11	9.61

B777-200LR	70.00	262.57	192.57	11.46
B777-300	65.66	262.57	196.91	10.14
B777-300ER	70.41	262.57	192.16	11.46
B777-8	64.82	262.57	197.75	8.11
B777-9	64.82	262.57	197.75	8.11
B787-8	61.03	265.52	204.49	7.80
B787-9	64.42	265.52	201.10	8.91
B787-10	64.42	265.52	201.10	9.47