Project

Calculating the Wing Lift Distribution with the Diederich Method in Microsoft Excel

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Abstract

Aim of this project is to provide the Diederich Method for calculating the lift distribution of a wing in a Microsoft Excel spreadsheet based on didactic considerations. The Diederich Method is described based on primary and secondary literature. Diagrams are digitized so that the method can run automatically. To optimize the lift distribution of the wing, the elliptical and triangular lift distribution as well as Mason's lift distribution are offered for comparison. A method for calculating the maximum lift coefficient of the wing is integrated into the Diederich Method. To do this, the maximum lift coefficients of the airfoils at the wing root and at the wing tip must be entered in the program. The calculation assumes a trapezoidal wing. Both wing sweep and linear wing twist can be taken into account. The aspect ratio must not assume values that are too small. Subsonic flow and unseparated flow are assumed. Since only the wing is described, all other influences such as from the fuselage or from the engines are not taken into account. The Excel workbook was created for teaching in aircraft preliminary design. At the moment, the Diederich Method is apparently nowhere offered as a spreadsheet. With this work, this gap can be closed.

DEPARTMENT OF AUTOMOTIVE AND AERONAUTICAL ENGINEERING

Calculating the Wing Lift Distribution with the Diederich Method in Microsoft Excel

Task for a Project (Bachelor studies)

Background

The lift distributions on the wing are essential for the induced drag, loads on the wing, therefore, for the wing mass, the stall behavior, and the maximum lift coefficient of the wing. The Diederich method is a semi-empirical method for determining the lift distribution of wings. The method is described in DIEDERICH, Franklin W., 1952. *A Simple Approximate Method for Calculating Spanwise Lift Distributions and Aerodynamic Influence Coefficients at Subsonic Speeds.* Washington: NACA (Technical Note 2751). For a better approach, see TORENBEEK, Egbert, 1988. *Synthesis of Subsonic Airplane Design*. Delft: University Press. Available at: https://bit.ly/3m8KIIV. Priyanka Barua and Dieter Scholz have already created an Excel workbook with the method, this can be used, but it still needs a didactic revision.

Task

The Diederich method should be made available as an Excel spreadsheet, adapted to existing tools for aircraft design at HAW Hamburg. These are the work steps:

- Reworking the mathematical basics of the method.
- Functional and optical revision of the existing Excel workbook.
- Creating a website and a user manual to make the method available online in English.

The report has to be written in English based on German or international standards on report writing.

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List of Symbols

A	aspect ratio
a	speed of sound
b	span
c	chord
C_L	lift coefficient of the wing
$C_{L,max}$	maximum lift coefficient of the wing
c_g	mean geometric chord
c_l	lift coefficient of the airfoil
$c_{l,max}$	maximum lift coefficient of the airfoil
C_{1-4}	Diederich´s Factor
c_{l_α}	lift curve slope of the airfoil
$(c_{l\alpha})_{\text{theory}}$	theoretical lift curve slope of the airfoil
$c_{l_{\alpha_i}}$	experimental lift curve slope of the airfoil
E	Jones Edges Velocity Factor
F	pan view parameter
f	sweep angle correction function
L	lift
$L_{a/b}$	lift distribution function
M	Mach number
S	wing area
(t/c)	relative airfoil thickness
V	airspeed
y	wing coordinate

Greek Symbols

α	angle of attack
α_{0_1}	Factor
β	Prandtl-Glauert Factor
ε	twist
φ_β	effective sweep angle
φ'_{TE}	airfoil trailing edge angle
φ_{25}	sweep angle of the 25% line
Γ	lift distribution
η	normalized spanwise wing coordinate (η = 1 at wing tip)
λ	taper

ν	kinematic viscosity
ρ	density

Indices

a	additional
b	basic
r	wing root
t	wing tip

List of Abbreviations

HTML	Hypertext Markup Language
ISA	International Standard Atmosphere
NACA	National Advisory Committee for Aeronautics
PDF	Portable Document Format
URL	Universal Resource Locator

1 Introduction

1.1 Motivation

The design of a new aircraft covers many subject areas, which all directly influence the final aircraft individually. One of these design areas is the development of the airplane wing. It represents an essential element in the design process. Ultimately, the wing design decisively determines vital properties such as fuel consumption, flight characteristics, and aircraft weight. Special attention is paid to the lift distribution for a comparative measure of different wing designs. The lift distribution gives a direct indication of the wing properties. Among other things, it is possible to see how the structural loading of the wing itself and the bending moment load applied at the fuselage transition turns out. Also, statements about drag arising on the wing can be made, which have importance in economic efficiency. Since the calculation of a lift distribution, when considering many physical variables as well as wing and atmospheric parameters, is associated with an increased effort, the question arises of how this work can be minimized while maintaining the accuracy of the results. Companies have complex computational programs at this point, but these are not publicly available due to secrecy or cost reasons. Indeed, students are often very interested in understanding the processes on a wing and, if necessary, applying them to their calculations or designs. It is desirable to integrate all the possibilities in one program compactly and understandably. In this way, this work aims to create such a program, which guarantees an intuitive use as possible under the keeping of a didactically considered structure. The program, which will be available in the form of an Excel workbook, is mainly addressed to students in the field of aeronautical Engineering. However, it is also available to any other interested user.

1.2 Title Terminology

Important words from the title "Calculating the Wing Lift Distribution with the Diederich Method in Microsoft Excel" are defined.

To Calculate
"To determine by mathematical processes" (Merriam-Webster 2023).

Wing
"A main supporting surface of an aircraft. This may be divided into inner, outer and wing-tip sections." (AGARD 1980)

Lift Distribution
"The spanwise disposition of the lift on a wing or other lifting surface." (AGARD 1980)

Lift (Noun)
"A component of the total aerodynamic force acting on an airfoil which causes an airplane to fly. In level flight, a lift force equal to the weight must be produced." (Crocker 2005)

Diederich Method
Franklin W. Diederich describes the method in 1952 in NACA Technical Note 2751 "A Simple Approximate Method for Calculating Spanwise Lift Distributions and Aerodynamic Influence Coefficients at Subsonic Speeds" (Diederich 1952). An easier approach can be found in Egbert Torenbeek's textbook from 1988 (Torenbeek 1988). The "Diederich Method" calculates the lift distribution, the distribution of the local lift coefficient, and the maximum lift coefficient of a wing based on geometric input and simple functions provided by the method.

Microsoft Excel
"Microsoft Excel is a spreadsheet developed by Microsoft for Windows, macOS, Android, iOS and iPadOS. It features calculation or computation capabilities, graphing tools, pivot tables, and a macro programming language called Visual Basic for Applications (VBA). Excel forms part of the Microsoft 365 suite of software." (Wikipedia 2023).

1.3 Literature Review

The most important literature source for this work is from Torenbeek (1988). Torenbeek reiterates the techniques of the Diederich method, published by Diederich (1952), and combines them with the findings of Anderson (1936). Due to the conflation and the simple formula usability of the procedure given by Torenbeek (1988), the mathematical basics of this work are based mainly on his book. The design of the Graphical User Interface (GUI) of the Excel file follows the standard set by Wolf (2009, Chapter 4) and Montarnal (2015, Chapter 1.4). This standard has been applied to all major Excel software under the supervision of Professor Scholz. Examples are:

- http://PreSTo.ProfScholz.de,
- http://OpenVSP.ProfScholz.de,
- http://SAS.ProfScholz.de.

As such this standard is also applied here.

1.4 Structure of the Work

This report is structured as follows:

Section 2	deals with the lift distributions and treats the effects of certain distribution forms.
Section 3	deals with the mathematical fundamentals for calculating the lift distributions based on the Diederich method.
Section 4	incurs the surface design of a teaching tool and subsequently describes the creation and functionality of the Excel workbook.
Section 5	elucidates the structure and design of the website.
Section 6	summarizes the achieved results of the project.
Appendix A	includes the Equations for determining needed atmospheric data.
Appendix B	contains the source codes of the functions created in Visual Basic.

2 Lift Distributions on Aircraft Wings

A lift distribution is the local distribution of the lift forces acting on the entire aircraft. The lift of the fuselage, empennage and other aircraft components is considered. These factors are often neglected for preliminary sizing and conceptual design; instead, the whole wing area, including the areas covered by the fuselage, is considered (Torenbeek 1988). The Greek letter gamma, Γ often refers to the lift distribution. It can be defined by the two-dimensional lift coefficient of the airfoil section, c_l multiplied by the existing local chord, $c(\eta)$. The chord, $c(\eta)$ is related to the mean geometric chord, c_g of the airplane wing to get a size-independent distribution. The wing coordinate, η describes the normalized spanwise wing coordinate (η = 1 at wing tip). The included variables will be discussed again in detail in Section 3.1. The lift distribution can now be described as

$$\Gamma(\eta) = \frac{c_l(\eta) \cdot c(\eta)}{c_g} \tag{2.1}$$

2.1 Forms of Distribution and Their Significance

Lift distributions can take different shapes depending on the choice of wing parameters. The shape of the distribution decisively determines the technical aspects of the aircraft and should therefore be a subject of close observation. The following distributions have different characteristics, which are briefly explained here.

2.1.1 Triangular Lift Distribution

The triangular lift distribution is the simplest assumption of a lift function. It often does not correspond to reality and is found in a similar form, only in tapered wings with a high aspect ratio. However, the triangular distribution positively affects the bending moment load at the wing root due to the comparatively close accumulation of lift forces at the fuselage.

2.1.2 Elliptical Lift Distribution

The elliptical lift distribution for all angles of attack will be achieved with an elliptical chord distribution, no twist, and the presence of geometrically equal wing sections (Dubs 1979). If the distribution is elliptical, the local vertical speed at the wing's trailing edge has a constant distribution over the entire span. The induced drag, resulting from the wing's downwash, reaches a minimum value for this case. Low-induced resistances are especially important due to the economic aspects because the fuel consumption of an aircraft depends on the generated resistances. Therefore, the elliptical distribution is often considered the optimum lift distribution, especially for achieving high lift-to-drag ratios.

2.1.3 Lift Distribution According to Mason

The lift distribution, according to Mason, represents a hybrid form of the two distributions explained above. It considers the higher structural loading of a wing with an elliptical lift distribution compared to one with a triangular lift distribution. When the elliptical distribution is adapted to the triangular distribution, the induced drag is increased, but on the other hand, the structural weight decreases because less bending moment load must be taken at the wing root. Mason was now concerned with the question of to what point the weight reduction of the structure provides more significant advantages than the simultaneous increase of induced drag brings disadvantages. The results are summarized by Mason (2001). According to Mason, the lift distribution is shown in blue in Figure 2.1, with an 11% reduction of the bending moment load at the wing root. The elliptical distribution is shown in red, and the triangular distribution is shown in green.

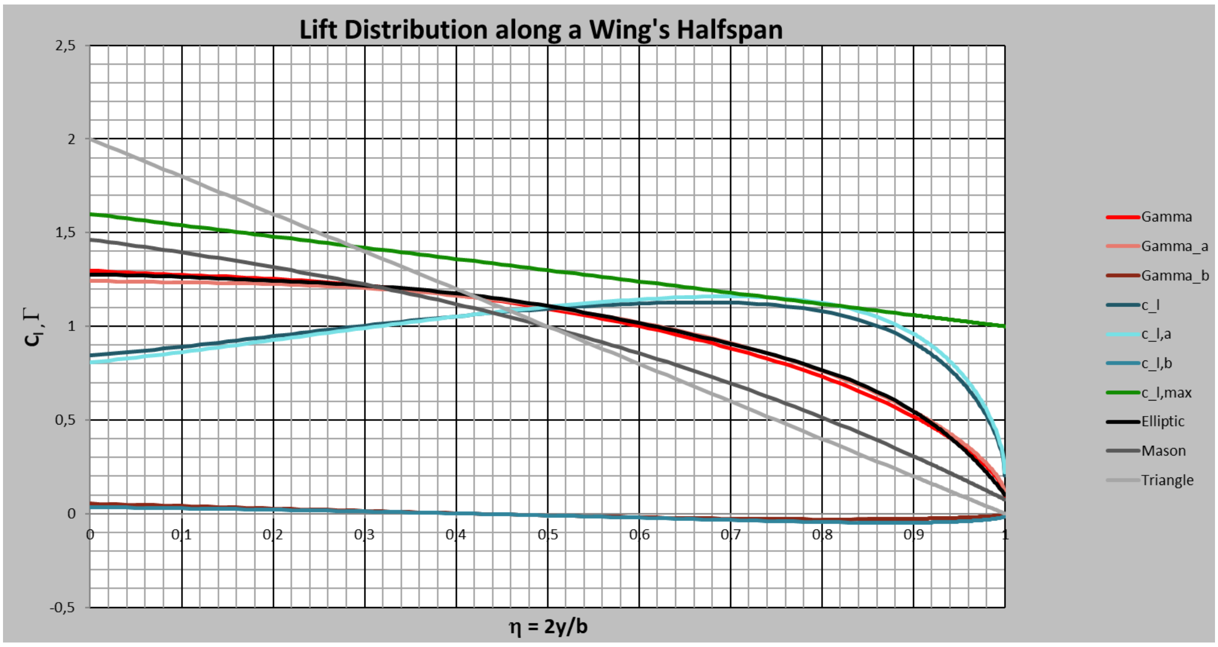


Figure 2.1 Lift distribution, according to Mason, with triangular and elliptical distribution as a comparison

2.2 Calculation Opportunities

Diverse techniques exist for the calculation of lift distributions. Probably the best-known and currently used method for the preliminary design of an airplane is the calculation according to Prandtl's wing theory. It assumes a circular distribution acting around the wing, which is, according to the Kutta Joukowski condition, proportional to the lift distribution of the wing (Oertel 2017). The calculation is performed according to the so-called Prandtl's integrodifferential Equation. Solving the integrodifferential Equation with simple calculus proves difficult; therefore, the Diederich method offers a different technique for calculating the lift distribution. It is dividing the lift distribution into two components and calculates them separately. In the end, both distributions are superposed to the total lift distribution. The Diederich method will be used in the following chapters due to the better integration into an Excel workbook and the easier understanding of the technique, which is especially important for the use within a teaching aid.

3 Lift Distribution According to Diederich

3.1. Theoretical Foundations

To understand the calculation technique according to the Diederich method, it is initially necessary to explain the most important theoretical foundations. The equations are taken from Torenbeek (1988) unless otherwise indicated.

3.1.1 Wing Coordinate

To describe a half-span in this work, the coordinate η is used, which is normalized with respect to the span, b and defined as

$$\eta = \frac{2 \cdot y}{b} \quad . \tag{3.1}$$

The origin (zero-point) is thereby located on the longitudinal axis of the fuselage. Thus, η describes the span of a wing in the range from -1 to 1.

3.1.2 Wing Area

The wing area, S is the plan area of the wing and can be determined with the following formula according to Scholz (2015)

$$S = 2 \cdot \int_0^1 c(\eta)\, dy \quad . \tag{3.2}$$

Here is $c(\eta)$ the chord distribution along the wing.

3.1.3 Aspect Ratio

The aspect ratio, A is defined as follows and describes the wing's slenderness ratio

$$A = \frac{b^2}{S} \quad . \tag{3.3}$$

A high aspect ratio points to an elongated narrow wing, whereas a low aspect ratio characterizes a short and wide wing.

3.1.4 Taper Ratio

The taper ratio, λ describes the ratio of the chord at the wing tip to the chord at the wing root and is thus defined as

$$\lambda = \frac{c_t}{c_r} \quad . \tag{3.4}$$

A low taper ratio occurs on very pointed wings, while a taper ratio of $\lambda = 1$ equates to a rectangular wing. According to Scholz (2015), an almost optimally elliptical lift distribution is achieved for $\lambda \approx 0.45$.

3.1.5 Chord Designations

The mean geometric chord, c_g is the chord of an equivalent unswept rectangular wing having the same base area as the initial wing. It can be determined using the following approach, derived from (3.3).

$$c_g = \sqrt{\frac{S}{A}} \tag{3.5}$$

The chord at the wing root, c_r and the chord at the wing tip, c_t can be determined by

$$c_r = \frac{2 \cdot c_g}{1 + \lambda} \quad , \tag{3.6}$$

$$c_t = \lambda \cdot c_r \quad . \tag{3.7}$$

In case of a linear distribution of the chord over the half span, $c(\eta)$ it can be described by a simple linear function.

$$c(\eta) = (c_t - c_r) \cdot \eta + c_r \tag{3.8}$$

3.1.6 Wing Sweep

The sweep angle, φ_{25} is defined between a line perpendicular to the fuselage longitudinal axis and the 25% line. The 25% line connects the points of every airfoil section, each located at $1/4$ of the chord, starting from the airfoil nose. If the sweep angle of the wing is not given by the 25% line, it can be converted from an m-line to an n-line using the following equation, according to Scholz (2015). The values *m* and *n* are stated as a percentage.

$$\varphi_n = \arctan\left[\tan(\varphi_m) - \frac{4}{A} \cdot \left(\frac{n-m}{100} \cdot \frac{1-\lambda}{1+\lambda}\right)\right] \tag{3.9}$$

3.1.7 Twist

The twist, ε describes the change of incidence angle over the half span. If the twist is negative, the airfoil nose slowly lowers towards the wing tip. It is called a washout. If there is a positive twist, this is called a wash-in. When describing the twist, the incidence angle of the wing root is selected as the reference. This is provided to be zero degrees in the following calculations. From this, the wing tip twist follows with the help of both angles of attack, α_t and α_r, according to Torenbeek (1988)

$$\varepsilon_t = \alpha_t - \alpha_r \quad . \tag{3.10}$$

If there is a linear twist over the half span, it can be described as a function of, η as

$$\varepsilon(\eta) = \varepsilon_t \cdot \eta \quad . \tag{3.11}$$

3.1.8 Relative Airfoil Thickness

The relative airfoil thickness, t/c is the ratio of the thickest place of an airfoil and the corresponding chord. It can be seen in Abbott (1959) for a wide range of airfoils. The relative airfoil thickness can be identified by the last two digits of the 4-digit NACA airfoils and is given as a percentage.

3.1.9 Mach Number

The dimensionless Mach number, M describes the airspeed based on the local speed of sound. The speed of sound depends on the flight altitude and can be determined by using the atmospheric equations of the ISA, which are shown in Appendix A (Scholz 2022). For the Mach number follows finally

$$M = \frac{V}{a} \ . \tag{3.12}$$

3.1.10 Lift Coefficient

The lift coefficient, C_L is a dimensionless number to describe the lifting behaviour of a wing.

$$C_L = \frac{2 \cdot L}{\rho \cdot V^2 \cdot S} \tag{3.13}$$

The lift, L can be equated with the aircraft weight for horizontal flight. V is the airspeed, the same as in the previous section for the Mach number determination, and S is the wing area, as in Section 3.1.2. The density, ρ can also be determined using the equations in Appendix A.

3.1.11 Reynolds Number

The Reynolds number, Re is a dimensionless aerodynamics parameter which is used, besides other things, to distinguish between the laminar and turbulent flow. It is defined as the ratio of the product of body length, l and flow speed, V to the kinematic viscosity ν.

$$Re = \frac{V \cdot l}{\nu} \tag{3.14}$$

For the applied case on the wing, the flowed body length can be equated with the mean geometric chord, c_g from Section 3.1.5. The kinematic viscosity, ν is also determined using equations in Appendix A.

3.2 Calculation of the Lift Distribution

The method used in this work to calculate the lift distribution is a slightly modified version of the method described by Diederich (1952). Because of the better didactics, the procedure described by Torenbeek (1988) will be used with small insertions in the fundamental principle. Torenbeek's work is based on that of Diederich and combines it with the findings of Anderson (1936).

The Diederich method is a semi-empirical method for determining the lift distribution of wings. The method assumes splitting the lift distribution into two components. One is the basic lift distribution, and the other is the additional lift distribution. The basic lift distribution is defined as the lift distribution for the twisted wing with a lift coefficient of $C_L = 0$. The additional lift distribution is the lift distribution on the untwisted wing with the standardized lift coefficient of $C_L = 1$. The superposition of both distributions and adjustment to the existing lift coefficient provides the lift distribution of the wing.

According to Torenbeek (1988), the following conditions must be fulfilled so that the method delivers target-oriented results.

- The airspeed is within the subsonic area. According to Roskam (1997), the transonic range starts from Mach numbers of $M > 0.8$. This value should not be exceeded to ensure that there are no or negligible compressibility effects.
- The angles of attack are relatively small so that the flow is always present. Separation phenomena will not be considered.
- The influence of the fuselage on the wing will not be considered. It is assumed that the properties of the fuselage at relevant points are equal to the properties of the wing when it is on its own.
- The twisting of the wing is assumed to be linear. (See Section 3.1.7)
- The leading and trailing edges of the wing run in a straight line.
- The sweep angle is in the range of $-35° < \varphi_{25} < 35°$.
- The minimum aspect ratio, A of the wing can be determined as a function of the sweep angle using (3.15).

$$A \geq \frac{4}{\cos(\varphi_{25})} \tag{3.15}$$

- Engine, ground, and aeroelasticity effects will not be considered.

3.2.1 Additional Lift Distribution

For the calculation of the additional lift distribution, $\Gamma_a(\eta)$ the lift distribution function, $L_a(\eta)$ is to be calculated.

$$L_a(\eta) = C_1 \cdot \frac{c(\eta)}{c_g} + C_2 \cdot \frac{4}{\pi} \cdot \sqrt{1-\eta^2} + C_3 \cdot f(\eta) \tag{3.16}$$

In a special case of an unswept wing ($\varphi_{25} = 0$), it results in an elliptical distribution of the sweep angle correction function, $f(\eta)$ see Figure 3.3. Subsequently, the additional lift distribution function can be simplified to

$$L_a(\eta) = C_1 \cdot \frac{c(\eta)}{c_g} + (C_2 + C_3) \cdot \frac{4}{\pi} \cdot \sqrt{1-\eta^2} \quad . \tag{3.17}$$

The needed chords, $c(\eta)$ and c_g are already known from Section 3.1.5.

The coefficients C_1, C_2, and C_3 can be determined from Figure 3.1 with the help of the plan view parameter. This is defined as

$$F = \frac{2\pi \cdot A}{c_{l_\alpha} \cdot \cos(\varphi_{25})} \quad . \tag{3.18}$$

The needed lift curve slope, c_{l_α} describes the local lift increase of a wing section. According to Torenbeek (1988) the value, $c_{l_\alpha} = 6.1\ 1/\mathrm{rad}$ can be used if no exact data from the airfoil are available. This value gives satisfactory results by relative thicknesses of $0.1 < t/c < 0.2$. If airfoil data are available, c_{l_α} can be determined with additional consideration of the Mach number influence, as per Datcom (1978).

$$c_{l_\alpha} = \frac{1.05}{\beta} \cdot \left[\frac{c_{l_{\alpha_i}}}{\left(c_{l_\alpha}\right)_{theory}} \right] \cdot \left(c_{l_\alpha}\right)_{theory} \tag{3.19}$$

with

$$\left(c_{l_\alpha}\right)_{theory} = 2\pi + 4.7 \cdot \frac{t}{c} \cdot (1 + 0.00375 \cdot \varphi'_{TE}) \quad . \tag{3.20}$$

The angle of the airfoil trailing edge, φ'_{TE} can be determined as described in Figure 3.2. Also, the required ratio of the experimental lift curve slope, $c_{l_{\alpha_i}}$ to the theoretical lift curve slope, $\left(c_{l_\alpha}\right)_{theory}$ can be determined with the aid of the Reynolds number explained in Section 3.1.11 and the determined airfoil trailing edge angle.

The Mach number, M can determine the still missing Prandtl-Glauert factor β

$$\beta = \sqrt{1 - M^2} \tag{3.21}$$

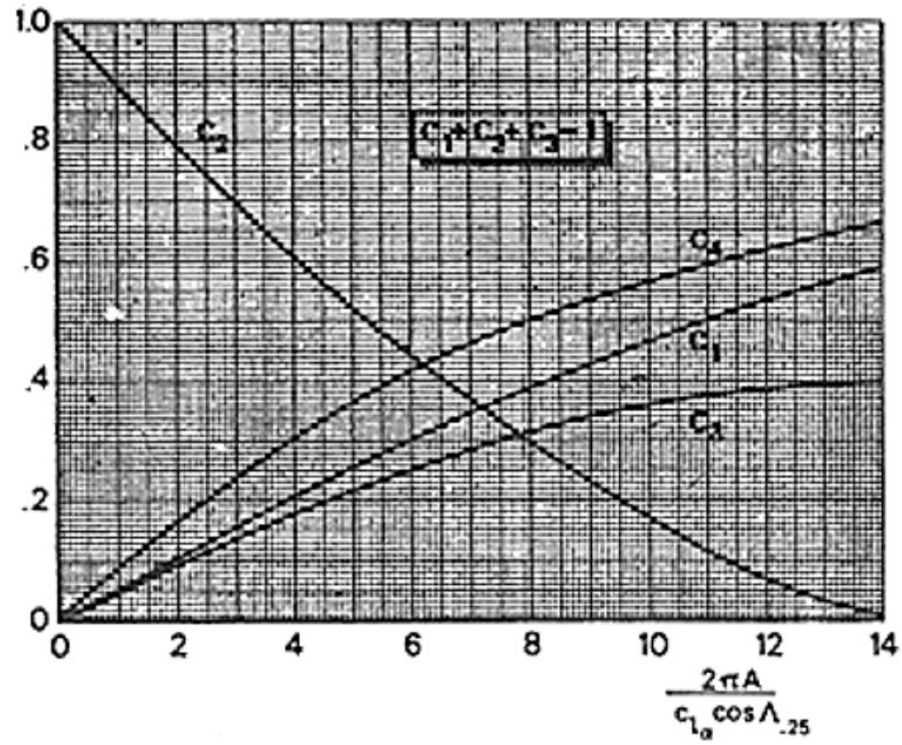


Figure 3.1 C_1, C_2, C_3 *and* C_4 as a function of the plan view parameter F (Torenbeek 1988)

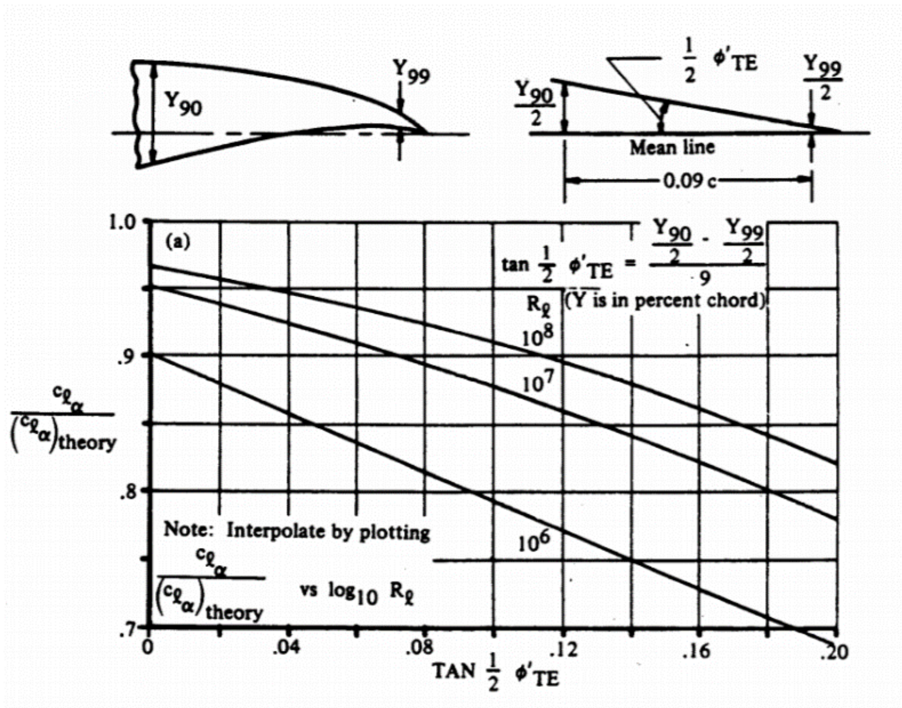


Figure 3.2 Definition of the airfoil trailing edge angle φ'_{TE} and determination of the lift curve slope ratio (Datcom 1978)

The still-needed sweep angle correction function, $f(\eta)$ is shown in Figure 3.3. The choice of the curve depends on the effective sweep angle

$$\varphi_\beta = \arctan\left(\frac{\tan(\varphi_{50})}{\beta}\right) \ . \tag{3.22}$$

Since the diagram only provides a curve representation for certain effective sweep angles, it is necessary to perform interpolation for special angles.

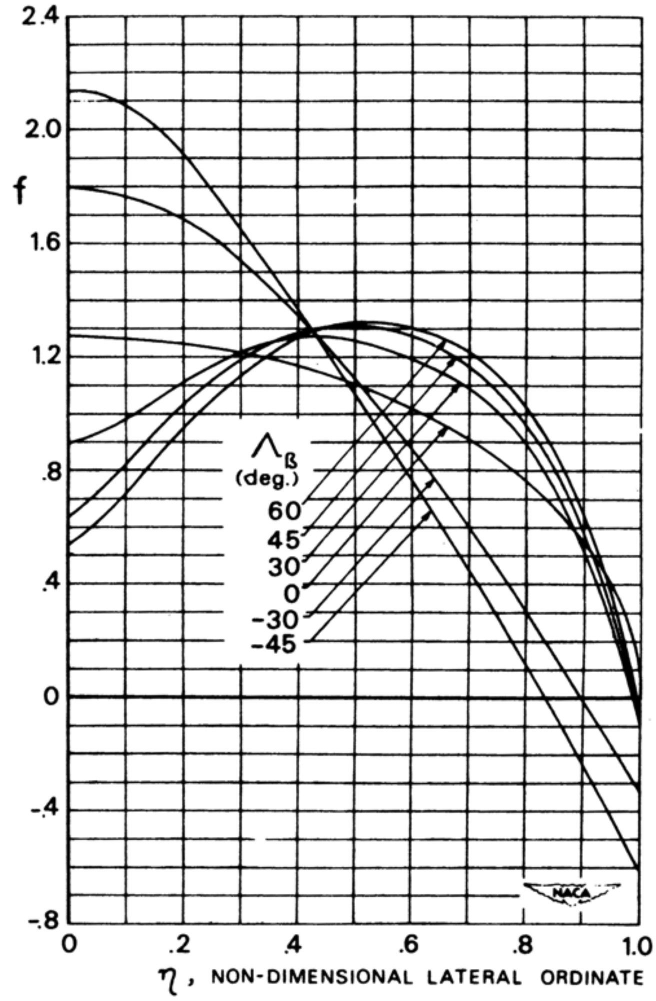


Figure 3.3 Sweep angle correction function, $f(\eta)$ according to Diederich (1952) from Torenbeek (1988), Λ_β is in this work φ_β

The lift distribution function, $L_a(\eta)$ can now be determined using (3.16) or (3.17). Since there is a standardization to the lift coefficient of $C_L = 1$, the function must still be multiplied by the existing lift coefficient of the wing. The result is the additional lift distribution $\Gamma_a(\eta)$.

$$\Gamma_a(\eta) = \frac{c_{l_a}(\eta) \cdot c(\eta)}{c_g} = L_a(\eta) \cdot C_L \tag{3.23}$$

3.2.2 Basic Lift Distribution

For the basic lift distribution, $\Gamma_b(\eta)$ the basic lift distribution function $L_b\,(\eta)$, will first be determined.

$$L_b\,(\eta) = L_a\,(\eta) \cdot C_4 \cdot \cos(\varphi_\beta) \cdot \left(\frac{\varepsilon(\eta)}{\varepsilon_t} + \alpha_{0_1}\right) \cdot \beta \cdot E \tag{3.24}$$

Due to the assumption of a linear twist, with the wing root as the reference the origin (zero-point), the proportion $\varepsilon(\eta)/\varepsilon_t$ is reduced to

$$\frac{\varepsilon(\eta)}{\varepsilon_t} = \frac{\varepsilon_t \cdot \eta}{\varepsilon_t} = \eta \quad . \tag{3.25}$$

The coefficient, C_4 can also be determined from Figure 3.1 using the plan view parameter, F which is determined according to (3.18).

The Jones Edges Velocity factor, E can be described by the following approximation

$$E = 1 + \frac{2 \cdot \lambda}{A \cdot (1 + \lambda)} \quad . \tag{3.26}$$

Using the additional lift distribution function according to (3.16) or (3.17) and assuming a linear twist, the factor α_{0_1} becomes

$$\alpha_{0_1} = -\int_0^1 \eta \cdot L_a\,(\eta)\, d\eta \quad . \tag{3.27}$$

Similarly, to the assumption for (3.17), the Equation for unswept wings can be simplified to

$$-\alpha_{0_1} = C_1 \cdot \frac{1 + 2\lambda}{3 \cdot (1 + \lambda)} + (C_2 + C_3)\frac{4}{3\pi} \quad . \tag{3.28}$$

The basic lift distribution, $\Gamma_b(\eta)$ can be calculated using the basic lift distribution function, $L_b\,(\eta)$ and the already determined factors.

$$\Gamma_b(\eta) = \left(\frac{c_{l_b}(\eta) \cdot c(\eta)}{c_g}\right) = \frac{L_b\,(\eta) \cdot \varepsilon_t \cdot c_{l_\alpha}}{E} \tag{3.29}$$

3.2.3 Total Lift Distribution

The additional and basic lift distributions from (3.23) and (3.29) will be added to represent the total lift distribution.

$$\Gamma(\eta) = \Gamma_a(\eta) + \Gamma_b(\eta) \tag{3.30}$$

3.3 Distribution of Local Lift Coefficients

Among other things, the distribution of the lift coefficients, $c_l(\eta)$ is important to localize in critical flight attitudes where the flow first separates along the wing. According to Torenbeek (1988), the following relation is available to determine this.

$$\Gamma(\eta) = \frac{c_l(\eta) \cdot c(\eta)}{c_g} = \frac{c_{l_b}(\eta) \cdot c(\eta)}{c_g} + \frac{c_{l_a}(\eta) \cdot c(\eta)}{c_g} \tag{3.31}$$

From this, the following relationship can be formulated with the help of (3.23) and (3.29).

$$c_{l_a}(\eta) = \frac{c_g}{c(\eta)} \cdot L_a(\eta) \cdot C_L \tag{3.32}$$

$$c_{l_b}(\eta) = \frac{c_g}{c(\eta)} \cdot \frac{L_b(\eta) \cdot \varepsilon_t \cdot c_{l_\alpha}}{E} \tag{3.33}$$

The distribution of the local lift coefficients can now be determined from the determined quantities.

$$c_l(\eta) = c_{l_a}(\eta) + c_{l_b}(\eta) \tag{3.34}$$

3.4 Maximum Lift Coefficient

The calculation of the maximum lift coefficient of the wing follows the procedure described in Torenbeek (1988). This is achieved when at any point along the wing, the local lift coefficient of the airfoil exceeds the maximum lift coefficient for this section. An example of this is shown in Figure 3.4. By further increasing the angle of attack, the flow starts to separate at this point, and the lift performance of the wing decreases. With the quantities determined in the previous sections, the maximum lift coefficient of the wing $C_{L,max}$ can be determined from the minimum of the term in brackets and the existing lift coefficient of the wing C_L .

$$C_{L,max} = \min\left[\frac{\left(c_{l,max}(\eta) - c_{l_b}(\eta)\right)}{c_{l_a}(\eta)}\right] \cdot C_L \tag{3.35}$$

This means that the term in brackets has to be evaluated at each spanwise station and the minimum value of all of them is determined.

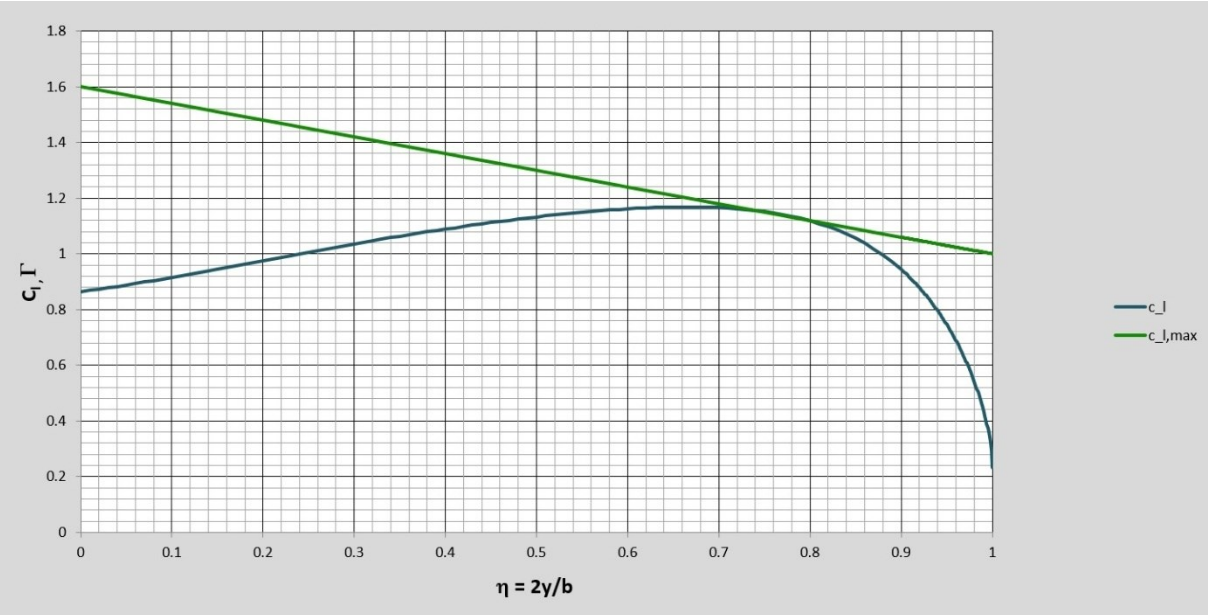


Figure 3.4 The maximum lift coefficient of the wing is reached when at one spanwise position on the wing, the local lift coefficient (blue line) reaches the maximum lift coefficient of the airfoil (green line)

4 Creating the Excel Workbook

4.1 General

A possibility to introduce students to the contents of wing design without having to deal intensively with the literature is the provision of an Excel workbook. Here, the basic principles can be understood quickly via parameter changes, and the subject matter can be learned "playfully". This work is based on an already existing Excel workbook, which resulted from the cooperation of Professor Scholz and Priyanka Barua. The basic calculation procedures for the lift distribution have already been presented in this file. Some additions and essential didactic reworking have been made to make the teaching aid usable for students without direct knowledge of the subject. In addition, the entire scope of the teaching aid was created in English to make it usable for non-German-speaking students. The following sections explain the Excel workbook's functionality and interface design basics. For details see "User Guide for the Diederich Method Excel-File" by Scholz (2023).

4.2 Design Basics

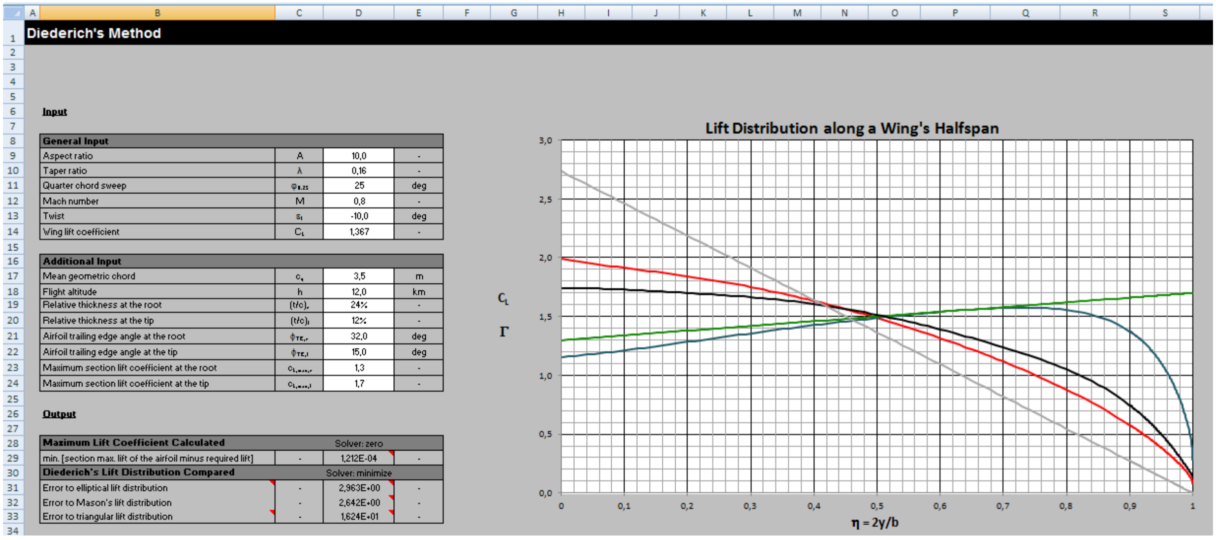
The user interface of the Excel workbook is adapted to the layout definition by Wolf (2009) and Montarnal (2015), which consists of three field types:

- Module title
- Sub-module title
- Calculation & Presentation

The field module title has a black background and white font. The sub-module title field has a 40% grey fill and black font. The Calculation & Presentation field has a 25% grey fill and black font. Further details on formatting can be found in Wolf (2009) and Montarnal (2015).

The Calculation & Presentation section contains input and output fields of the calculation, tables, parameter names, formula symbols and units. We divide the Calculation & Presentation section into a right and a left section. The left side should contain the fixed values and inputs, and the right should contain the calculation and outputs. Due to the structure, it is impossible to realize this division in every application, as was the case here.

The input fields have a white background so the user can see immediately where entries should be made.

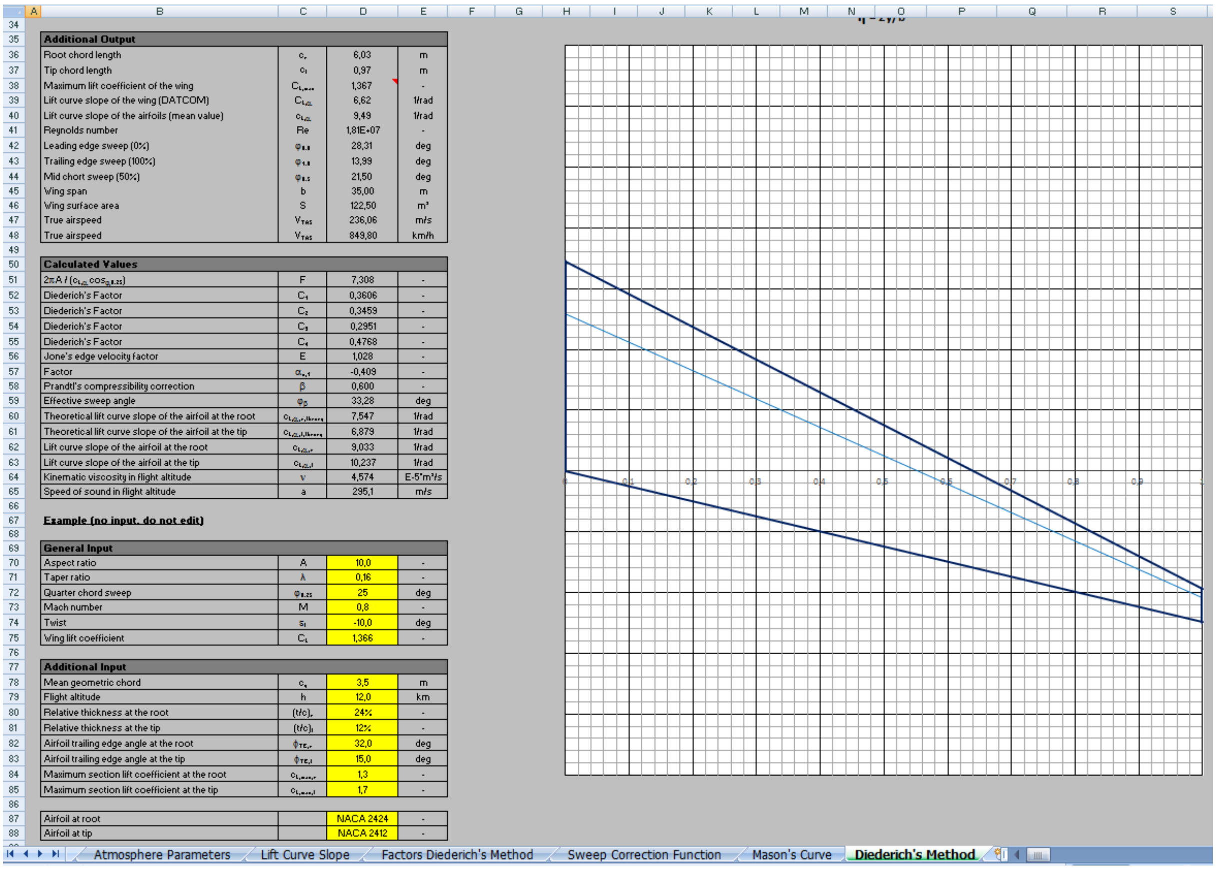


General Input			
Aspect ratio	A	10,0	-
Taper ratio	λ	0,16	-
Quarter chord sweep	$\varphi_{0.25}$	25	deg
Mach number	M	0,8	-
Twist	ε_t	-10,0	deg
Wing lift coefficient	C_L	1,367	-

Additional Input			
Mean geometric chord	c_g	3,5	m
Flight altitude	h	12,0	km
Relative thickness at the root	$(t/c)_r$	24%	-
Relative thickness at the tip	$(t/c)_t$	12%	-
Airfoil trailing edge angle at the root	$\phi_{TE,r}$	32,0	deg
Airfoil trailing edge angle at the tip	$\phi_{TE,t}$	15,0	deg
Maximum section lift coefficient at the root	$c_{L,max,r}$	1,3	-
Maximum section lift coefficient at the tip	$c_{L,max,t}$	1,7	-

Maximum Lift Coefficient Calculated		Solver: zero	
min. [section max. lift of the airfoil minus required lift]	-	1,212E-04	-
Diederich's Lift Distribution Compared		Solver: minimize	
Error to elliptical lift distribution	-	2,963E+00	-
Error to Mason's lift distribution	-	2,642E+00	-
Error to triangular lift distribution	-	1,624E+01	-

Figure 4.1 User interface of the Excel workbook (upper part)



Additional Output			
Root chord length	c_r	6,03	m
Tip chord length	c_t	0,97	m
Maximum lift coefficient of the wing	$C_{L,max}$	1,367	-
Lift curve slope of the wing (DATCOM)	$C_{L,\alpha}$	6,62	1/rad
Lift curve slope of the airfoils (mean value)	$c_{L,\alpha}$	9,49	1/rad
Reynolds number	Re	1,81E+07	-
Leading edge sweep (0%)	$\varphi_{0.0}$	28,31	deg
Trailing edge sweep (100%)	$\varphi_{1.0}$	13,99	deg
Mid chort sweep (50%)	$\varphi_{0.5}$	21,50	deg
Wing span	b	35,00	m
Wing surface area	S	122,50	m²
True airspeed	V_{TAS}	236,06	m/s
True airspeed	V_{TAS}	849,80	km/h

Calculated Values			
$2\pi A / (c_{L,\alpha} \cos_{\varphi 0.25})$	F	7,308	-
Diederich's Factor	C_1	0,3606	-
Diederich's Factor	C_2	0,3459	-
Diederich's Factor	C_3	0,2951	-
Diederich's Factor	C_4	0,4768	-
Jone's edge velocity factor	E	1,028	-
Factor	$\alpha_{0,1}$	-0,409	-
Prandtl's compressibility correction	β	0,600	-
Effective sweep angle	φ_β	33,28	deg
Theoretical lift curve slope of the airfoil at the root	$c_{L,\alpha,r,theory}$	7,547	1/rad
Theoretical lift curve slope of the airfoil at the tip	$c_{L,\alpha,t,theory}$	6,879	1/rad
Lift curve slope of the airfoil at the root	$c_{L,\alpha,r}$	9,033	1/rad
Lift curve slope of the airfoil at the tip	$c_{L,\alpha,t}$	10,237	1/rad
Kinematic viscosity in flight altitude	ν	4,574	E-5*m²/s
Speed of sound in flight altitude	a	295,1	m/s

General Input			
Aspect ratio	A	10,0	-
Taper ratio	λ	0,16	-
Quarter chord sweep	$\varphi_{0.25}$	25	deg
Mach number	M	0,8	-
Twist	ε_t	-10,0	deg
Wing lift coefficient	C_L	1,366	-

Additional Input			
Mean geometric chord	c_g	3,5	m
Flight altitude	h	12,0	km
Relative thickness at the root	$(t/c)_r$	24%	-
Relative thickness at the tip	$(t/c)_t$	12%	-
Airfoil trailing edge angle at the root	$\phi_{TE,r}$	32,0	deg
Airfoil trailing edge angle at the tip	$\phi_{TE,t}$	15,0	deg
Maximum section lift coefficient at the root	$c_{L,max,r}$	1,3	-
Maximum section lift coefficient at the tip	$c_{L,max,t}$	1,7	-

Airfoil at root		NACA 2424	-
Airfoil at tip		NACA 2412	-

Figure 4.2 User interface of the Excel workbook (lower part)

4.3 Use

The following section describes how to use the Excel workbook and how it works. As the theoretical principles have already been dealt with in Section 3, references will be made to this in the calculation designations. The Excel workbook consists of several worksheets. The most interesting page for the user is the "Diederich's Method" worksheet. The other worksheets explain how the main page works or provide intermediate results, which can thus be viewed separately. But these are not necessary for the use and are only an additional source of information. Additional information for understanding the Excel workbook is widely deposited in fields with red triangles by notes.

4.3.1 Diederich's Method

The Diederich method is given on the main worksheet of the Excel table called "Diederich's Method". The worksheet shows parameters on the left side and two diagrams on the right side. Input values are entered into white fields. Output values have the standard gray background. The upper part of the worksheet shows on the right a diagram with the lift distributions (Figure 4.1). The respective curve is assigned by name in the legend to the right of the diagram. Below is the second diagram (Figure 4.2). It is a visualization of the right part of the wing corresponding to the input parameters. The most important facts for use are on the far right-hand side, explained briefly once again. This way, correct use can be ensured even without a detailed user guide, which will be provided as a file in PDF format. The content of this section is not explained in detail here since it is a summary of the most important contents of the following sections. Further down the page is a large table of values containing the plot data of all graphs and auxiliary values. It is positioned outside the field of view because it is unnecessary for immediate use. However, it can be viewed anytime if certain numerical values are needed, or a better understanding of the calculation is desired.

Input

The input fields are divided into General and Additional. The General Input section contains all the values needed to determine the lift distribution. The Additional Input section provides additional adjustments that adapt the distributions even better to the aircraft configuration but are not absolutely necessary. The values shown here represent an exemplary standard airfoil NACA 2424 and NACA 2412. Also, in the initial condition, a flight altitude, h of 12 km and a mean chord, c_g of 3.5 m are assumed. This allows meaningful qualitative values of the lift distributions. However, if more detailed data are available, it is strongly recommended to adapt the additional input to the specific case.

The input parameters of the **General Input** are based on the guidelines mentioned in Section 3.2. The sweep angle, φ_{25} and the Mach number, M specifications are clearly listed to the right of the diagrams in the "How to use" field. The minimal aspect ratio, A as shown there, changes with the help of (3.15) depending on the input of the sweep angle φ_{25}. The taper, λ of the wing should be in the range $0 < \lambda \leq 1$ for conventional wing configurations. The first three parameters of the general input are mainly responsible for the appearance of the wing planform. How individual changes of these affect the wing can therefore be seen very well in the lower part of the diagram. The angle input of the twisting at the wing tip, ε_t should include realistic values that exclude flow separation at the airfoil. The lift coefficient of the wing, C_L should already be known. Otherwise, a calculation is made using (3.13). A representation of the general input is shown in Figure 4.2.

General Input			
Aspect ratio	A	10,0	-
Taper ratio	λ	0,16	-
Quarter chord sweep	$\varphi_{0.25}$	25	deg
Mach number	M	0,8	-
Twist	ε_t	-10,0	deg
Wing lift coefficient	C_L	1,367	-

Figure 4.3 Input parameters of the General Input

The first parameter of the **Additional Input** is the flight altitude, h. Input values up to 20 km provide the correct atmospheric data required for the determination of the Reynolds number using (3.14). The second parameter of the mean geometric chord, c_g also affects the Reynolds number of the wing. The relative airfoil thicknesses, (t/c) and the trailing edge angles, φ'_{TE} for the wing tip and wing root can be taken from the NACA catalogue in Abbott (1959). The values are adjusted accordingly in case of a different distribution over the half span. In the case of a constant airfoil, the same values are entered for the wing tip and the wing root. Depending on these values the lift curve slope, c_{l_α} of a wing section is finally determined, as discussed in Section 3.2.1. The maximum lift coefficient of the airfoil can also be taken from the NACA catalogue in Abbott (1959). As with the previous values, it is possible to enter the same values for the wing tip and the wing root or different values in the case of different airfoils. From this, the maximum lift coefficient, $C_{L,max}$ of the wing is determined according to the procedure in Section 3.3. The parameters of the Additional Input are shown in Figure 4.3.

Additional Input			
Mean geometric chord	c_g	3,5	m
Flight altitude	h	12,0	km
Relative thickness at the root	$(t/c)_r$	24%	-
Relative thickness at the tip	$(t/c)_t$	12%	-
Airfoil trailing edge angle at the root	$\phi_{TE,r}$	32,0	deg
Airfoil trailing edge angle at the tip	$\phi_{TE,t}$	15,0	deg
Maximum section lift coefficient at the root	$c_{L,max,r}$	1,3	-
Maximum section lift coefficient at the tip	$c_{L,max,t}$	1,7	-

Figure 4.4 Input parameters of the Additional Input

Output

The output fields are divided into three areas. The "Comparison Output", the "Additional Output" and the "Calculated Values" output.

The **Comparison Output** provides a direct measure for comparing the lift distribution determined by the Diederich method with the comparison distributions discussed in Section 2.1. The deviation is determined within the value table via the sum of the residual squares, according to Barot (2020). A small value in the output represents a well-approximated curve. On the other hand, large values represent large deviations. The smallest of the three values thus determines the most obvious property which the determined lift distribution exhibits. The properties of the respective distributions can be seen in Section 2.1. Figure 4.4 shows the Comparison Output.

Diederich's Lift Distribution Compared		Solver: minimize	
Error to elliptical lift distribution	-	2,963E+00	-
Error to Mason's lift distribution	-	2,642E+00	-
Error to triangular lift distribution	-	1,624E+01	-

Figure 4.5 Comparison Output with deviation measure

The **Additional Output** provides intermediate and additional results that are potentially interesting for the user. Among other things, depending on the mean geometric chord, c_g the chords of the wing tip, c_r and the wing root, c_t are output for the entered parameters of the general input. Furthermore, the maximum lift coefficient, $C_{L,max}$ of the wing and the mean value of the lift curve slope, c_{l_α} can be read from the lift curve slopes of the wing tip, $c_{l_{\alpha,t}}$ and the wing root $c_{l_{\alpha,r}}$. The Reynolds number of the airfoil is necessary to get the airfoil data from the NACA catalogues. The Reynolds number is output dependent on the flight altitude, h and the mean geometric chord, c_g and can thus be used for the airfoil data in the Additional Input.

For some calculations, especially in the field of aircraft design, the angles of the leading edge and the wing's trailing edge are needed. These can also be read for the selected sweep angle of the 25% line. Furthermore, the wingspan, *b* the wing area, *S* and the airspeed are stated in m/s and km/h. The listing of the Additional Output can be seen in Figure 4.5.

Additional Output			
Root chord length	c_r	6,03	m
Tip chord length	c_t	0,97	m
Maximum lift coefficient of the wing	$C_{L,max}$	1,367	-
Lift curve slope of the wing (DATCOM)	$C_{L,\alpha}$	6,62	1/rad
Lift curve slope of the airfoils (mean value)	$c_{L,\alpha}$	9,49	1/rad
Reynolds number	Re	1,81E+07	-
Leading edge sweep (0%)	$\varphi_{0.0}$	28,31	deg
Trailing edge sweep (100%)	$\varphi_{1.0}$	13,99	deg
Mid chort sweep (50%)	$\varphi_{0.5}$	21,50	deg
Wing span	b	35,00	m
Wing surface area	S	122,50	m²
True airspeed	V_{TAS}	236,06	m/s
True airspeed	V_{TAS}	849,80	km/h

Figure 4.6 Output values of the Additional Output

The **Calculated Values Output** presents numerous intermediate results of the lift distribution calculation in a bundled form. Results can be tracked here and are available as intermediate values in case of a manual calculation. Most results can also be found in the additional worksheets with the respective product graphs. Here can be seen once again where the results come from. The Calculated Values Output is shown in Figure 4.6.

Calculated Values			
2πA / ($c_{L,\alpha}$ $\cos_{\varphi,0.25}$)	F	7,308	-
Diederich's Factor	C_1	0,3606	-
Diederich's Factor	C_2	0,3459	-
Diederich's Factor	C_3	0,2951	-
Diederich's Factor	C_4	0,4768	-
Jone's edge velocity factor	E	1,028	-
Factor	$\alpha_{o,1}$	-0,409	-
Prandtl's compressibility correction	β	0,600	-
Effective sweep angle	φ_β	33,28	deg
Theoretical lift curve slope of the airfoil at the root	$c_{L,\alpha,r,theory}$	7,547	1/rad
Theoretical lift curve slope of the airfoil at the tip	$c_{L,\alpha,t,theory}$	6,879	1/rad
Lift curve slope of the airfoil at the root	$c_{L,\alpha,r}$	9,033	1/rad
Lift curve slope of the airfoil at the tip	$c_{L,\alpha,t}$	10,237	1/rad
Kinematic viscosity in flight altitude	ν	4,574	E-5*m²/s
Speed of sound in flight altitude	a	295,1	m/s

Figure 4.7 Calculated Values Output

Lift Distribution Diagram over the Half Span

The lift distribution diagram contains numerous curve progressions plotted over the wing coordinate, η as defined in Section 3.1.1, allowing a direct comparison. The reddish curves are the lift distribution according to the Diederich method. The total lift distribution, labelled as Gamma in the diagram, results from the additional lift distribution, γ_a and the basic lift distribution, γ_b as already discussed in Section 3.2. The local lift coefficients along the half span $c_l(\eta)$, $c_{l_a}(\eta)$ and $c_{l_b}(\eta)$ are represented by the blue-shaded curves. They are, among other things, the input variables for the lift distributions, as can be seen in (3.31). This shows that the lift distribution takes the chord over the half span into account. With a taper ratio of, $\lambda = 1$ and thus a constant distribution of the chord are the curves, $\Gamma(\eta)$ and $c_l(\eta)$ identical. The diagram also shows the distribution of the maximum local lift coefficient, $c_{l,max}$ over the half span. The curve is shown in green and is determined by a linear function, depending on the parameters in the Additional Input, for the maximum lift coefficients at the wing root and wing tip. The grey-shaded curves represent the comparison lift distributions listed in the Comparison Output. Here you can check again directly at which points the deviations occur and how you can adjust them to the desired distribution by the possible configuration of the wing. With a central click on the diagram, three options appear on the right. Curves can be selected and deselected by selecting the lower field to make them visible in the diagram. This allows a direct comparison of the two curves to each other.

The half span below the lift distribution diagram is, as already mentioned, a visualization of the first three General Input parameters. The dark blue lines represent the plan view of the half-wing. The light blue line represents the 25% line of the wing at which the sweep angle is also measured. For an easier understanding and a more direct factual reference, the effects of the wing parameters on the curves can be found by comparing the lift distributions. The overview of the diagrams can be seen again in Figure 4.1.

4.3.2 Mason's Curve

The "Mason's Curve" worksheet is basically used to create the curve according to Mason in the lift distribution diagram on the main page. Here, the lift distribution according to Mason (2001), already discussed in Section 2.1.3, was plotted via a table of values and made usable for Excel with a subsequent approximation by a polynomial of degree 3. As with the other worksheets, the table of values is at the bottom of the page. To directly compare the lift distributions on the main page, the scale between the distribution according to Diederich and the distribution according to Mason had to be defined. According to Mason, the optimally elliptical distribution in the diagram, as seen in Figure 4.7, serves as a reference point for this, which can be equated to the sweep angle correction function, $f(\eta)$ for an effective sweep angle, $\varphi_\beta = 0$. The quotient of the intercepts of both elliptical distributions defines the scale factor. This is shown in the green colored output field. In addition to the scale factor, the Mason polynomial is multiplied by the lift coefficient, C_L of the airfoil since a normalization to $C_L = 1$ is present, as with the other distribution functions.

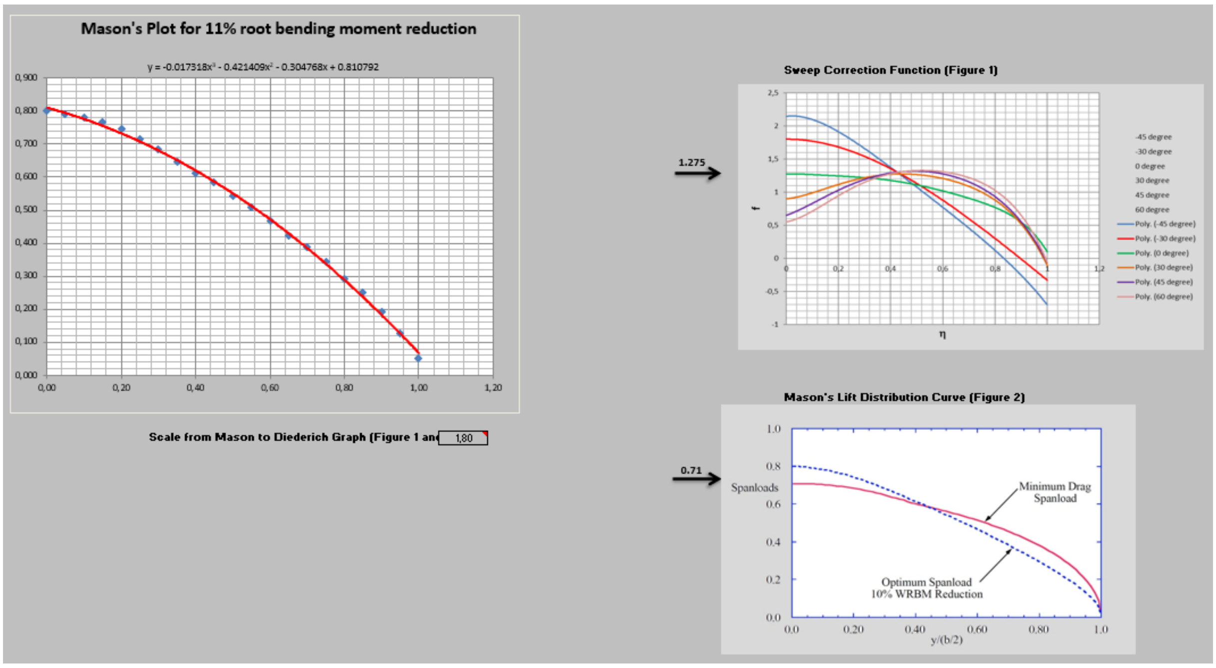


Figure 4.8 Worksheet „Mason's Curve " with diagram from Mason (2001)

4.3.3 Sweep Correction Function

The worksheet "Sweep Correction Function" is a reconstruction of the sweep angle correction function, $f(\eta)$ according to Diederich (1952), which is already shown in Section 3.2.1 in Figure 3.3. This was approximated using 6th-degree polynomials for accuracy. Here, depending on the input value of the effective sweep angle, φ_β according to (3.22) the function, $f(\eta)$ is passed on to the value table of the main page. The function created in Visual Basic interpolates the needed output values for effective sweep angles between the given functions. The source code for the function can be found in Appendix B. The function for the sweep angle, $\varphi_\beta = 0$ also provides the basis for the optimally elliptical distribution in the lift distribution diagram of the main page and, like the Mason curve, is multiplied by the lift coefficient, C_L of the airfoil for fitting. The surface of the worksheet is shown in Figure 4.8.

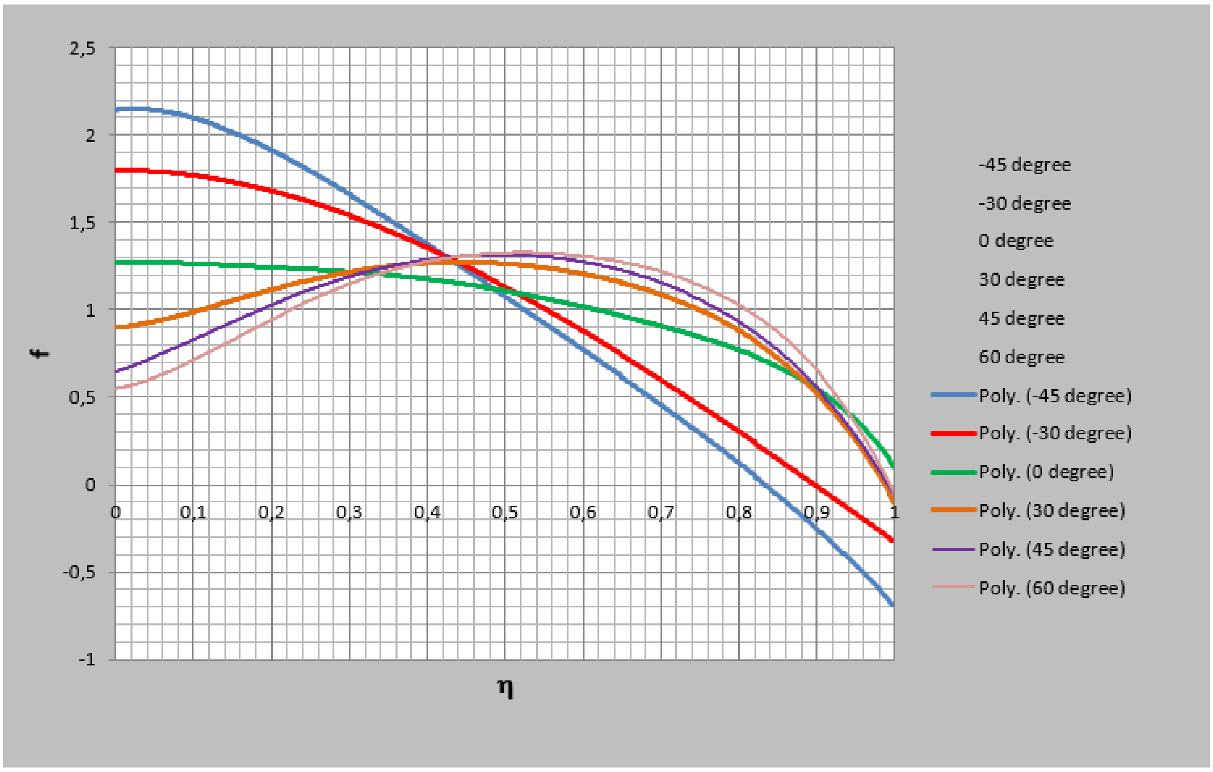


Figure 4.9 Worksheet „Sweep Correction Function “

4.3.4 Factors of Diederich's Method

The "Factors of Diederich's Method" worksheet, shown in Figure 4.9, is used to output the Diederich factors, C_1 to C_4, which were covered in Section 3.2. Using 3rd-degree polynomials, the curves from Figure 3.1 were approximated and transferred to Visual Basic. They output the needed factors depending on the plan view parameter F. The results for the selected input parameters are listed here but can also be found in the Calculated Values Output on the main page. The source code of the function can be found in Appendix B.

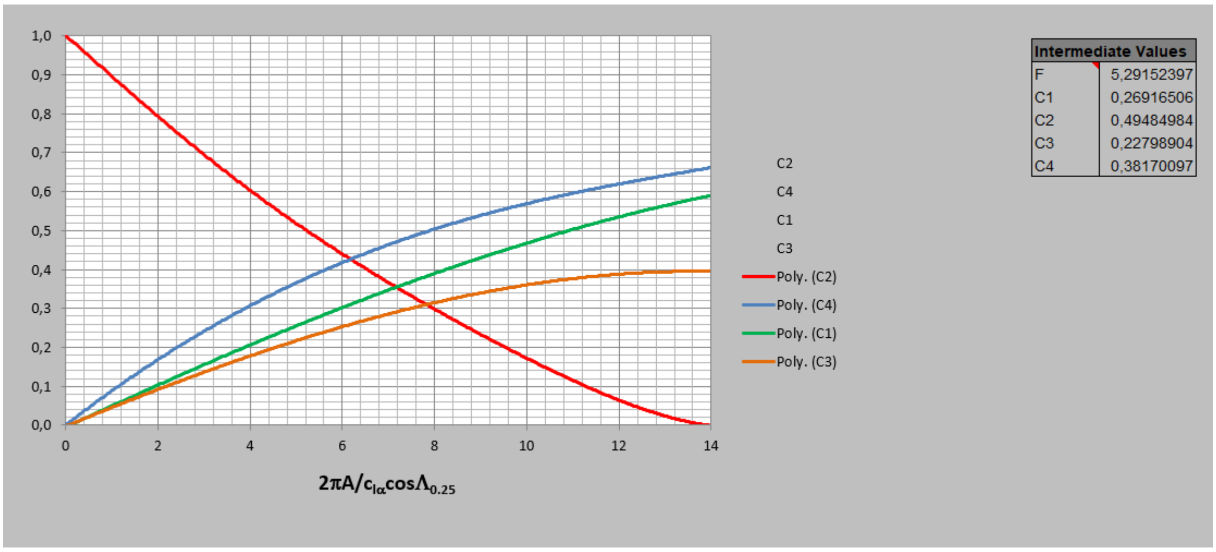


Figure 4.10 Worksheet „ Factors of Diederich's Method "

4.3.5 Lift Curve Slope

In this worksheet, depending on the airfoil trailing edge angle, φ'_{TE} selected on the main page, an associated ratio of experimental lift curve slope, $c_{l\alpha i}$ to theoretical lift curve slope, $(c_{l\alpha})_{\text{theory}}$ is issued. This ratio is needed to calculate the lift curve slope, $c_{l\alpha}$ of the airfoil, whose calculation has already been shown in Section 3.2.1 by (3.19) and (3.20). The output occurs as a function of the Reynolds number, as shown in Figure 3.2. The approximation polynomials of the three curves for Reynolds numbers 10^6, 10^7 and 10^8 were linked in Visual Basic with an interpolation function to provide meaningful values even for Reynolds numbers between the given curves. The calculation is done here for the lift curve slope of the wing tip, $c_{l\alpha,t}$ and the lift curve slope of the wing root, $c_{l\alpha,r}$. For further calculation within the Diederich method, the mean value of the airfoil's lift curve slopes is calculated and given in the Additional Output. The user interface of the worksheet is shown in Figure 4.10. The source code for this feature can be found in Appendix B.

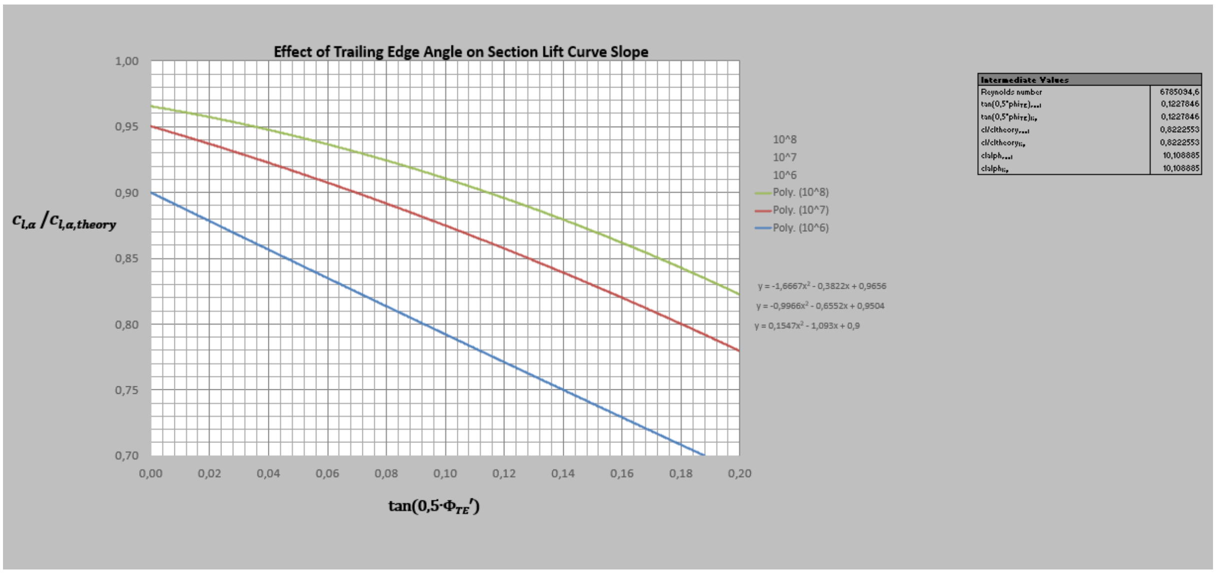


Figure 4.11 Worksheet "Lift Curve Slope"

4.3.6 Atmosphere Parameters

The "Atmosphere Parameters" worksheet provides the basis for calculating the Reynolds number as a function of the flight altitude. For flight altitudes, *h* between 0 km and 20 km, the required data are issued and visualized by diagrams. The calculation and the generation of the diagrams, seen in Figure 4.11, were performed using the equations in Appendix A. The intermediate results for the altitude selected on the main page are shown in the green output box on the right, but they are also available in the Calculated Values Output on the main page.

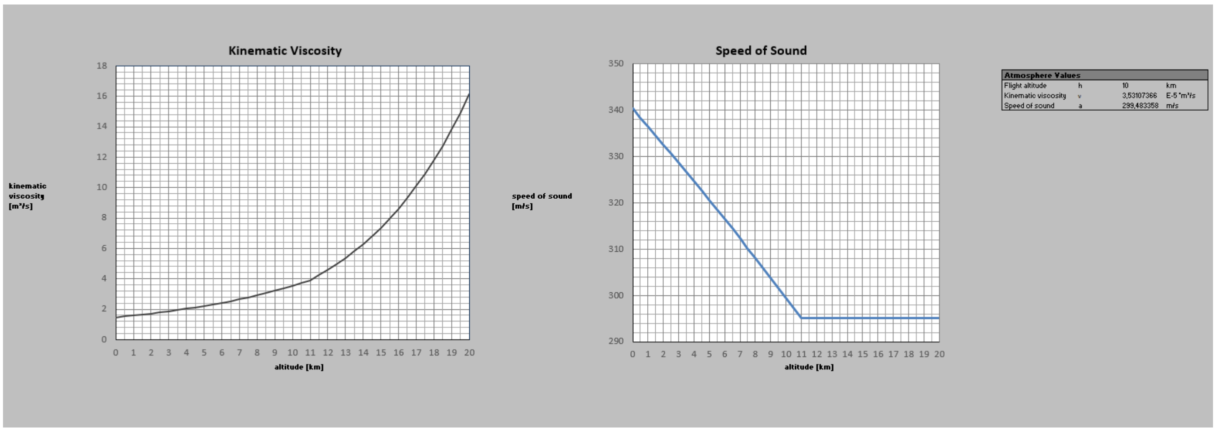


Figure 4.12 Worksheet "Atmosphere Parameters"

5 Webpage Creation

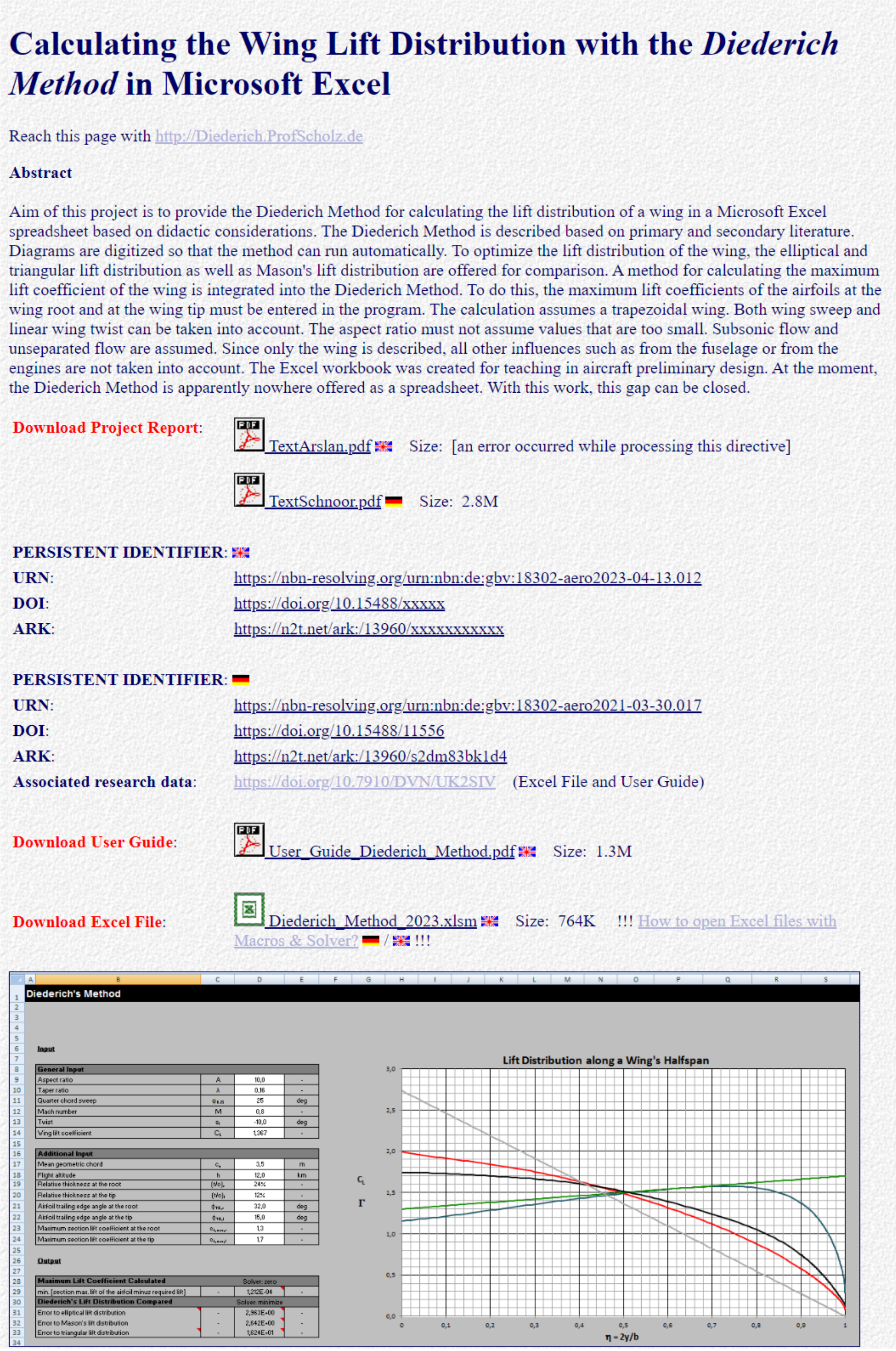
Creating a web page helps make the Excel workbook available to interested users. With a mouse click, it allows immediate access to the file and the user's guide. At the same time, it provides a place that is always accessible to reach the desired file. Also, in case of data loss or incorrect editing of the Excel workbook, a new copy can be loaded onto the computer.

5.1 Main Idea of the Design

The design of the website should follow a simple structure. The Excel workbook and the user guide are offered separately as downloads. The most important specifications and instructions are integrated into the Excel workbook on the user interface. The interested user is allowed to download the user guide to understand every aspect of the parameters. The user manual can be downloaded by the link in the next section.

5.2 Setting Up the Website

The website was programmed by Professor Scholz. The interface of the website is shown in Figure 5.1.



Calculating the Wing Lift Distribution with the *Diederich Method* in Microsoft Excel

Reach this page with http://Diederich.ProfScholz.de

Abstract

Aim of this project is to provide the Diederich Method for calculating the lift distribution of a wing in a Microsoft Excel spreadsheet based on didactic considerations. The Diederich Method is described based on primary and secondary literature. Diagrams are digitized so that the method can run automatically. To optimize the lift distribution of the wing, the elliptical and triangular lift distribution as well as Mason's lift distribution are offered for comparison. A method for calculating the maximum lift coefficient of the wing is integrated into the Diederich Method. To do this, the maximum lift coefficients of the airfoils at the wing root and at the wing tip must be entered in the program. The calculation assumes a trapezoidal wing. Both wing sweep and linear wing twist can be taken into account. The aspect ratio must not assume values that are too small. Subsonic flow and unseparated flow are assumed. Since only the wing is described, all other influences such as from the fuselage or from the engines are not taken into account. The Excel workbook was created for teaching in aircraft preliminary design. At the moment, the Diederich Method is apparently nowhere offered as a spreadsheet. With this work, this gap can be closed.

Download Project Report: TextArslan.pdf Size: [an error occurred while processing this directive]

TextSchnoor.pdf Size: 2.8M

PERSISTENT IDENTIFIER:
URN: https://nbn-resolving.org/urn:nbn:de:gbv:18302-aero2023-04-13.012
DOI: https://doi.org/10.15488/xxxxx
ARK: https://n2t.net/ark:/13960/xxxxxxxxxxx

PERSISTENT IDENTIFIER:
URN: https://nbn-resolving.org/urn:nbn:de:gbv:18302-aero2021-03-30.017
DOI: https://doi.org/10.15488/11556
ARK: https://n2t.net/ark:/13960/s2dm83bk1d4
Associated research data: https://doi.org/10.7910/DVN/UK2SIV (Excel File and User Guide)

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Figure 5.1 Interface of the created website

The website can be accessed as a subdomain of Professor Scholz using the following link:

http://Diederich.ProfScholz.de

6 Summary

An Excel spreadsheet has been created, which offers, especially for students, an easy introduction to the theme of wing aerodynamics. Besides the essential contents of the Diederich method, further interesting calculations have been included in this workbook. Atmosphere or wing geometry results can be read, which can also be read off, which can be of interest. The compact but well-considered number of hints of the user guide will quickly lead any interested person to the desired success. Providing the results in English also enables foreign interested parties to use the Diederich method offered with the Excel spreadsheet.

7 Closing Remarks

This project's scope was to translate the project "Die Diederich-Methode zur Berechnung der Auftriebsverteilung am Tragflügel in Microsoft Excel" by Schnoor (2021) from German into English. In addition, the Graphical User Interface (GUI) of the Excel workbook was adapted to the standard layout definition by Wolf (2009) and Montarnal (2015) to ensure the same layout as the other tables in the digital library of Professor Dieter Scholz. Since the original project used a different layout definition, the content of Chapter 4.2 had to be changed in this project.

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Appendix A

Equations for the ISA

Equations for the troposphere

Troposphere from 0 m = 0 ft **to** 11000 m = 36089 ft (geopotential height)

$T = T_0 - L \cdot H$

H		geopotential height
T_0	=	288.15 K
L	=	0.0065 K/m = 6.5 K/km = 1.9812 · 10^{-3} K/ft

$$\delta = \frac{p}{p_0} = \left(1 - k_a \cdot H\right)^{5.25588}$$

$$\sigma = \frac{\rho}{\rho_0} = \left(1 - k_a \cdot H\right)^{4.25588}$$

p_0	=	101325 Pa = 1013.25hPa = 1.01325 bar
ρ_0	=	1.225 kg/m³
k_a	=	2.2558 · 10^{-5} 1/m = 0.022558 1/km = 6.8756 · 10^{-6} 1/ft

Equations for the Stratosphere

Stratosphere from 11000 m = 36089 ft **to** 20000 m = 65617 ft (geopotential height)

$T = T_S$ = 216.65 K = -56.5 °C = const

$$\frac{\sigma}{\sigma_T} = \frac{\rho}{\rho_T} = \frac{\delta}{\delta_T} = \frac{p}{p_T} = e^{-k_b\left(H - H_T\right)}$$

H_T	=	11000 m = 11 km = 36089 ft
k_b	=	1.57688 · 10^{-4} 1/m = 0.157688 1/km = 4.80634 · 10^{-5} 1/ft
σ_T	=	0.297070
ρ_T	=	0.3639 kg/m³
δ_T	=	0.223356
p_T	=	22632 Pa = 226.32 hPa = 0.22632 bar

Figure A.1 Troposphere and stratosphere Equations according to the ISA (Scholz 2012) speed of sound $a = \sqrt{\gamma \cdot R} \cdot \sqrt{T}$ $\sqrt{\gamma \cdot R} = 20.0468\ 1/\sqrt{\mathrm{K}} \cdot \frac{\mathrm{m}}{\mathrm{s}}$

$R = 287.053\ \frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}}$ $\gamma = 1.4$

$a_0 = 340.294\ \mathrm{m/s} = 1225.06\ \mathrm{km/h} = 661.48\ \mathrm{kt}$

dynamic viscosity $\mu = \frac{\beta_S \cdot T^{3/2}}{T + S}$ $\beta_S = 1.458 \cdot 10^{-6}\ \frac{\mathrm{kg}}{\mathrm{m} \cdot \mathrm{s} \cdot \sqrt{\mathrm{K}}}$

$\mu_0 = 1.7894 \cdot 10^{-5}\ \frac{\mathrm{kg}}{\mathrm{m} \cdot \mathrm{s}}$ $S = 110.4\ \mathrm{K}$

kinematic viscosity $\nu = \frac{\mu}{\rho}$ $\nu_0 = 1.4607 \cdot 10^{-5}\ \frac{\mathrm{m}^2}{\mathrm{s}}$

relative density $\sigma = \frac{\rho}{\rho_0}$

relative pressure $\delta = \frac{p}{p_0}$

relative temperature $\theta = \frac{T}{T_0}$

equation of state for a perfect gas $\frac{p}{\rho} = R \cdot T$ $R = 287.053\ \frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}}$

$$\frac{\delta}{\sigma} = \theta$$

Figure A.2 General Equations for the atmosphere data according to the ISA (Scholz 2012)

Appendix B

Program Code of the Functions Created in Visual Basic

```
(Allgemein)                                             C_two

Function C_two(x As Double) As Double

C_two = 0.000003 * (x ^ 5) - 0.000094 * (x ^ 4) + 0.0010118 * (x ^ 3) - 0.0015527 * (x ^ 2) - 0.1042267 * x + 1.001183

End Function

Function C_one(x As Double) As Double

C_one = -0.0000264 * (x ^ 3) - 0.000564 * (x ^ 2) + 0.0556219 * x - 0.0054559

End Function

Function C_three(x As Double) As Double

C_three = -0.0000506 * (x ^ 3) - 0.0007669 * (x ^ 2) + 0.0490423 * x - 0.002549

End Function

Function C_four(x As Double) As Double

C_four = 0.0000034 * (x ^ 4) - 0.0000064 * (x ^ 3) - 0.0037242 * (x ^ 2) + 0.0913507 * x + 0.0008776

End Function


Function F(Factor_5 As Double, x As Double) As Double

Dim y1, y2, y3, y4, y5, y6 As Double
Dim dy1, dy2, dy3, dy4, dy5, dy6 As Double
Dim slope1, slope2, slope3, slope4, slope5 As Double
Dim interp1, interp2, interp3, interp4, interp5 As Double

y1 = -45
y2 = -30
y3 = 0
y4 = 30
y5 = 45
y6 = 60

dy1 = -3.553922 * (x ^ 6) + 12.825226 * (x ^ 5) - 22.261029 * (x ^ 4) + 21.497862 * (x ^ 3) - 11.867439 * (x ^ 2) + 0.515369 * x + 2.143885
dy2 = -0.131119 * (x ^ 4) + 1.329643 * (x ^ 3) - 3.354167 * (x ^ 2) + 0.027496 * x + 1.798392
dy3 = -17.156863 * (x ^ 6) + 41.534691 * (x ^ 5) - 37.075792 * (x ^ 4) + 14.199146 * (x ^ 3) - 2.736741 * (x ^ 2) + 0.062062 * x + 1.274186
dy4 = -11.538462 * (x ^ 5) + 25.670163 * (x ^ 4) - 22.38345 * (x ^ 3) + 6.815559 * (x ^ 2) + 0.433846 * x + 0.897133
dy5 = 2.124183 * (x ^ 6) - 15.667421 * (x ^ 5) + 28.172448 * (x ^ 4) - 22.782548 * (x ^ 3) + 6.031885 * (x ^ 2) + 1.406838 * x + 0.64797
dy6 = 2.369281 * (x ^ 6) - 25.53733 * (x ^ 5) + 50.684703 * (x ^ 4) - 41.851819 * (x ^ 3) + 13.047359 * (x ^ 2) + 0.677491 * x + 0.549916

slope1 = Factor_5 - y1
slope2 = Factor_5 - y2
slope3 = Factor_5 - y3
slope4 = Factor_5 - y4
slope5 = Factor_5 - y5

interp1 = (dy2 - dy1) * (slope1 / 15) + dy1
interp2 = (dy3 - dy2) * (slope2 / 30) + dy2
interp3 = (dy4 - dy3) * (slope3 / 30) + dy3
interp4 = (dy5 - dy4) * (slope4 / 15) + dy4
interp5 = (dy6 - dy5) * (slope5 / 15) + dy5

If Factor_5 < y1 Then
     F = dy1
         ElseIf Factor_5 < y2 Then
         F = interp1
             ElseIf Factor_5 < y3 Then
             F = interp2
                 ElseIf Factor_5 < y4 Then
                 F = interp3
                     ElseIf Factor_5 < y5 Then
                     F = interp4
                          ElseIf Factor_5 < y6 Then
                          F = interp5
                            Else
                            F = dy6


End If

End Function
```

Figure B.1 Output functions for the factors C_1 - C_4 and the sweep angle correction function $f(\eta)$

(Allgemein) | CLA

```
Function CLA(Factor_3 As Double, x As Double) As Double

Dim y1, y2, y3 As Double
Dim dy1, dy2, dy3 As Double
Dim slope1, slope2 As Double
Dim interp1, interp2 As Double

y1 = 1000000
y2 = 10000000
y3 = 100000000

dy1 = -0.1547 * (x ^ 2) - 1.093 * x + 0.9
dy2 = -0.9966 * (x ^ 2) - 0.6552 * x + 0.9504
dy3 = -1.6667 * (x ^ 2) - 0.3822 * x + 0.9656

slope1 = Factor_3 - y1
slope2 = Factor_3 - y2

interp1 = (dy2 - dy1) * (slope1 / 9000000) + dy1
interp2 = (dy3 - dy2) * (slope2 / 90000000) + dy2

If Factor_3 < y1 Then
     CLA = dy1
         ElseIf Factor_3 < y2 Then
         CLA = interp1
             ElseIf Factor_3 < y3 Then
             CLA = interp2
                    Else
                    CLA = dy6


End If

End Function
```

Figure B.2 Output function for the lift curve slope ratio