10 Mass and Center of Gravity

Minimizing the operating empty mass is **a top priority in aircraft construction**. The operating empty mass accounts for approximately 45% to 65% of the maximum take-off mass (compare with Fig. 5.15). To this must be added the considerable mass fraction for fuel, especially in the case of long-haul aircraft. Only the (small) remainder is usable mass. If the operating empty mass increases in relation to the take-off mass by a certain percentage, the usable mass decreases by a considerably higher percentage. On this basis, it becomes clear that the viability of an **aircraft project** as a whole can be seriously **jeopardized if** it turns out that the **mass specifications cannot be met**.

In the case of large aircraft with **JAR-25 certification**, the design objective is to achieve the required **flight performance with low mass**. Smaller aircraft can be certified more easily according to other certification regulations. Aircraft with an MTOW ≤ 5700 kg can be certified according to **JAR-23** or FAR Part 23 as a "normal, utility or aerobatic aeroplane". Aircraft with an MTOW ≤ 8600 kg can be certified according to JAR-23 or FAR Part 23 as "commuter aeroplanes" (compare Section 3). The aircraft manufacturer often tries to fully utilize the maximum permitted MTOW in these cases. Therefore, the **flight performance** must then be **optimized for the given MTOW**.

As the mass increases, so does the required lift and therefore the drag, required thrust, fuel consumption and, finally, the aircraft's fuel and **operating costs**. For this reason, aircraft optimization was for a long time virtually synonymous with minimizing operating empty mass and fuel mass. The current operating situation for aircraft is characterized by low fuel prices, rising personnel and capital costs as well as a growing demand by passengers for a punctual service (departure reliability). In this context it is important to keep a firm eye on the price of the aircraft and the maintenance costs in addition to the mass (see also Section 14). Therefore, optimization of the aircraft with the aim of achieving low operating empty mass and low fuel mass should not be conducted at the expense of the price of the aircraft and the maintenance costs.

It is important to bear in mind that a *local* increase in mass causes a *global* increase in mass. Therefore, if some aircraft components are substantially heavier than planned, this requires an increase in the size of the wing and the engines. The total influence of the increase in mass is therefore greater than the increase that was originally established on the detailed design. This phenomenon is called the **snowball effect**.

The further one is into the design phase, the more expensive it will be if a **mass reduction program** becomes necessary. Despite the additional development costs, mass reduction programs can prove to be very advantageous overall during development.

In addition to “**mass**”, the term “**weight**” is often used. Of course, this is virtually the same thing via the interrelationship $F = m \cdot g$. In German, the equivalent term for “weight” was used in the past, but “mass” is now the correct expression according to Germany's “Law on Units in Metrology”. Nevertheless, terms such as “weights department” have stayed. In English too, the term “weight” is generally used more often than “mass”.

The **certification regulations** JAR-25 and FAR Part 25 do not directly restrict mass and center of gravity, but rather require that mass and center of gravity must remain within the limits of what is permitted taking into account stability and safe flight operations.

JAR 25.25 **Weight Limits**
(a) **Maximum weights** ... must be established so that they are not more than -
(2) The highest weight at which compliance with each applicable **structural loading** and **flight requirement** is shown.
(b) **Minimum weight** ... must be established so that it is not less than -
(2) ... the lowest weight at which compliance with each **structural loading** condition of this JAR-25 is shown; or
(3) The lowest weight at which compliance with each applicable **flight requirement** is shown.

JAR 25.27 **Centre of gravity limits**
The **extreme forward** and the **extreme aft centre of gravity** limitations must be established for each practicably separable operating condition. No such limit may lie beyond -
(b) The extremes within which the **structure** is proven; or
(c) The extremes within which compliance with each applicable **flight requirement** is shown.

To ensure that these limits can be observed during flight operations, the flight manual must contain the data in a form that is easy to use.

JAR 25.1583 **Operating limitations**
(c) ... The weight and centre of gravity limitations established under JAR 25.1519 must be furnished in the aeroplane **Flight Manual** ... or in a separate weight and balance control and loading document which is incorporated by reference in the aeroplane Flight Manual

ACJ 25.1583(c) **Centre-of-Gravity Limitations**

Indication should be given in tabular or graphic form of the **c.g. limits** for take-off and landing and **for any** other **practicably separable flight condition**, as appropriate for the range of weights **between the maximum take-off weight and the minimum landing weight** presented in accordance with JAR 25.1583(c). The landing gear position appropriate to each condition should be shown, or, alternatively, data should be presented for landing-gear-extended position only and should include the moment change due to gear retraction. **C.g. limits should be presented in terms** of both distance-from-datum and percentage **of the mean aerodynamic chord (MAC)**. The datum for the former should be defined and the length and location of the MAC should be stated.

ACJ 25.1519 **Weight, Centre of Gravity and Weight Distribution**

A statement of the **maximum** certificated **take-off** and **landing weights**, and the **minimum** certificated **take-off** and **landing weights**, should be established, together with the maximum ramp or **taxying weight**, the maximum **zero-fuel weight** and any other fixed limit on weight, including weight limitations resulting from such factors as **brake energy limits**, **tyre limits**, **etc.**, established in accordance with the airworthiness standards of JAR-25...

10.1 Mass forecasts

The **aircraft’s mass is required** to **calculate flight performance** and to **assess the design.** Individual masses are required to determine the center of gravity and the **positioning** of the **landing gear** and the **wings**. In the case of design from statistics, estimated individual masses serve as a **target** for components that have to be constructed in detail by specialist departments. Masses are also often the starting point for a **cost estimate**.

Effective work with masses starts with a clear **mass breakdown**. **In the *operation* of civil aircraft mass breakdown and mass designations according to [ATA 100] have established themselves** (see Appendix A). However, the manufacturer's empty mass breakdown according to [ATA 100] is not detailed enough to forecast and record masses. Although there are various standards and regulations for the detailed breakdown of masses, firms often use their own mass breakdowns in practice. Standardized mass breakdowns are **[DIN 9020**] (Germany) (**Fig. 10.1**), and for military aircraft **MIL-STD-1374** (USA). **Fig. 10.2** shows a comparison of the mass breakdowns according to ATA, DIN and MIL. In the case of Airbus, masses are broken down according to so-called "*Weight Chapters*" (**Fig. 10.3**).

In order to carry out elementary center-of-gravity calculations and calculations for the positioning of landing gear and wings, the mass breakdown should at least have the level of detail described by the simple mass breakdown according to **Table 10.1,** based on [DIN 9020] and [ATA 100].

In addition to the mass designations according to [ATA 100], the **gross weight** and the **design gross weight** are often used in aircraft design. The ***gross weight*** is defined as "*The total airplane weight at any moment during the flight*" [TORENBEEK 88]. The ***design gross weight*** is defined as "*The aircraft weight at which the structure will withstand the design load factor*" [RAYMER 89]. In fighter aircraft the MTOW can be higher than the *design gross weight* during flight. In this case, fuel must be consumed after take off until the MTOW decreases down to the *design gross weight*, or below, before the aircraft allowed to fly with maximum load factor.

<table>
<tr><th>Nr</th><th colspan="2">Massehauptgruppen</th><th colspan="8">Massebegriffe – Kurzzeichen</th></tr>
<tr><td>1</td><td colspan="2">Struktur</td><td rowspan="4">Standard-Leermasse m_{LST} (Hersteller-Leermasse)</td><td rowspan="5">Leermasse m_L</td><td rowspan="6">Grundmasse m_G</td><td rowspan="7">Betriebsleermasse m_{BL}</td><td rowspan="8">Leertankmasse m_{Lt}</td><td rowspan="10">Flügel-Leertankmasse m_{FLt}</td><td rowspan="11">Startmasse m_{ST}</td><td rowspan="12">Rollmasse m_{Roll}</td></tr>
<tr><td>2</td><td colspan="2">Antriebsanlage</td></tr>
<tr><td>3</td><td colspan="2">Standardausrüstung</td></tr>
<tr><td>4</td><td colspan="2">Masseabweichungen</td></tr>
<tr><td>5</td><td colspan="2">Sonderausrüstung
(feste Einsatzausrüstung)</td><td></td></tr>
<tr><td>6</td><td colspan="2">Bewegliche Einsatzausrüstung</td><td colspan="2" rowspan="6">Zuladung L_Z</td></tr>
<tr><td>7</td><td colspan="2">Besatzung und Dienstlast</td><td rowspan="5">Betriebsladung L_B</td></tr>
<tr><td>8</td><td colspan="2">Nutzlast</td><td rowspan="4">Freiladung L_F
(verfügbare Ladung L_v)</td></tr>
<tr><td>9</td><td rowspan="3">Betriebsstoffe</td><td>Einspritzflüssigkeit</td><td rowspan="3"></td></tr>
<tr><td>10</td><td>Kraftstoff in Innenbehältern und in Außenbehältern am Rumpf</td></tr>
<tr><td>11</td><td>Kraftstoff in Flügelbehältern und in Außenbehältern am Flügel</td><td></td></tr>
<tr><td>12</td><td colspan="2">Rollkraftstoff</td><td colspan="7"></td></tr>
<tr><td colspan="11">Betriebsleermasse + Betriebsstoffe (ausschließlich Rollkraftstoff) = Betriebsmasse
Der Gesamtschmierstoff, sowie die Kraftstoff- und Einspritzflüssigkeitsreste sind in der Massehauptgruppe Antriebsanlage enthalten.</td></tr>
</table>

Fig. 10.1: Main mass groups and mass designations according to [DIN9020] Part 1

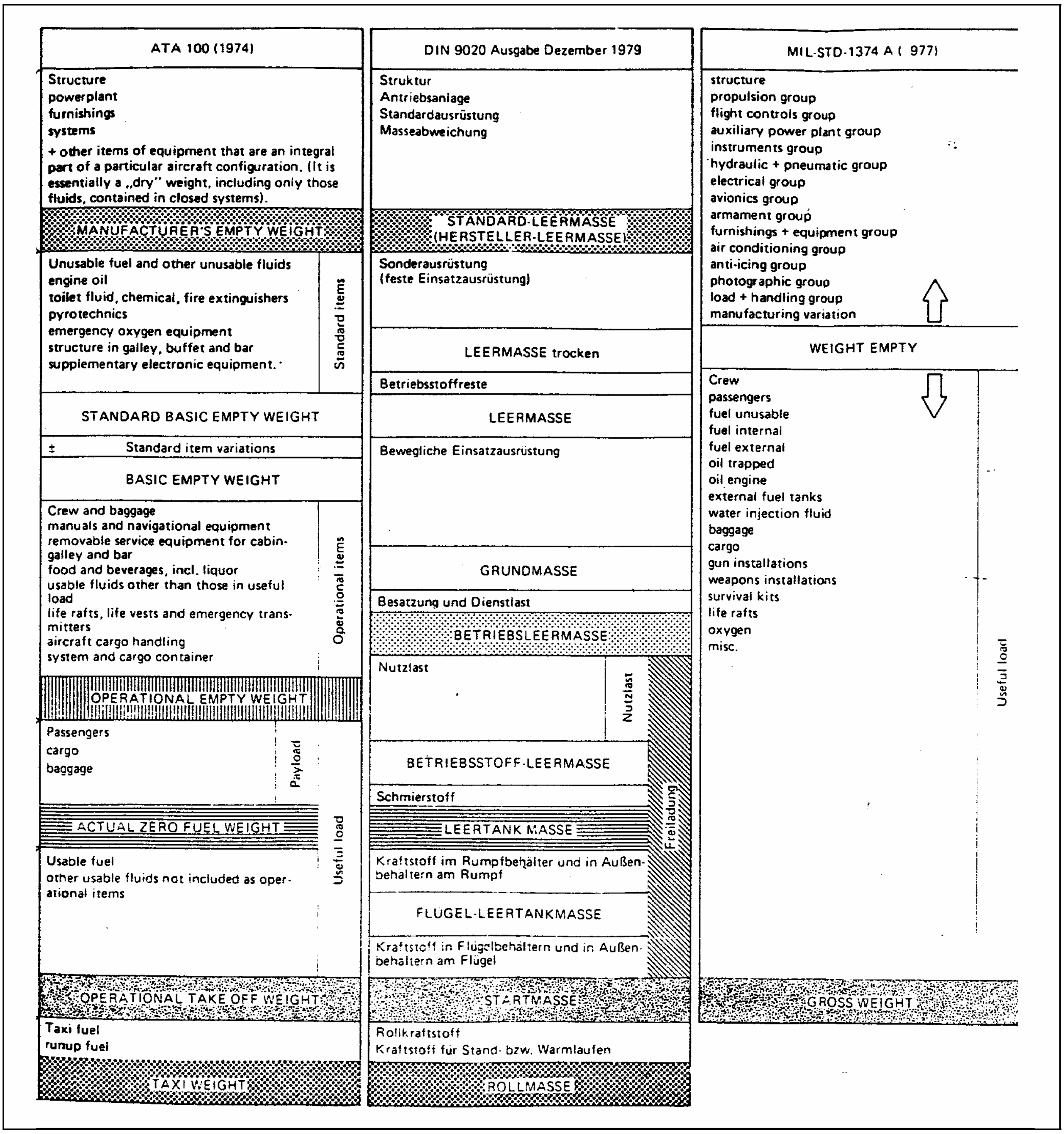


Fig. 10.2: Comparison of mass breakdowns according to [ATA 100], [DIN 9020] and MIL-STD-1374. Source: [DIN 9020]

```
-----------------------------------
10  WING STRUCTURE
11  FUSELAGE
13  HORIZONTAL TAIL PLANE
14  VERTICAL TAIL
15  LANDING GEAR
16  PYLONS

STRUCTURE                 -----------------

20  EQUIPPED ENGINES
21  BLEED AIR SYSTEM
22  ENGINES CONTROL
25  FUEL SYSTEM

POWER UNITS               -----------------

30  AUXILIARY POWER UNIT
31  HYDRAULIC GENERATION
32  HYDRAULIC DISTRIBUTION
33  AIR CONDITIONING
34  DE ICING
35  FIRE PROTECTION
36  FLIGHT CONTROLS
37  INSTRUMENTS PANELS
38  AUTOMATIC FLIGHT SYSTEM
39  NAVIGATION
40  COMMUNICATION
41  ELECTRICAL GENERATION
42  ELECTRICAL DISTRIBUTION

SYSTEMS                   -----------------

50  FURNISHINGS
51  FIXED EMERGENCY OXYGEN
52  LIGHTING
53  WATER SYSTEM

FURNISHINGS               -----------------

   ----MANUFACTURER WEIGHT EMPTY----


60  OPERATOR EQUIPMENT
61  OPERATIONAL EQUIPMENT

OPERATORS ITEMS           -----------------

  ----OPERATING WEIGHT EMPTY----
```

Fig. 10.3:
In-house main mass groups (*Weight Chapters*) at Airbus

Table 10.1: Mass groups of a very simple mass breakdown for the design based on the mass breakdowns according to [DIN 9020] and [ATA 100]

			Flügel (wing) m_W ,
		+	Rumpf (fuselage) m_F ,
		+	Höhenleitwerk (horizontal tail) m_H ,
		+	Seitenleitwerk (vertical tail) m_V ,
		+	Bugfahrwerk (nose landing gear) $m_{LG,N}$,
		+	Hauptfahrwerk (main landing gear) $m_{LG,M}$,
		+	Triebwerksgondel (nacelle) m_N
	=	Struktur (structure)	
	+	Triebwerk, installiert (power plant, installed) $m_{E,inst}$	
	+	Flugzeugsysteme (aircraft systems) m_{SYS}	
=	Hersteller-Leermasse (⇒ manufacturer's empty weight, MEW) m_{ME}		
	+	Ausrüstung und Besatzung (⇒ standard and operational items)	
=	Betriebsleermasse (⇒ operational empty weight, OEW) m_{OE}		

Mass can be **forecasted to various degrees of accuracy**. A distinction is made between *Class I* methods and *Class II* methods. *Class II* methods are more precise than *Class I* methods. However, an exact definition of what Class I or Class II is depends on the view of the person involved. The *Class I method* [BOEING 68] would definitely be categorized as a *Class II* method in a university setting. Nowadays, forecasts of mass in the industry are carried out with complex computer programs (**Fig. 10.4**).

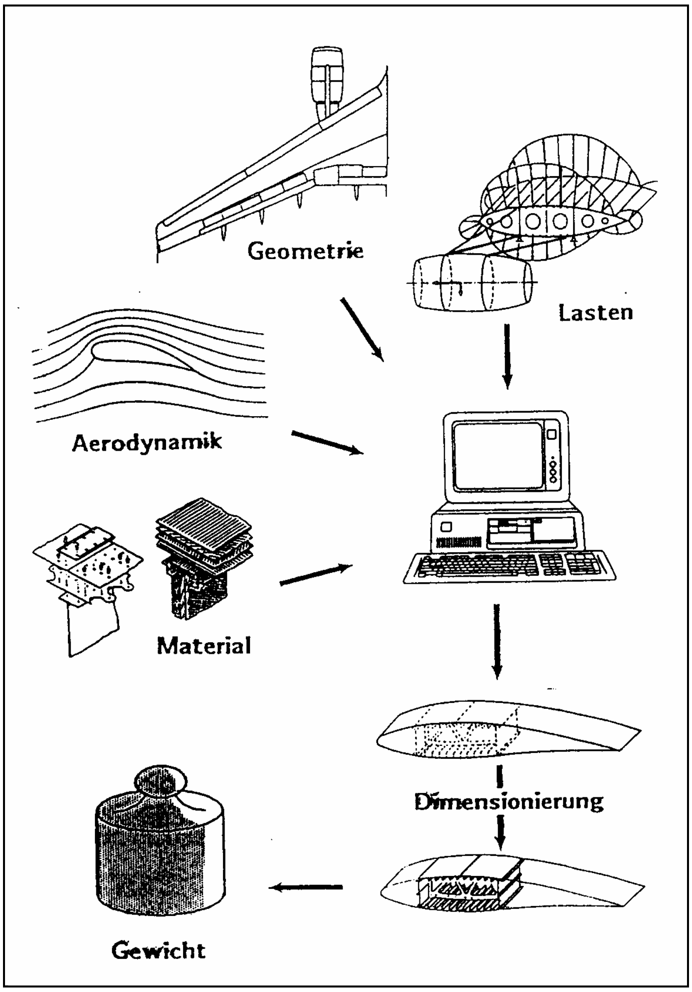


Fig. 10.4:
Example of the approach adopted by an in-house computer program to forecast masses (Airbus)

Two simple *Class I* methods for forecasting mass and mass breakdown are to be presented here.

Class I method for forecasting mass breakdown according to [ROSKAM V]

Due to the simple procedure used, this method does not provide a forecast of mass, but rather a **forecast of the mass breakdown**. The following are required as **input values**:

1. Forecast of the **operating empty mass** m_{OE} **according to Section 5**,
2. A **mass breakdown of a constructed aircraft** that is similar to the aircraft being designed in terms of design and the certification basis.

Step 1: Look for a **mass breakdown** of an aircraft that is as similar as possible to the aircraft being designed in terms of design and the certification basis. A good source for such mass breakdowns is [ROSKAM V] Appendix A.

Step 2: From the mass breakdown (Step 1), calculate the **relative mass breakdown** in relation to the operating empty mass m_{OE}.

Step 3: Calculate the required **masses of the mass groups** according to Table 10.1 by multiplying the relative mass breakdown (Step 2) by the operating empty mass m_{OE} estimated in Section 5

Class I Method for forecasting the mass and the mass breakdown according to data from [RAYMER 89]

Table 10.2 contains the calculation data for the *Class I* method according to [RAYMER 89] converted to SI units and adapted to the mass groups according to Table 10.1. In each mass group (wing, fuselage, ...) the valid factor (transport aircraft, general aviation aircraft) must be multiplied by the value of the reference parameter. The mass of the nacelles is already included in the other mass groups. The masses of the nose landing gear and main landing gear act like 15:85. The sum of the masses, formed from the structure groups, the powerplant group, the system group, the crew and the items, provides the operating empty mass m_{OE}.

The *Class 1* mass forecast according to [RAYMER 89] could also be used to calculate a mass breakdown from the operating empty mass m_{OE} according to Section 5. In this case, the mass breakdown determined here according to [RAYMER 89] would have to be used as the input parameter for Step 1 of the mass breakdown according to [ROSKAM V] (see above).

Table 10.2: Calculation system for a Class I mass forecast based on data from [RAYMER 89]

	factor		reference parameter		mass [kg]
	transports	gen. aviation	name	value	transports or gen. aviation
wing	49	12,2	$S_{exposed}$ [m²]		"factor" • "value"
fuselage	24	6,8	S_{wetted} [m²]		"factor" • "value"
horizontal tail	27	9,8	$S_{exposed}$ [m²]		"factor" • "value"
vertical tail	27	9,8	$S_{exposed}$ [m²]		"factor" • "value"
nose gear	0,006	0,009	m_{MTO} [kg]		"factor" • "value"
main gear	0,037	0,048	m_{MTO} [kg]		"factor" • "value"
nacelle	-	-	-	-	0
structure	-	-	-	-	sum
power plant	1,3	1,4	m_E [kg]		"factor" • "value"
systems & items	0,17	0,10	m_{MTO} [kg]		"factor" • "value"
m_{OE}	-	-	-	-	sum

Class II method for forecasting mass and mass breakdown with equations from [TORENBEEK 88]

TORENBEEK obtains his data from a large number of different publications. In particular, publications from the International Society of Allied Weight Engineers, SAWE (http://gumbus.jsc.nasa.gov/sawe/default.html) have been used.

INTERNATIONAL SOCIETY OF ALLIED WEIGHT ENGINEERS, INC.

Only the key equations from [TORENBEEK 88], focusing on transport aircraft, are reproduced in this script.

Wing mass m_W

The equations for calculating wing mass require several parameters, which are defined here. The reference value is:

$$b_{ref} = 1.905\ \mathrm{m} \quad . \qquad (10.1)$$

The structural span:

$$b_s = \frac{b}{\cos\varphi_{50}} \quad \text{possibly with } \varphi_{50} \text{ according to equation (7.12) from } \varphi_{25} . \qquad (10.2)$$

The ultimate load factor:

$$n_{ult} \approx 1.5 \cdot n_{lim} \quad . \qquad (10.3)$$

The limit load factor n_{lim} is taken from in JAR-23 and JAR-25:

JAR **23**.337	**Limit manoeuvring load factors**
(a)	The positive limit manoeuvring load factor n may not be less than -
(1)	$2.1+\left(\frac{24000}{W+10000}\right)$ for **normal** and **commuter** category aeroplanes (where W = design maximum take-off weight lb), except that n need not be more than 3·8;
(2)	**4·4** for **utility** category aeroplanes; or
(3)	**6·0** for **aerobatic** category aeroplanes.

JAR **25**.337	**Limit manoeuvring load factors**
(b)	The positive limit manoeuvring load factor 'n' for any speed up to VD may not be less than $2.1+\left(\frac{24000}{W+10000}\right)$ except that 'n' may not be less than 2·5 and need not be greater than 3·8 - where 'W' is the design maximum take-off weight (lb).

This means for *normal* and *commuter category aeroplanes* according to JAR-23 and for aircraft according to JAR-25:

$$
\begin{array}{lll}
m_{MTO} \le 1868\ \text{kg} & \Rightarrow & n_{lim} = 3.8 \\
1868\ \text{kg} < m_{MTO} < 22680\ \text{kg} & \Rightarrow & n_{lim} = 2.1 + \dfrac{24000}{2.205 \cdot m_{MTO}[\text{kg}] + 10000} \\
m_{MTO} \ge 22680\ \text{kg} & \Rightarrow & n_{lim} = 2.5
\end{array}
\qquad \textbf{(10.4)}
$$

With these parameters the **wing mass for aircraft with MTOW ≤ 5700 kg** can be calculated from:

$$
\frac{m_W}{m_{MTO}} = 4.90 \cdot 10^{-3} \cdot b_s^{\,0.75} \cdot \left(1 + \sqrt{\frac{b_{ref}}{b_s}}\right) \cdot n_{ult}^{\,0.55} \cdot \left(\frac{b_s / t_r}{m_{MTO} / S_W}\right)^{0.30} \qquad \textbf{(10.5)}
$$

The **wing mass for aircraft with MTOW > 5700 kg** can be calculated from:

$$
\frac{m_W}{m_{MZF}} = 6.67 \cdot 10^{-3} \cdot b_s^{\,0.75} \cdot \left(1 + \sqrt{\frac{b_{ref}}{b_s}}\right) \cdot n_{ult}^{\,0.55} \cdot \left(\frac{b_s / t_r}{m_{MZF} / S_W}\right)^{0.30} \qquad \textbf{(10.6)}
$$

The equations for calculating wing mass include the high lift system and the ailerons. It is assumed that the landing gear is mounted on the wing, but not the engines. The following **corrections** are necessary:

+ 2%	for a wing with spoilers,
- 5%	for 2 engines on the wing,
- 10%	for 4 engines on the wing,
- 5%	if the landing gear is <u>not</u> mounted on the wing,
- 30%	if the wing is braced; then the wing mass also includes the mass of the brace, which accounts for approximately 10% of the wing mass.

m_W	wing mass in **kg**,
b_s	structural span in **m**,
t_r	thickness of the wing root – thickness close to the fuselage in **m**,
S_W	wing area in **m²**,
m_{MTO}	maximum take-off mass in **kg**,
m_{MZF}	maximum zero fuel mass in **kg**.

Comments on equations (10.5) and (10.6):

- The relative wing mass increases according to the ratio of the structural span b_s and the thickness of the wing root t_r. The b_s / t_r ratio is called the *cantilever ratio*. Typical values are:

  $b_s / t_r = 40$ for cantilever wings,

  $b_s / t_r = 70$ for braced wings.

- If a specific aircraft mass is to be supported by the wing, the relative wing mass falls if a small wing area S_W is chosen and therefore a large wing loading m_{MTO} / S_W or m_{MZF} / S_W.

- The relative wing mass increases with the structural span b_s - i.e. given a constant aspect ratio and sweep, it increases with the size of the aircraft.
- The relative wing mass increases with the load factor *n*.
- The influence of the taper ratio λ is not taken into account in the equations (10.5) and (10.6). Other, corresponding equations from [ROSKAM V], for example, show that the relative wing mass rises with increasing λ.

Fuselage mass m_F

For dive speeds V_D > 250 kts (=128.6 m/s) EAS is

$$m_F = 0.23 \cdot \sqrt{V_D \cdot \frac{l_H}{w_F + h_F}} \cdot S_{F,wet}^{\ 1.2} \tag{10.7}$$

+ 8%	for a pressure cabin,
+ 4%	for engines at the rear of the fuselage,
+ 7%	for main landing gear that is mounted on the fuselage,
- 4%	if the fuselage does not contain a landing gear bay,
+ 10%	for a cargo aircraft with a reinforced cabin floor.

Equation (10.7) can be used to estimate the separate mass of a tail boom. If a main landing gear leg is accommodated in the **tail boom**, the mass is increased by 7%.

V_D Dive speed in **m/s** equivalent airspeed, V_{EAS}. V_{EAS} is a function of the true airspeed, V_{TAS}

$V_{EAS} = V_{TAS} \cdot \sqrt{\sigma}$ with $\sigma = \rho / \rho_0$,

$V_D = M_D \cdot a$ with a the speed of sound according to equation (5.30),

M_D dive Mach number.

According to JAR-23.335(b) or JAR-25.335(b) and practical experience, M_D is 0.05 to 0.09 higher than M_C or M_{MO},

l_H lever arm of the horizontal tailplane (see Section 9.4),

w_F maximum fuselage width,

h_F maximum fuselage height,

$S_{F,wet}$ fuselage wetted area in **m²**.

For aircraft with airspeeds V_C < 200 kts see *Cessna Method* from [ROSKAM V].

<u>Horizontal tailplane mass</u> m_H and **<u>vertical tailplane mass</u>** m_V

For dive speeds V_D > 250 kts (=128.6 m/s) EAS is

$$m_H = k_H \cdot S_H \cdot \left(62 \cdot \frac{S_H{}^{0.2} \cdot V_D}{1000 \cdot \sqrt{\cos \varphi_{H,50}}} - 2.5 \right) \qquad \textbf{(10.8)}$$

$$m_V = k_V \cdot S_V \cdot \left(62 \cdot \frac{S_V{}^{0.2} \cdot V_D}{1000 \cdot \sqrt{\cos \varphi_{V,50}}} - 2.5 \right) \qquad \textbf{(10.9)}$$

m_H Mass of the horizontal tailplane in **kg**,

m_V Mass of the vertical tailplane in **kg**,

$k_H = 1$ for a fixed fin,

$k_H = 1.1$ for a trimmable fin,

$k_V = 1 + 0.15 \cdot \frac{S_H \cdot z_H}{S_V \cdot b_V}$

S_H Horizontal tailplane area in **m²**,

S_V Vertical tailplane area in **m²**,

z_H vertical distance from the root of the vertical tailplane to the place where the horizontal tailplane is attached to the vertical tailplane,

b_V Span of the vertical tailplane,

V_D Dive speed in **m/s** (see above),

$\varphi_{H,50}$ Horizontal tailplane sweep of the 50% line,

$\varphi_{V,50}$ Vertical tailplane sweep of the 50% line.

For dive speeds $V_D \leq$ 250 kts (=128.6 m/s) EAS is

$$m_H + m_V = 0.64 \cdot \left(n_{ult} \cdot \left(S_H + S_V \right)^2 \right)^{0.75} \qquad \textbf{(10.10)}$$

with designations and units as above.

<u>Landing gear mass</u> m_{LG}

$$m_{LG,N} \text{ bzw. } m_{LG,M} = k_{LG} \cdot \left(A_{LG} + B_{LG} \cdot m_{MTO}{}^{3/4} + C_{LG} \cdot m_{MTO} + D_{LG} \cdot m_{MTO}{}^{3/2} \right) \qquad \textbf{(10.11)}$$

m_{LG} Landing gear mass in **kg**, $m_{LG} = m_{LG,N} + m_{LG,M}$

$m_{LG,N}$	Nose landing gear in **kg**,
$m_{LG,M}$	Mass of the main landing gear in **kg**,
m_{MTO}	Take-off mass in **kg**,
$k_{LG} = 1$	for low wing aircraft,
$k_{LG} = 1.08$	for high wing aircraft,
A_{LG} ... D_{LG}	from **Table 10.3**.

Table 10.3: Coefficients for calculating the landing gear mass [TORENBEEK 88]

airplane type	gear type	gear component	A_{LG}	B_{LG}	C_{LG}	D_{LG}
jet trainers and business jets	retractable gear	main gear	15.0	0.033	0.0210	-
		nose gear	5.4	0.049	-	-
other civil types	fixed gear	main gear	9.1	0.082	0.0190	-
		nose gear	11.3	-	0.0024	-
		tail gear	4.1	-	0.0024	-
	retractable gear	main gear	18.1	0.131	0.0190	$2.23 \cdot 10^{-5}$
		nose gear	9.1	0.082	-	$2.97 \cdot 10^{-6}$
		tail gear	2.3	-	0.0031	-

Comments on the equation (10.11):

- The equation is applied separately for the main landing gear and the nose landing gear. The sum of the two masses gives the landing gear mass m_{LG}.
- The landing gear mass m_{LG} accounts for approximately 7% of the take-off mass m_{MTO} in small aircraft.
- On aircraft weighing more than approx. 30,000kg the landing gear mass is around 4.5% of the take-off mass m_{MTO}.

Mass of the nacelle m_N

For turbo jets:

$$m_N = \frac{0.055 \cdot T_{TO}}{g} \qquad \textbf{(10.12)}$$

For turbo fans:

$$m_N = \frac{0.065 \cdot T_{TO}}{g} \qquad \textbf{(10.13)}$$

m_N	Mass of all nacelles combined,

T_{TO}	Take-off thrust of all engines combined,
g	Acceleration due to gravity.

For corresponding calculation equations for propeller aircraft see [ROSKAM V] and [TORENBEEK 88].

Mass of the installed engines $m_{E,inst}$

$$m_{E,inst} = k_E \cdot k_{thr} \cdot n_E \cdot m_E \qquad \textbf{(10.14)}$$

$k_E = 1.16$	for single-engine propeller aircraft,
$k_E = 1.35$	for multi-engine propeller aircraft,
$k_E = 1.15$	for jet-propelled passenger aircraft with nacelles,
$k_E = 1.40$	for aircraft with buried engines
$k_{thr} = 1.00$	without reverse thrust,
$k_{thr} = 1.18$	with reverse thrust,
n_E	number of engines,
m_E	mass of an engine without add-on components for engine integration.

Mass of systems m_{SYS}

$$m_{SYS} = k_{EQUIP} \cdot m_{MTO} + 0.768 \cdot k_{F/C} \cdot m_{MTO}^{2/3} \qquad \textbf{(10.15)}$$

m_{SYS}	Mass of systems in **kg**,
$k_{EQUIP} = 0.08$	single-engine propeller aircraft,
$k_{EQUIP} = 0.11$	twin-engine propeller aircraft,
$k_{EQUIP} = 0.13$	jet trainer,
$k_{EQUIP} = 0.14$	short-haul transport aircraft,
$k_{EQUIP} = 0.11$	medium-haul transport aircraft,
$k_{EQUIP} = 0.08$	long-haul transport aircraft,
m_{MTO}	maximum take-off mass in **kg**,
$k_{F/C} = 0.23$	for aircraft with simple surface controls,
$k_{F/C} = 0.44$	for transport aircraft with manual surface controls,
$k_{F/C} = 0.64$	for transport aircraft with primary surface controls by means of secondary energy (e.g. hydraulics) and flap drive,

$k_{F/C}$ = 0.74 for transport aircraft with primary surface controls – including spoilers – by means of secondary energy (e.g. hydraulics) and flap drive,

$k_{F/C}$ = 0.77 for transport aircraft with primary surface controls by means of secondary energy (e.g. hydraulics) and flap and slat drive,

$k_{F/C}$ = 0.88 for transport aircraft with primary surface controls – including spoilers – by means of secondary energy (e.g. hydraulics) and flap and slat drive.

Comment on equation (10.15):

- [TORENBEEK 88] differentiates between equipment and surface controls. In this case both groups have been summarized in a calculation equation and give the weight of the aircraft systems.
- For simplicity's sake, it is assumed here that equation (10.15) also contains the mass of the equipment and crew (⇒ standard and operational items). Addition to the group masses presented here then provides the operating empty mass m_{OE} .

Other *Class II* methods for forecasting mass and mass breakdown

A mass forecast is as good as the available mass data on which the statistical methods are based. New aircraft exhibit other mass characteristics as a result of technical progress. **Equations for forecasting mass must therefore be adapted to the state of the art.** Methods for forecasting mass have largely been created at aircraft construction companies. They represent part of the company's know-how and are not released externally, as a rule – or are only released when the methods are very old and their correct applicability is therefore questionable.

Examples of **methods for forecasting mass** which have been published:

- Cessna [ROSKAM V],
- General Dynamics (GD) [NICOLAI 75], [ROSKAM V],
- US Air Force [NICOLAI 75], [ROSKAM V],
- IABG [LTH-GEWICHTE 86].

A method is also available from

- Boeing [BOEING 68].

As no methods can claim to give the "correct" result, individual masses **should be** calculated, **compared** and averaged with different **methods**.

It can be observed that **individual or group masses** are **partly** calculated **with a large error**. However, the errors of the individual masses have different algebraic signs, so that the **total mass** is calculated **with better of accuracy** than the individual masses.

Mass forecast for engines

The dry mass of jet engines can be estimated with **Fig. 10.5**. **Fig. 10.6** provides the dry mass for turbo-prop engines. With data from JANE'S, [RAYMER 89] devises an equation for estimating the mass of jet engines with a bypass ratio *BPR* of between 0 and 6:

$$m_E = \frac{0.0724}{g} \cdot T_{TO}^{\ \ 1.1} \cdot e^{-0.045 \cdot BPR} \qquad \textbf{(10.16)}$$

m_E	Mass of an engine in **kg**,
g	Acceleration due to gravity in **m/s²**,
T_{TO}	Take-off thrust in **N**,
BPR	Bypass ratio.

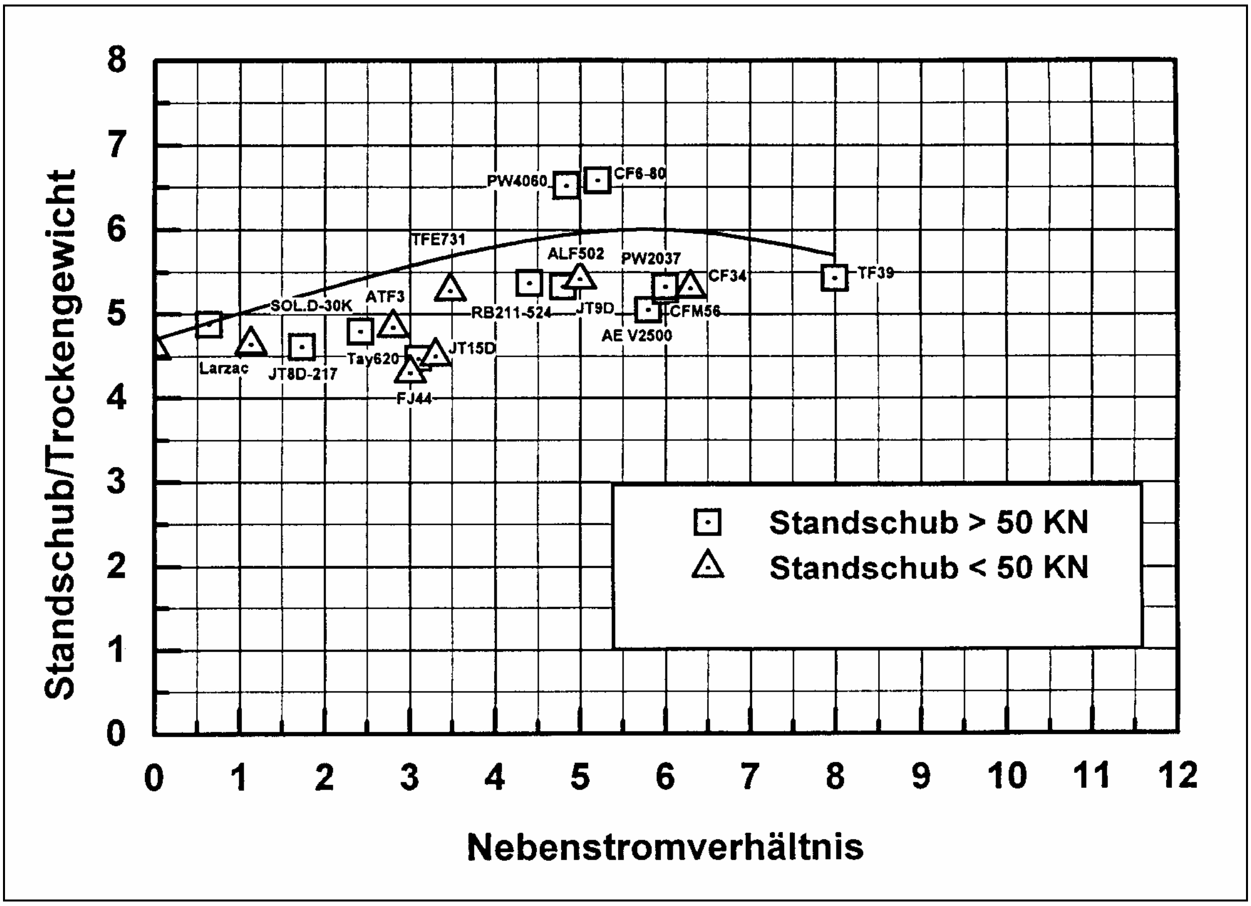


Fig 10.5: Estimate of mass m_E of jet engines [MARCKWARDT 98a]

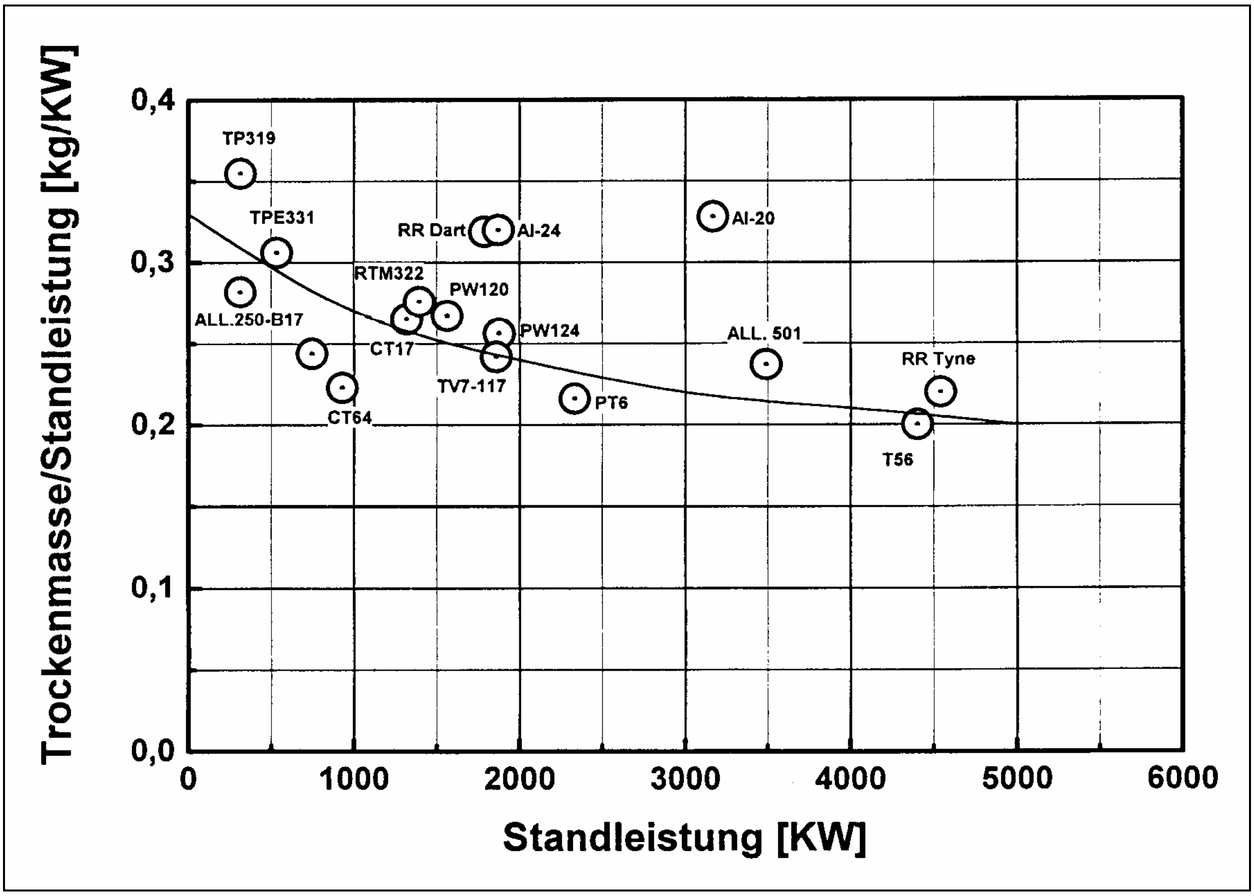


Fig. 10.6: Estimate of mass m_E of turbo prop engines [MARCKWARDT 98a]

Iterative calculation of masses

For individual or group masses the percentage of the total mass (MTOW, OEW, MEW or MZFW) is partly calculated. However, the total mass is itself only an estimated figure, as the **masses** only arise **from an iteration**:

Step 1: Calculate the **individual masses** with the equations (10.1) to (10.16).

Step 2: **Add together** all the individual masses for operating empty mass m_{OE}.

Step 3: Calculate the **maximum take-off mass** m_{MTO} with M_{ff} according to Step 5.9.2:

$$m_{MTO} = \frac{m_{MPL} + m_{OE}}{M_{ff}} \quad . \qquad \textbf{(10.17)}$$

Step 4: **Continue the (inner) iteration** by returning to Step 1. Keep going through Steps 1 to 3 until the maximum take-off mass m_{MTO} changes by no more than 0.5% from one iteration step to another.

Step 5: If, after the (inner) iteration, the maximum take-off mass m_{MTO} differs by more than 5% from the value m_{MTO}, with which the wing area S_W and take-off thrust T_{TO} were calculated, S_W **and** T_{TO} must be **recalculated**. This is carried out on the basis of the wing loading or the thrust-to-weight ratio determined in Section 5.

Step 7: If the wing area S_W and take-off thrust T_{TO} have been modified, **continue the (outer) iteration** by returning to Step 1. Determine the changes to the mass of the wing and the engines **until** the mass of the wing and the engines only changes marginally from one iteration step to another.

10.2 Centre of gravity calculations

Centre of gravity of the mass groups

A center of gravity (CG) must be allocated to each of the mass groups according to Table 10.1. The center of gravity of **engines, nose and main landing gear** occurs at the point where they are mounted on the aircraft. The point where the nose and main landing gear are mounted has not been specified by calculation up to now. However, a comparison with other aircraft allows the position to be initially estimated with sufficient accuracy for the center of gravity calculation. The center of gravity of the **systems** and **equipment** can be determined at 40% to 50% of the length of the fuselage. The center of gravity positions of the other mass groups are shown in **Fig. 10.7** and **Fig. 10.8**.

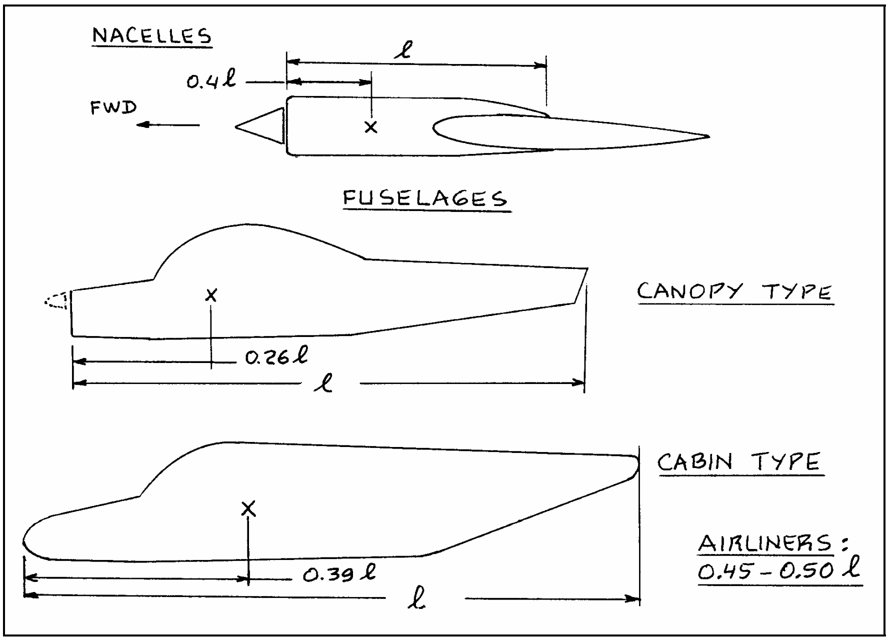
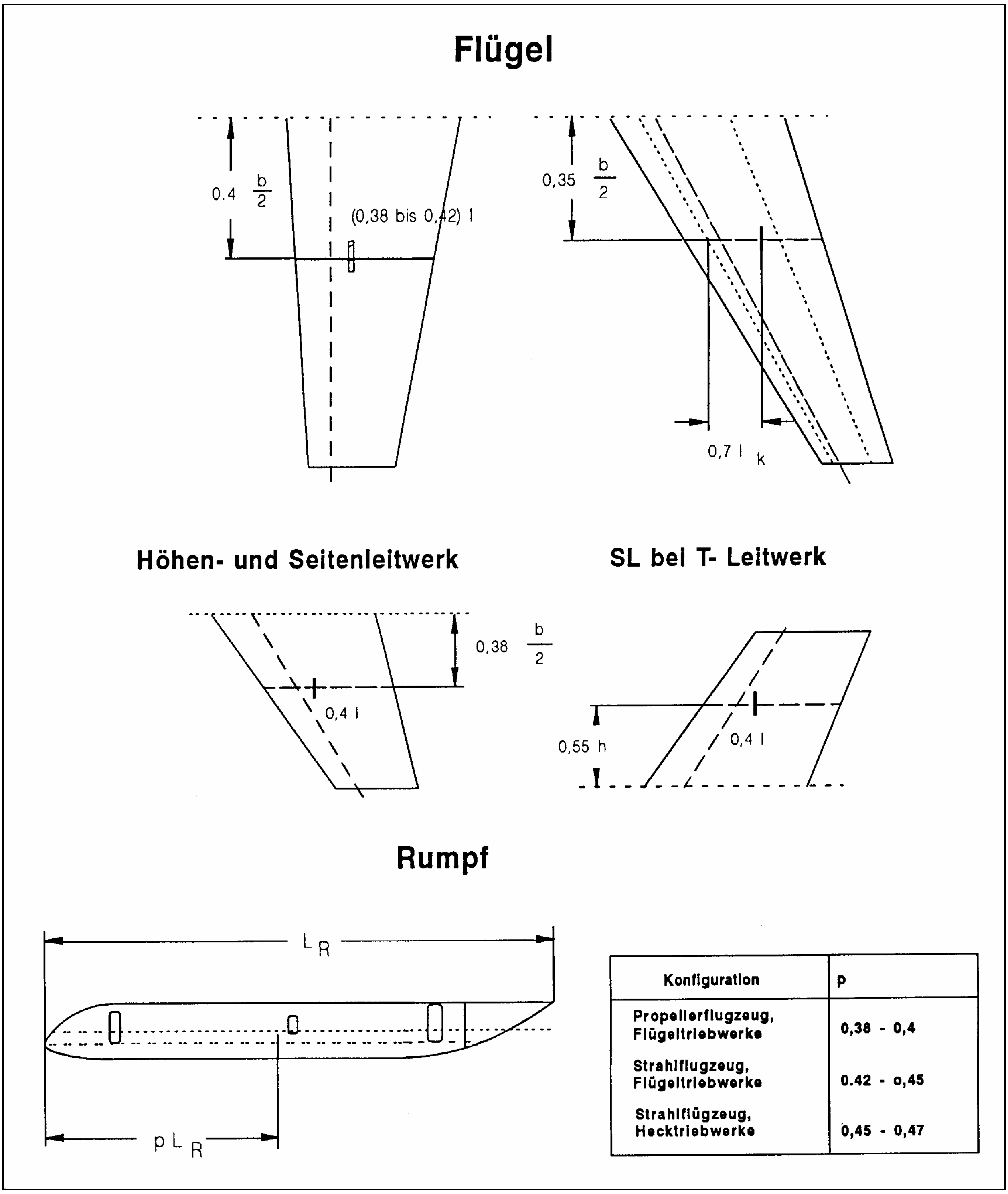


Fig. 10.7:
The position of the centers of gravity of mass groups according to [ROSKAM II] based on data from [TORENBEEK 88]



Konfiguration	p
Propellerflugzeug, Flügeltriebwerke	0,38 - 0,4
Strahlflugzeug, Flügeltriebwerke	0.42 - o,45
Strahlflügzeug, Hecktriebwerke	0,45 - 0,47

Fig. 10.8: The position of the centers of gravity of mass groups according to [MARCKWARDT 98a] based on data from [TORENBEEK 88]

Four basic questions apply when calculating center of gravity

Question 1: Where is the center of gravity?

The center of gravity along the length of the aircraft x_{CG} is of special interest. The zero point to which all lever arms x_i of the individual masses m_i relate is moved so far in front of the aircraft that the lever arms always remain positive, even if subsequent changes are made. This convention means that sign errors are avoided.

$$x_{CG} = \frac{\sum m_i \cdot x_i}{\sum m_i} \qquad \textbf{(10.18)}$$

Question 2: How far does the center of gravity x_{CG} shift if a mass m_i increases by Δm_i?

Equation (10.18) derived according to m_i gives

$$\frac{dx_{CG}}{dm_i} = \frac{x_i}{\sum m_i} \quad . \qquad \textbf{(10.19)}$$

If the component *i* becomes heavier by the mass Δm_i then the center of gravity shifts as follows:

$$\Delta x_{CG} = x_i \cdot \frac{\Delta m_i}{\sum m_i} \quad . \qquad \textbf{(10.20)}$$

Question 3: How far does the center of gravity x_{CG} shift if a mass m_i is displaced by Δx_i?

Equation (10.18) derived according to x_i gives:

$$\frac{dx_{CG}}{dx_i} = \frac{m_i}{\sum m_i} \quad . \qquad \textbf{(10.21)}$$

If the component *i* with mass m_i is moved by Δx_i then the center of gravity is displaced as follows:

$$\Delta x_{CG} = \Delta x_i \cdot \frac{m_i}{\sum m_i} \quad . \qquad \textbf{(10.22)}$$

Components which are normally relocated: batteries, parts of the air conditioning and computers. Only minimal shifts in the aircraft's center of gravity can be achieved by moving these parts.

Question 4: How far must the wing be moved to find a suitable position for the aircraft's center of gravity?

Step 1: The aircraft is split into 2 main groups:

- The **fuselage group**, *FG,* consists of: horizontal tailplane, vertical tailplane, fuselage, sum of all the systems, and rear engines, if fitted.
- **Wing group**, *WG,* consists of: wing, landing gear, and engines, if wing-mounted.

The mass and center of gravity is determined for both groups (see Question 1).

Step 2: The moment around the leading edge, LE, of the mean aerodynamic chord MAC: LEMAC is established.

$$\left(m_{WG} + m_{FG}\right) \cdot x_{CG,LEMAC} = m_{WG} \cdot x_{WG,LEMAC} + m_{FG} \cdot \left(x_{FG} - x_{LEMAC}\right) \qquad \textbf{(10.23)}$$

$x_{CG,LEMAC}$ The distance from the LE on the MAC (LEMAC) of the entire aircraft up to the CG. $x_{CG,LEMAC}$ is predefined, as required, e.g. $x_{CG,LEMAC} = 0.25 \cdot c_{MAC}$,

m_{WG} Mass of wing group,

m_{FG} Mass of fuselage group,

$x_{WG,LEMAC}$ Distance from LEMAC to CG of the wing group,

x_{FG} Distance from zero point to CG of fuselage group,

x_{LEMAC} Distance from zero point to LEMAC.

$$\boxed{x_{LEMAC} = x_{FG} - x_{CG,LEMAC} + \frac{m_{WG}}{m_{FG}}\left(x_{WG,LEMAC} - x_{CG,LEMAC}\right) \ . \qquad \textbf{(10.24)}}$$

The load and trim sheet

Fig. 10.9 shows a load and trim sheet. It contains the admissible range for a combination of aircraft mass and center of gravity position. The migration of the center of gravity during loading and unloading is also included in the load and trim sheet. The load and trim sheet is used both in flight operations and in aircraft design.

Fig. 10.10 explains the **model for passengers to board** the aircraft. The starting point in the diagram is the OEW and the center of gravity of the OEW. It is assumed that passengers will occupy the window seats (A → C) first. Once the window seats are full, the seats next to the window seats will be filled (C → D) and then the seats next to the aisle (D → E). If *the rear window seats* are filled first, the weight and center of gravity migrate upwards via the right curve in the diagram, for example in the case of window seats (A → B2 → C). If the *front window seats* are filled first, the left curve is followed (A → B1 → C).

The other arrows in Fig. 10.9 show the loading with cargo and fuel. As can be seen in the diagram, the **loading sequence** is important. If the normal loading sequence is, for example: 1. Passengers, 2. Cargo, 3. Fuel, then the following sequences must also be checked for their influence: 1-3-2, 2-1-3, 2-3-1, 3-1-2, 3-2-1. Attention must also be paid to the **unloading sequence**. **Fig. 10.11** shows clearly the influence of the configuration on center of gravity migration during loading.

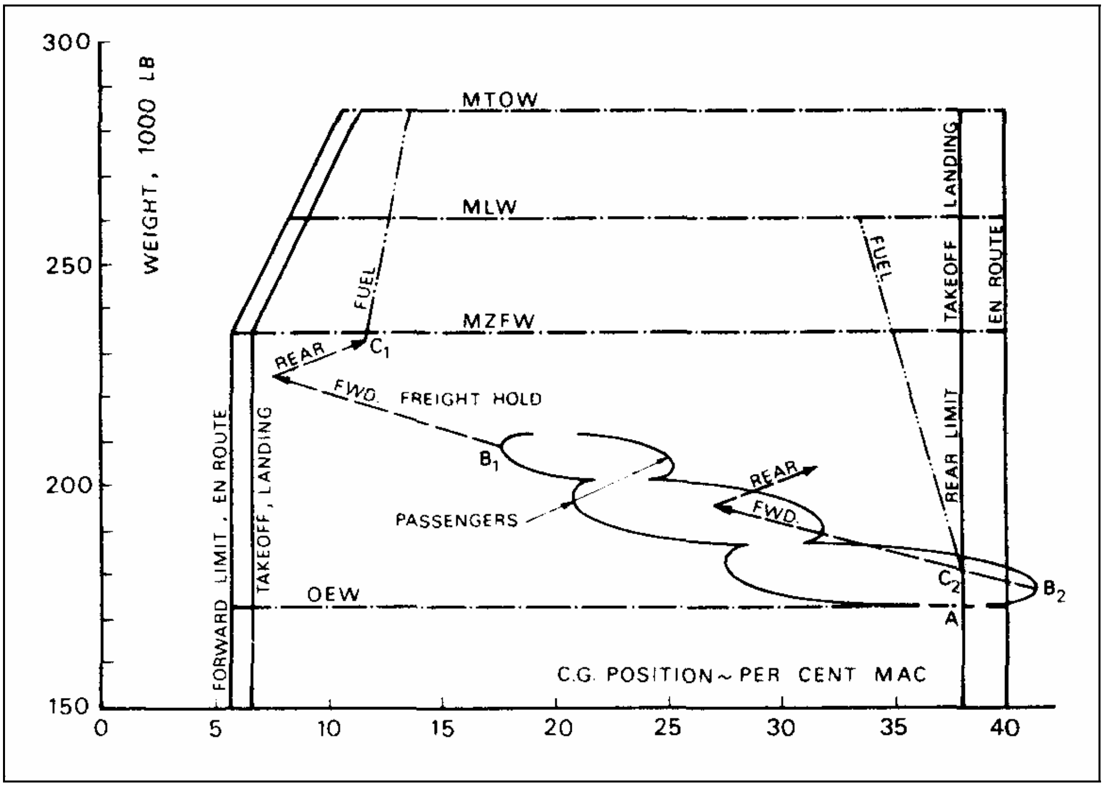


Fig. 10.9: Load and trim sheet [TORENBEEK 88]

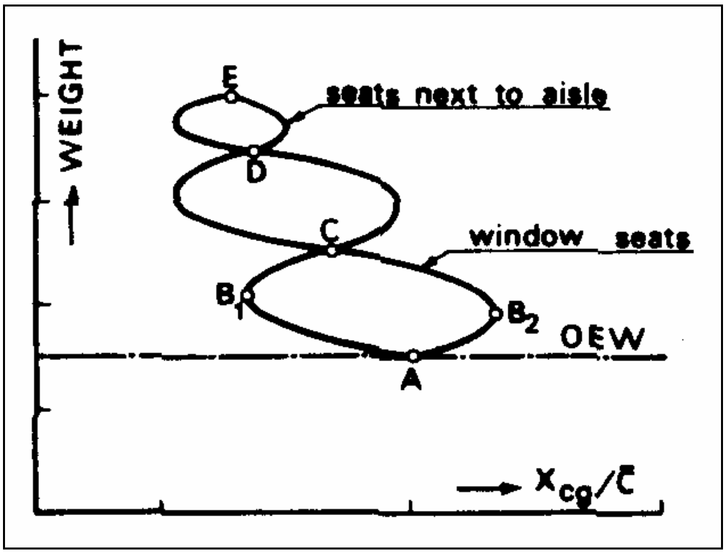


Fig. 10.10:
Passengers boarding, shown in the load and trim sheet [TORENBEEK 88]

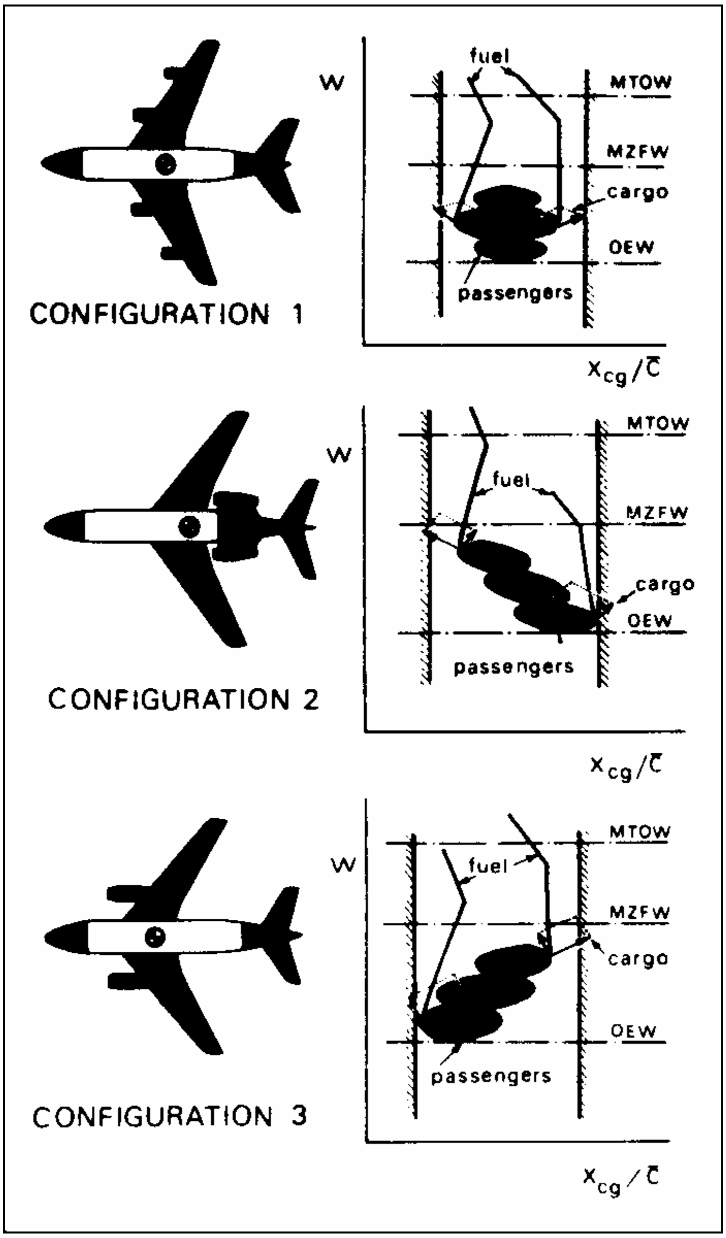


Fig. 10.11:
The influence of the configuration on the load and trim sheet [TORENBEEK 88]

The required center of gravity range (c.g. range) is determined from the differing loading scenarios. The usual center of gravity range of various types of aircraft is shown in **Table 10.4**.

Table 10.4: Center of gravity range of various types of aircraft [ROSKAM II]

Type	C.G. Range	Type	C.G. Range
	fr.$\bar{c}_W$		fr.$\bar{c}_W$
Homebuilts	0.10	Military Trainers	0.10
Single Engine Prop. Driven	0.06-0.27	Fighters	0.20
Twin Engine Prop. Driven	0.12-0.22	Mil.Patr. Bomb and Transp.	0.30
Ag. Airpl.	0.10	Fl.Boats, Amph. and Float Amph. and	0.25
Business Jets	0.10-0.21		
Regional TBP	0.14-0.27		
Jet Transp.	0.12-0.32	Supersonic Cruise	0.30